Tutorial: Working with categorical data with R and the vcd package

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Abstract

This tutorial describes the creation of frequency and contingency tables from categorical variables, along with tests of independence, measures of association, and methods for graphically displaying results. The framework is provided by the R package **vcd**, but other packages are used to help with various tasks.

Keywords: contingency tables, mosaic plots, sieve plots, categorical data, independence, conditional independence, R.

1. Introduction

This tutorial, part of the **vcdExtra** package, describes the creation of frequency and contingency tables from categorical variables, along with tests of independence, measures of association, and methods for graphically displaying results. It borrows structure and some ideas from from Robert Kabakoff's *Quick-R* web page, http://www.statmethods.net/stats/frequencies.html.

There is much more to the analysis of categorical data than is described here, where the emphasis is on crosstabulated tables of frequencies ("contingency tables"), statistical tests, associated loglinear models, and visualization of *how* variables are related.

A more general treatment of graphical methods for categorical data is contained in my book, *Visualizing Categorical Data* (Friendly 2000), for which **vcd** is a partial R companion, covering topics not otherwise available in R. A more complete theoretical description of statistical methods is provided in Agresti's (2002) "Categorical Data Analysis," for which see the R companion by Laura Thompson, https://home.comcast.net/~lthompson221/Splusdiscrete2.pdf.

2. Creating and manipulating frequency tables

R provides many methods for creating frequency and contingency tables. Several are described below. In the examples below, we use some real examples and some anonymous ones, where the variables `A`, `B`, and `C` represent categorical variables, and `X` represents an arbitrary R data object.

The first thing you need to know is that categorical data can be represented in three different forms in R, and it is sometimes necessary to convert from one form to another, for carrying out statistical tests, fitting models or visualizing the results. Once a data object exists in R, you can examine its structure with the `str()` function.

case form a data frame containing individual observations, with one or more factors, used as the classifying variables. In case form, there may also be numeric covariates. The total number of observations is `nrow(X)`.

Example: The `Arthritis` data is available in case form in the **vcd** package. There are two explanatory factors: `Treatment` and `Sex`. `Age` is a covariate, and `Improved` is the response—an ordered factor, with levels None < Some < Marked. Excluding `Age`, we have a $2 \times 2 \times 3$ contingency table for `Treatment`, `Sex` and `Improved`.

```
> str(Arthritis)      # show the structure

'data.frame':        84 obs. of  5 variables:
 $ ID       : int  57 46 77 17 36 23 75 39 33 55 ...
 $ Treatment: Factor w/ 2 levels "Placebo","Treated": 2 2 2 2 2 2 2 2 2 2 ...
 $ Sex      : Factor w/ 2 levels "Female","Male": 2 2 2 2 2 2 2 2 2 2 ...
 $ Age      : int  27 29 30 32 46 58 59 59 63 63 ...
 $ Improved : Ord.factor w/ 3 levels "None"<"Some"<..: 2 1 1 3 3 3 1 3 1 1 ...

> Arthritis[1:5,]     # first 5 observations

  ID Treatment  Sex Age Improved
1 57   Treated Male  27     Some
2 46   Treated Male  29     None
3 77   Treated Male  30     None
4 17   Treated Male  32   Marked
5 36   Treated Male  46   Marked
```

frequency form a data frame containing one or more factors, and a frequency variable, often called `Freq` or `count`. The total number of observations is `sum(X[,"Freq"])` or some equivalent form.

Example: For small frequency tables, it is often convenient to enter them in frequency form using `expand.grid()` for the factors and `c()` to list the counts in a vector. The example below, from Agresti (2002) gives results for the 1991 General Social Survey, with respondents classified by sex and party identification.

```
> # Agresti (2002), table 3.11, p. 106
> GSS <- data.frame(
+   expand.grid(sex=c("female", "male"),
+               party=c("dem", "indep", "rep")),
+   count=c(279,165,73,47,225,191))
> GSS

     sex party count
1 female   dem   279
2   male   dem   165
3 female indep    73
4   male indep    47
5 female   rep   225
6   male   rep   191

> str(GSS)

'data.frame':        6 obs. of  3 variables:
 $ sex  : Factor w/ 2 levels "female","male": 1 2 1 2 1 2
 $ party: Factor w/ 3 levels "dem","indep",..: 1 1 2 2 3 3
 $ count: num  279 165 73 47 225 191
```

```
> sum(GSS$count)

[1] 980
```

table form a matrix, array or table object, whose elements are the frequencies in an n-way table. The variable names (factors) and their levels are given by `dimnames(X)`. The total number of observations is `sum(X)`. The number of dimensions of the table is `length(dimnames(X))`, and the table sizes are given by `sapply(dimnames(X), length)`.

