PHYS-GA 2000 Computational Physics PS5

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1 Problem 1

1.1 a

With given function :

$$\Gamma(a) = \int_0^\infty x^{a-1} e^{-x} dx \tag{1}$$

Besides adopting the integral terms inside, we simply used the gaussxwab method again to do the integral, which is mentioned in the former PS. Following is the gamma function plotted, which shows it's functioning well.

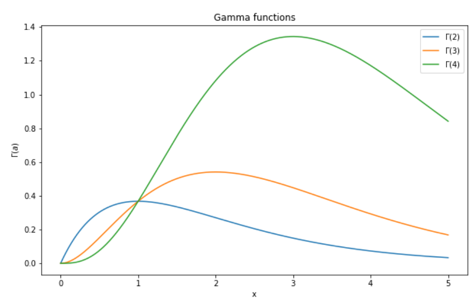

Figure 1: This is the desired Gamma function plot for given equation

1.2 b

Let's start with the gamma function:

$$\Gamma(a) = \int_0^\infty x^{a-1} e^{-x} dx \tag{2}$$

Derive the term $x^{a-1}e^{-x}$:

$$\frac{d}{dx} x^{a-1} e^{-x} = (a-1)x^{a-2}e^{-x} - x^{a-1}e^{-x} = 0 \tag{3}$$

Zero point for derivation is $x = 0$ or $x = a - 1$.

For the two zero points, we take it in, it is obvious that $x = a - 1$ is the one we want.

1.3 c

Here we take $x = c$:

$$z = \frac{c}{2c} = \frac{1}{2} \tag{4}$$

Hence we have the result that the integrant max should be at the point $c = x = a - 1$ with conclusion in b.

1.4 d

Putting $x^{n-1} = x^{a-1ln(x)}$ in, we will have a new Γ function:

$$\Gamma(a) = \int_0^\infty x^{a-1}e^{-x} = \int_0^\infty e^{(a-1)ln(x)-x} \tag{5}$$

This function works better for extreme situations. Considering the original one, if x is a small number and even if a is large, it still grows relatively slow. New expression avoid this kind of problems: It is going to be either very large or very small.

1.5 e and f

In this part, we adopted the equation we got from the part c and d. As mentioned above, it should work well for the given numbers.

```
gammanew = gauss_quad_new(0,1,100,3/2)
print('The gamma value desired for this part is',gammanew)
```

```
The gamma value desired for this part is 0.8862269613087213
```

```
for a in [2, 6, 10]:
    print('The gamma value desired for gamma',a,'is',gauss_quad_new(0,1,10000,a))
```

```
The gamma value desired for gamma 2 is 0.9999999999999997
The gamma value desired for gamma 6 is 119.99999999999991
The gamma value desired for gamma 10 is 362879.9999999966
```

Figure 2: This is the result of the Gamma function using the revised gamma function we got in d.

The expecting numbers are 0.886 for c and 1,120,362880 for part d. By the result given above, it works well.

2 Problem 2

In this problem we used the SVD techniques on fitting the data given. SVD is given here:

$$A = UWV^T \tag{6}$$

Notice that U and V^T are orthonormal matrices.

2.1 a

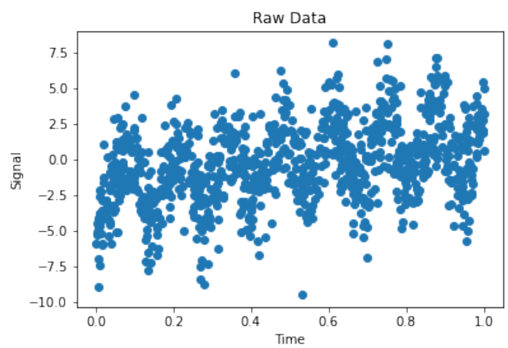

Figure 3: This is the Raw data of the given signal.

We simply plotted the all the points from the data set (Using the given code from recitation). It can be used for further comparithon to the fitting curves.

A small problem here: I noticed that I have to divide np.max terms on the time part. Otherwise all the fitting process will not be working.(Commented on the code) Worthy to be think of.