Example: The `HairEyeColor` is stored in table form in **vcd**.

```
> str(HairEyeColor)                       # show the structure

 table [1:4, 1:4, 1:2] 32 53 10 3 11 50 10 30 10 25 ...
 - attr(*, "dimnames")=List of 3
  ..$ Hair: chr [1:4] "Black" "Brown" "Red" "Blond"
  ..$ Eye : chr [1:4] "Brown" "Blue" "Hazel" "Green"
  ..$ Sex : chr [1:2] "Male" "Female"

> sum(HairEyeColor)                       # number of cases

[1] 592

> sapply(dimnames(HairEyeColor), length) # table dimension sizes

Hair  Eye  Sex
   4    4    2
```

Example: Enter frequencies in a matrix, and assign `dimnames`, giving the variable names and category labels.

```
> ## A 4 x 4 table  Agresti (2002, p. 57) Job Satisfaction
> JobSat <- matrix(c(1,2,1,0, 3,3,6,1, 10,10,14,9, 6,7,12,11), 4, 4)
> dimnames(JobSat) = list(income=c("< 15k", "15-25k", "25-40k", "> 40k"),
+                 satisfaction=c("VeryD", "LittleD", "ModerateS", "VeryS"))
> JobSat

        satisfaction
income   VeryD LittleD ModerateS VeryS
  < 15k      1       3        10     6
  15-25k     2       3        10     7
  25-40k     1       6        14    12
  > 40k      0       1         9    11
```

`JobSat` is a matrix, not an object of `class("table")`, and some functions are happier with tables than matrices. You can coerce it to a table with `as.table()`. using

```
> JobSat <- as.table(JobSat)
> str(JobSat)

 table [1:4, 1:4] 1 2 1 0 3 3 6 1 10 10 ...
 - attr(*, "dimnames")=List of 2
  ..$ income      : chr [1:4] "< 15k" "15-25k" "25-40k" "> 40k"
  ..$ satisfaction: chr [1:4] "VeryD" "LittleD" "ModerateS" "VeryS"
```

2.1. Ordered factors and reordered tables

In table form, the values of the table factors are ordered by their position in the table. Thus in the `JobSat` data, both `income` and `satisfaction` represent ordered factors, and the positions of the values in the rows and columns reflects their ordered nature.

For the `HairEyeColor` data, hair color and eye color are ordered arbitrarily. For visualizing the data using mosaic plots and other methods described below, it turns out to be more useful to assure that both hair color and eye color are ordered from dark to light. Hair colors are actually ordered this way already, and it is easiest to re-order eye colors by indexing. Again `str()` is your friend.

```
> HairEyeColor <- HairEyeColor[, c(1,3,4,2), ]
> str(HairEyeColor)

 num [1:4, 1:4, 1:2] 32 53 10 3 10 25 7 5 3 15 ...
 - attr(*, "dimnames")=List of 3
  ..$ Hair: chr [1:4] "Black" "Brown" "Red" "Blond"
  ..$ Eye : chr [1:4] "Brown" "Hazel" "Green" "Blue"
  ..$ Sex : chr [1:2] "Male" "Female"
```

This is also the order for both hair color and eye color shown in the result of a correspondence analysis (Figure 4) below.

With data in case form or frequency form, when you have ordered factors represented with character values, you must ensure that they are treated as ordered in R.[1]

Imagine that the `Arthritis` data was read from a text file. By default the `Improved` will be ordered alphabetically: `Marked`, `None`, `Some`— not what we want. In this case, the function `ordered()` (and others) can be useful.

```
> Arthritis <- read.csv("arthritis.txt",header=TRUE)
> Arthritis$Improved <- ordered(Arthritis$Improved, levels=c("None", "Some", "Marked"))
```

With this order of `Improved`, the response in this data, a mosaic display of `Treatment` and `Improved` (Figure 1)shows a clearly interpretable pattern.

Finally, there are situations where, particularly for display purposes, you want to re-order the *dimensions* of an n-way table, or change the labels for the variables or levels. This is easy when the data are in table form: `aperm()` permutes the dimensions, and assigning to `names` and `dimnames` changes variable names and level labels respectively. We will use the following version of `UCBAdmissions` in Section 3.4 below.[2]

```
> UCB <- aperm(UCBAdmissions, c(2, 1, 3))
> dimnames(UCB)[[2]] <- c("Yes", "No")
> names(dimnames(UCB)) <- c("Sex", "Admit?", "Department")
> ftable(UCB)

              Department   A   B   C   D   E   F
Sex    Admit?
```

[1] In SAS, many procedures offer the option `order = data | internal | formatted` to allow character values to be ordered according to (a) their order in the data set, (b) sorted internal value, or (c) sorted formatted representation provided by a SAS format.

[2] Changing `Admit` to `Admit?` might be useful for display purposes, but it dangerous— because it is then difficult to use that variable name in a model formula.

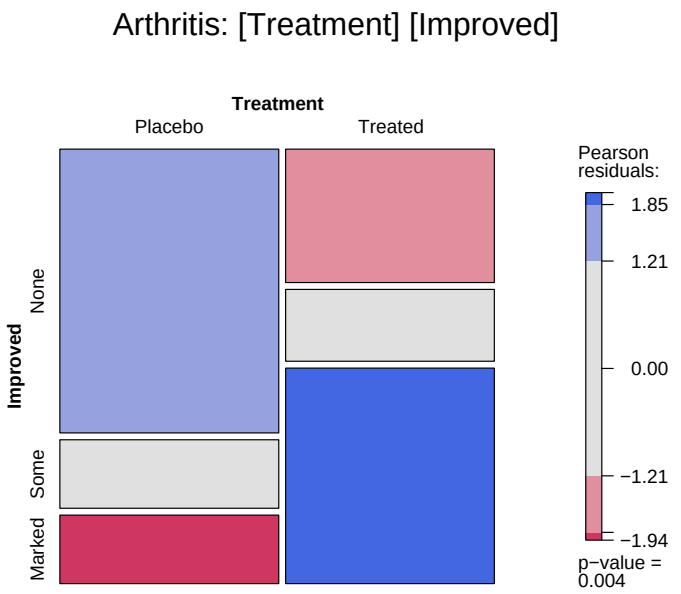


Figure 1: Mosaic plot for the `Arthritis` data, showing the marginal model of independence for Treatment and Improved. Age, a covariate, and Sex are ignored here.

```
Male   Yes              512 353 120 138  53  22
       No               313 207 205 279 138 351
Female Yes               89  17 202 131  94  24
       No                19   8 391 244 299 317
```

2.2. `structable()`

For 3-way and larger tables the `structable()` function in **vcd** provides a convenient and flexible tabular display. The variables assigned to the rows and columns of a two-way display to be specified by a model formula.

```
> structable(HairEyeColor)                    # show the table: default

             Eye Brown Hazel Green Blue
Hair  Sex
Black Male          32    10     3   11
      Female        36     5     2    9
Brown Male          53    25    15   50
      Female        66    29    14   34
Red   Male          10     7     7   10
      Female        16     7     7    7
Blond Male           3     5     8   30
      Female         4     5     8   64
```

```
> structable(Hair+Sex ~ Eye, HairEyeColor)   # specify col ~ row variables

      Hair Black        Brown         Red        Blond
      Sex   Male Female  Male Female Male Female  Male Female
Eye
Brown         32     36    53     66   10     16     3      4
Hazel         10      5    25     29    7      7     5      5
Green          3      2    15     14    7      7     8      8
Blue          11      9    50     34   10      7    30     64
```

It also returns an object of class `"structable"` which may be plotted with `mosaic()` (not shown here).

```
> HSE < - structable(Hair+Sex ~ Eye, HairEyeColor)   # save structable object
> mosaic(HSE)                                        # plot it
```