2.2 b

We use the SVD technique to fit the curves with the data given. We notice that it was too flat(nearly a linear function for the given data from the figure). But the points are rather discrete from each other. Therefore, it is not fitted enough for the 3rd order polynomials.

2.3 c

This residual further proved our conclusion in b. Comparing to the original data, the residual are still discrete from each other and the error is still on a rather high level. Therefore this 3rd order fitting is proved to be untrustworthy.

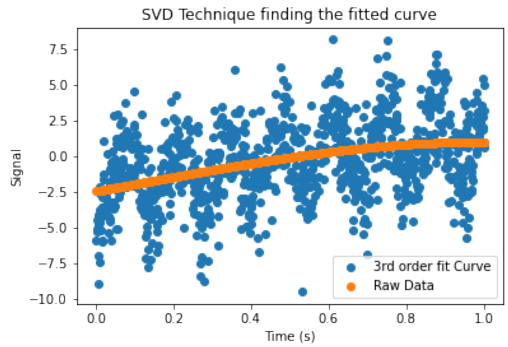

Figure 4: This is the Raw data of the given signal and the 3rd order fitting.

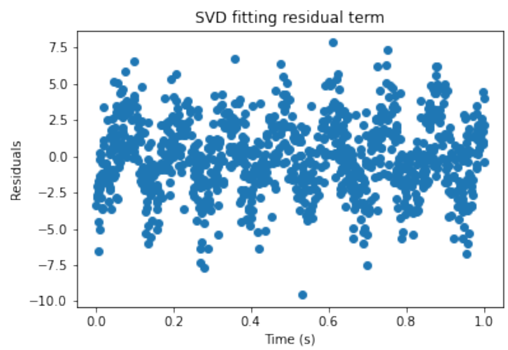

Figure 5: This is the residuals for the 3rd order.

2.4 d

I tried many different orders finding this one looks the best. If we go higher, it will be over-fitted, especially on the larger time part, all the scattering point are gathering, which means the curves has too much fluctuation there. But if it was too low, obviously it would be to flat (as part b), which is not fitted enough. Basically it is the best result with the polynomials we can get.

So for a expecting periodic signals, is polynomial the best way dealing with it? Obviously not. In order to get the periodic pattern, sometimes the polynomials need to be in really high orders. An ideal polynomial should reflect the fluctuation pattern of the data.

2.5 e

We adopted the similar ways in the recitation. This works great for this signal because trigonometric functions has the periodic pattern itself. You don't need high-order polynomials defining that. The period here is basically $\frac{T}{8}$ here.

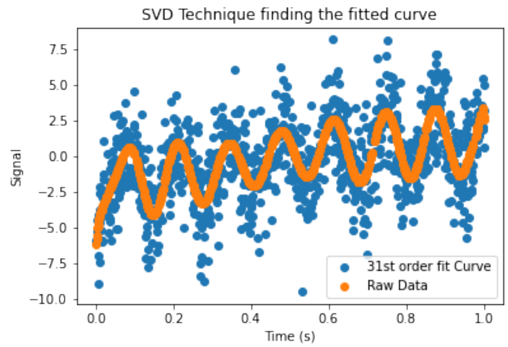

Figure 6: This is the fitting curves for the 31st order polynomials.

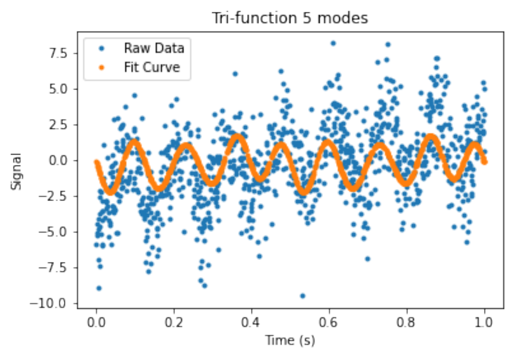

Figure 7: This is the fitting curves with the tri-functions.

In this given data obviously the trigonometric functions works the best. Using polynomials will ask for high order estimations to get the periodic pattern. We should deal with the different data with a proper way of fitting.

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