2.3. `table()` and friends

You can generate frequency tables from factor variables using the `table()` function, tables of proportions using the `prop.table()` function, and marginal frequencies using `margin.table()`.

```
>  n=500
>  A <- factor(sample(c("a1","a2"), n, rep=TRUE))
>  B <- factor(sample(c("b1","b2"), n, rep=TRUE))
>  C <- factor(sample(c("c1","c2"), n, rep=TRUE))
>  mydata <- data.frame(A,B,C)

> # 2-Way Frequency Table
> attach(mydata)
> mytable <- table(A,B)   # A will be rows, B will be columns
> mytable                 # print table
> margin.table(mytable, 1) # A frequencies (summed over B)
> margin.table(mytable, 2) # B frequencies (summed over A)
> prop.table(mytable)    # cell percentages
> prop.table(mytable, 1) # row percentages
> prop.table(mytable, 2) # column percentages
```

`table()` can also generate multidimensional tables based on 3 or more categorical variables. In this case, use the `ftable()` function to print the results more attractively.

```
> # 3-Way Frequency Table
> mytable <- table(A, B, C)
> ftable(mytable)
```

Table ignores missing values. To include `NA` as a category in counts, include the table option `exclude=NULL` if the variable is a vector. If the variable is a factor you have to create a new factor using `newfactor <- factor(oldfactor, exclude=NULL)`.

2.4. xtabs

The `xtabs()` function allows you to create crosstabulations using formula style input.

```
> # 3-Way Frequency Table
> mytable <- xtabs(~A+B+C, data=mydata)
> ftable(mytable)    # print table
> summary(mytable)   # chi-square test of indepedence
```

If a variable is included on the left side of the formula, it is assumed to be a vector of frequencies (useful if the data have already been tabulated).

```
> (GSStab <- xtabs(count ~ sex + party, data=GSS))

        party
sex      dem indep rep
  female 279    73 225
  male   165    47 191

> summary(GSStab)

Call: xtabs(formula = count ~ sex + party, data = GSS)
Number of cases in table: 980
Number of factors: 2
Test for independence of all factors:
        Chisq = 7.01, df = 2, p-value = 0.03005
```

2.5. Converting among frequency tables and data frames

As we've seen, a given contingency table can be represented equivalently in different forms, but some R functions were designed for one particular representation. Table 1 shows some handy tools for converting from one form to another.

Table 1: Tools for converting among different forms for categorical data

	To this		
From this	Case form	Frequency form	Table form
Case form	noop	`xtabs(~A+B)`	`table(A,B)`
Frequency form	`expand.dft(X)`	noop	`xtabs(count~A+B)`
Table form	`expand.dft(X)`	`as.data.frame(X)`	noop

A contingency table in table form (an object of `class(table)`) can be converted to a data.frame with `as.data.frame()`.[3] The resulting `data.frame` contains columns representing the classifying factors and the table entries (as a column named by the `responseName` argument, defaulting to `Freq`. This is the inverse of `xtabs()`.

Example: Convert the `GSStab` in table form to a data.frame in frequency form.

```
> as.data.frame(GSStab)

     sex party Freq
1 female   dem  279
```

[3] Because R is object-oriented, this is actually a short-hand for the function `as.data.frame.table()`.

```
2   male   dem  165
3 female indep   73
4   male indep   47
5 female   rep  225
6   male   rep  191
```

Example: Convert the `Arthritis` data in case form to a 3-way table of `Treatment` × `Sex` `Improved`.[4]

```
> Art.tab <-with(Arthritis, table(Treatment, Sex, Improved))
> str(Art.tab)

 'table' int [1:2, 1:2, 1:3] 19 6 10 7 7 5 0 2 6 16 ...
 - attr(*, "dimnames")=List of 3
  ..$ Treatment: chr [1:2] "Placebo" "Treated"
  ..$ Sex      : chr [1:2] "Female" "Male"
  ..$ Improved : chr [1:3] "None" "Some" "Marked"

> ftable(Art.tab)

                Improved None Some Marked
Treatment Sex
Placebo   Female           19    7      6
          Male             10    0      1
Treated   Female            6    5     16
          Male              7    2      5
```

There may also be times that you wil need an equivalent case form data.frame with factors representing the table variables rather than the frequency table. For example, the `mca()` function in package **MASS** only operates on data in this format. Marc Schwartz has provided code for `expand.dft()` on the Rhelp mailing list for converting a table back into a case form dataframe.

2.6. A complex example

If you've followed so far, you're ready for a more complicated example. The data file, `tv.dat` represents a 4-way table of size $5 \times 11 \times 5 \times 3$ where the table variables (unnamed in the file) are read as `V1` – `V4`, and the cell frequency is read as `V5`. The file, stored in the `data/` directory of **vcdExtra**, can be read as follows:

```
> tv.data<-read.table(system.file("data","tv.dat",package="vcdExtra"))
> head(tv.data,5)

  V1 V2 V3 V4 V5
1  1  1  1  1  6
2  2  1  1  1 18
3  3  1  1  1  6
4  4  1  1  1  2
5  5  1  1  1 11
```

[4] Unfortunately, `table()` does not allow a `data` argument to provide an environment in which the table variables are to be found. In the exanples in Section 2.3 I used `attach(mydata)` for this purpose, but `attach()` leaves the variables in the global environment, while `with()` just evaluates the `table()` expression in a temporary environment of the data.

For a local file, just use `read.table()` in this form:

```
> tv.data<-read.table("C:/R/data/tv.dat")
```

The data `tv.dat` came from the initial implementation of mosaic displays in R by Jay Emerson. In turn, they came from the initial development of mosaic displays (Hartigan and Kleiner 1984) that illustrated the method with data on a large sample of TV viewers whose behavior had been recorded for the Neilson ratings. This data set contains sample television audience data from Neilsen Media Research for the week starting November 6, 1995.

The table variables are:
`V1`– values 1:5 correspond to the days Monday–Friday;
`V2`– values 1:11 correspond to the quarter hour times 8:00PM through 10:30PM;
`V3`– values 1:5 correspond to ABC, CBS, NBC, Fox, and non-network choices;
`V4`– values 1:3 correspond to transition states: turn the television Off, Switch channels, or Persist in viewing the current channel.

We are interested just the cell frequencies, and rely on the facts that the (a) the table is complete—there are no missing cells, so `nrow(tv.data)`=825; (b) the observations are ordered so that `V1` varies most rapidly and `V4` most slowly. From this, we can just extract the frequency column and reshape it into an array.

```
> TV <- array(tv.data[,5], dim=c(5,11,5,3))
> dimnames(TV) <- list(c("Monday","Tuesday","Wednesday","Thursday","Friday"),
+                 c("8:00","8:15","8:30","8:45","9:00","9:15","9:30",
+                 "9:45","10:00","10:15","10:30"),
+                 c("ABC","CBS","NBC","Fox","Other"), c("Off","Switch","Persist"))
> names(dimnames(TV))<-c("Day", "Time", "Network", "State")
```

But this table is too large and awkward to work with. Among the networks, Fox and Other occur infrequently. We can also cut it down to a 3-way table by considering only viewers who persist with the current station.

```
> TV <- TV[,,1:3,]     # keep only ABC, CBS, NBC
> TV <- TV[,,,3]       # keep only Persist -- now a 3 way table
> structable(TV)
```

```
                 Time 8:00 8:15 8:30 8:45 9:00 9:15 9:30 9:45 10:00 10:15 10:30
Day       Network
Monday    ABC         146  151  156   83  325  350  386  340   352   280   278
          CBS         337  293  304  233  311  251  241  164   252   265   272
          NBC         263  219  236  140  226  235  239  246   279   263   283
Tuesday   ABC         244  181  231  205  385  283  345  192   329   351   364
          CBS         173  180  184  109  218  235  256  250   274   263   261
          NBC         315  254  280  241  370  214  195  111   188   190   210
Wednesday ABC         233  161  194  156  339  264  279  140   237   228   203
          CBS         158  126  207   59   98  103  122   86   109   105   110
          NBC         134  146  166   66  194  230  264  143   274   289   306
Thursday  ABC         174  183  197  181  187  198  211   86   110   122   117
          CBS         196  185  195  104  106  116  116   47   102    84    84
          NBC         515  463  472  477  590  473  446  349   649   705   747
```

```
Friday    ABC          294  281  305  239  278  246  245  138   246   232   233
          CBS          130  144  154   81  129  153  136  126   138   136   152
          NBC          195  220  248  160  172  164  169   85   183   198   204
```

Finally, for some purposes, we might want to collapse the 11 times into a smaller number. Here, we use `as.data.frame.table()` to convert the table back to a data frame, `levels()` to re-assign the values of `Time`, and finally, `xtabs()` to give a new, collapsed frequency table.

```
> TV.df <- as.data.frame.table(TV)
> levels(TV.df$Time) <- c(rep("8:00-8:59",4),rep("9:00-9:59",4), rep("10:00-10:44",3))
> TV2 <- xtabs(Freq ~ Day + Time + Network, TV.df)
> structable(Day ~ Time+Network,TV2)

                   Day Monday Tuesday Wednesday Thursday Friday
Time        Network
8:00-8:59   ABC             536     861       744      735   1119
            CBS            1167     646       550      680    509
            NBC             858    1090       512     1927    823
9:00-9:59   ABC            1401    1205      1022      682    907
            CBS             967     959       409      385    544
            NBC             946     890       831     1858    590
10:00-10:44 ABC             910    1044       668      349    711
            CBS             789     798       324      270    426
            NBC             825     588       869     2101    585
```

Whew!

3. Tests of Independence

3.1. CrossTable

OK, now we're ready to do some analyses. For tabular displays, the `CrossTable()` function in the **gmodels** package produces cross-tabulations modeled after `PROC FREQ` in SAS or `CROSSTABS` in SPSS. It has a wealth of options for the quantities that can be shown in each cell.

```
> # 2-Way Cross Tabulation
> library(gmodels)
> CrossTable(GSStab,prop.t=FALSE,prop.r=FALSE,prop.c=FALSE)

   Cell Contents
|-------------------------|
|                       N |
| Chi-square contribution |
|-------------------------|


Total Observations in Table:  980
```

```
             | party
         sex |       dem |     indep |       rep | Row Total |
-------------|-----------|-----------|-----------|-----------|
      female |       279 |        73 |       225 |       577 |
             |     1.183 |     0.078 |     1.622 |           |
-------------|-----------|-----------|-----------|-----------|
        male |       165 |        47 |       191 |       403 |
             |     1.693 |     0.112 |     2.322 |           |
-------------|-----------|-----------|-----------|-----------|
Column Total |       444 |       120 |       416 |       980 |
-------------|-----------|-----------|-----------|-----------|
```

There are options to report percentages (row, column, cell), specify decimal places, produce Chi-square, Fisher, and McNemar tests of independence, report expected and residual values (pearson, standardized, adjusted standardized), include missing values as valid, annotate with row and column titles, and format as SAS or SPSS style output! See `help(CrossTable)` for details.

3.2. Chi-square test

For 2-way tables you can use `chisq.test()` to test independence of the row and column variable. By default, the p-value is calculated from the asymptotic chi-squared distribution of the test statistic. Optionally, the p-value can be derived via Monte Carlo simulation.

```
> (HairEye <- margin.table(HairEyeColor, c(1, 2)))

       Eye
Hair    Brown Hazel Green Blue
  Black    68    15     5   20
  Brown   119    54    29   84
  Red      26    14    14   17
  Blond     7    10    16   94

> chisq.test(HairEye)

        Pearson's Chi-squared test

data:  HairEye
X-squared = 138.2898, df = 9, p-value < 2.2e-16
```

3.3. Fisher Exact Test

`fisher.test(X)` provides an exact test of independence. `X` must be a two-way contingency table in table form. Another form, `fisher.test(X, Y)` takes two categorical vectors of the same length. For tables larger than 2×2 the method can be computationally intensive (or can fail) if the frequencies are not small.

```
> fisher.test(GSStab)

        Fisher's Exact Test for Count Data
```

```
data:  GSStab
p-value = 0.03115
alternative hypothesis: two.sided

> fisher.test(HairEye)
```

But this does not work:

```
Error in fisher.test(HairEye) : FEXACT error 6.
LDKEY is too small for this problem.
Try increasing the size of the workspace.
```

3.4. Mantel-Haenszel test and conditional association

Use the `mantelhaen.test(X)` function to perform a Cochran-Mantel-Haenszel χ^2 chi test of the null hypothesis that two nominal variables are *conditionally independent*, $A \perp B \,|\, C$, in each stratum, assuming that there is no three-way interaction. `X` is a 3 dimensional contingency table, where the last dimension refers to the strata.

The `UCBAdmissions` serves as an example of a $2 \times 2 \times 6$ table, with `Dept` as the stratifying variable.

```
> ## UC Berkeley Student Admissions
> mantelhaen.test(UCBAdmissions)

        Mantel-Haenszel chi-squared test with continuity correction

data:  UCBAdmissions
Mantel-Haenszel X-squared = 1.4269, df = 1, p-value = 0.2323
alternative hypothesis: true common odds ratio is not equal to 1
95 percent confidence interval:
 0.7719074 1.0603298
sample estimates:
common odds ratio
        0.9046968
```

The results show no evidence for association between admission and gender when adjusted for department. However, we can easily see that the assumption of equal association across the strata (no 3-way association) is probably violated. For $2 \times 2 \times k$ tables, this can be tested using `woolf_test()` in **vcd**.

```
> oddsRatio <- function(x) (x[1,1]*x[2,2])/(x[1,2]*x[2,1])
> apply(UCBAdmissions, 3, oddsRatio)

        A         B         C         D         E         F
0.3492120 0.8025007 1.1330596 0.9212838 1.2216312 0.8278727

> woolf_test(UCBAdmissions)

        Woolf-test on Homogeneity of Odds Ratios (no 3-Way assoc.)

data:  UCBAdmissions
X-squared = 17.9017, df = 5, p-value = 0.003072
```

We can visualize the odds ratios of Admission for each department with fourfold displays using `fourfold()`. The cell frequencies n_{ij} of each 2×2 table are shown as a quarter circle whose radius is proportional to $\sqrt{n_{ij}}$, so that its area is proportional to the cell frequency. Confidence rings for the odds ratio allow a visual test of the null of no association; the rings for adjacent quadrants overlap *iff* the observed counts are consistent with the null hypothesis. In the extended version (the default), brighter colors are used where the odds ratio is significantly different from 1. The following lines produce Figure 2.[5]

```
> col <- c("#99CCFF", "#6699CC", "#F9AFAF", "#6666A0", "#FF0000", "#000080")
> fourfold(UCB,mfrow=c(2,3), color=col)
```

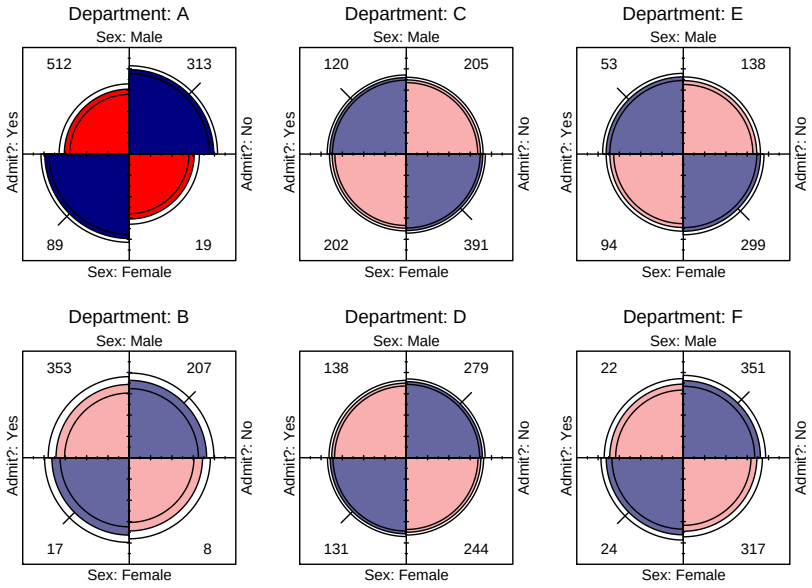


Figure 2: Fourfold display for the `UCBAdmissions` data. Where the odds ratio differs significantly from 1.0, the confidence bands do not overlap, and the circle quadrants are shaded more intensely.

Another **vcd** function, `cotabplot()`, provides a more general approach to visualizing conditional associations in contingency tables, similar to trellis-like plots produced by `coplot()` and lattice graphics. The `panel` argument supplies a function used to render each conditional subtable. The following gives a display (not shown) similar to Figure 2.

```
> cotabplot(UCB, panel = cotab_fourfold)
```

3.5. Measures of Association

There are a variety of statistical measures of *strength* of association for contingency tables— similar in spirit to r or r^2 for continuous variables. With a large sample size, even a small degree of association can show a significant χ^2, as in the example below for the `GSS` data.

[5] The color values `col[3:4]` were modified from their default values here.

The `assocstats()` function in **vcd** calculates the ϕ coefficient, contingency coefficient, and Cramer's V for an $r \times c$ table. The input must be in table form, a two-way $r \times c$ table. It won't work with `GSS` in frequency form, but by now you should know how to convert.

```
> assocstats(GSStab)

                    X^2 df P(> X^2)
Likelihood Ratio 7.0026  2 0.030158
Pearson          7.0095  2 0.030054

Phi-Coefficient   : 0.085
Contingency Coeff.: 0.084
Cramer's V        : 0.085
```

A web article by Richard Darlington, http://www.psych.cornell.edu/Darlington/crosstab/TABLE0.HTM gives further description of these and other measures of association.

3.6. Measures of Agreement

The `Kappa()` function in the vcd package calculates Cohen's κ and weighted κ for a square two-way table with the same row and column categories (Cohen 1960).[6] Normal-theory z-tests can be obtained by dividing κ by its asymptotic standard error (ASE).

```
> (K <- Kappa(SexualFun))

               value        ASE
Unweighted 0.1293303 0.06884553
Weighted   0.2373806 0.12646539

> (Z <- K$Weighted[1]/K$Weighted[2])

   value
1.877040
```

A visualization of agreement, both unweighted and weighted for degree of departure from exact agreement is provided by the `agreementplot()` function. Figure 3 shows the agreementplot for the `SexualFun` data, produced as shown below. The Bangdiwala measures represent the proportion of the shaded areas of the diagonal rectangles, using weights w_1 for exact agreement, and w_2 for partial agreement one step from the main diagonal.

```
> agree <- agreementplot(SexualFun, main="Is sex fun?")
> unlist(agree)

        Bangdiwala Bangdiwala_Weighted            weights1            weights2
         0.1464624           0.4981723           1.0000000           0.8888889
```

In other examples, the agreement plot can help to show *sources* of disagreement. For example, when the shaded boxes are above or below the diagonal (red) line, a lack of exact agreement can

[6] Don't confuse this with `kappa()` in base R that computes something entirely different (the condition number of a matrix).

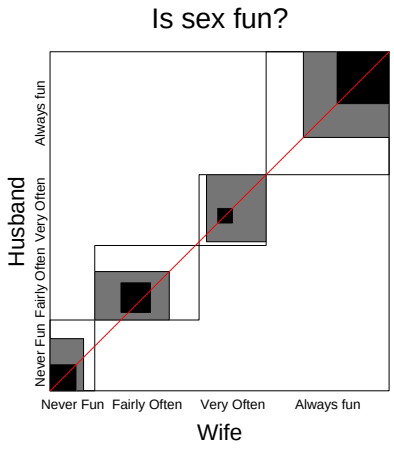


Figure 3: Agreement plot for the `SexualFun` data.

be attributed in part to different frequency of use of categories by the two raters– lack of *marginal homogeneity*.

3.7. Correspondence analysis

Use the **ca** package for correspondence analysis for visually exploring relationships between rows and columns in contingency tables. For an $r \times c$ table, the method provides a breakdown of the Pearson χ^2 for association in up to $M = \min(r-1, c-1)$ dimensions, and finds scores for the row (x_{im}) and column (y_{jm}) categories such that the observations have the maximum possible correlations.

Here, we carry out a simple correspondence analysis of the `HairEye` data. The printed results show that nearly 99% of the association between hair color and eye color can be accounted for in 2 dimensions.

```
> library(ca)
> ca(HairEye)

 Principal inertias (eigenvalues):
           1        2        3
Value      0.208773 0.022227 0.002598
Percentage 89.37%   9.52%    1.11%


 Rows:
            Black     Brown       Red    Blond
Mass     0.182432  0.483108  0.119932 0.214527
ChiDist  0.551192  0.159461  0.354770 0.838397
Inertia  0.055425  0.012284  0.015095 0.150793
Dim. 1  -1.104277 -0.324463 -0.283473 1.828229
Dim. 2   1.440917 -0.219111 -2.144015 0.466706


 Columns:
            Brown     Hazel     Green     Blue
```

```
Mass      0.371622  0.157095  0.108108 0.363176
ChiDist   0.500487  0.288654  0.385727 0.553684
Inertia   0.093086  0.013089  0.016085 0.111337
Dim. 1   -1.077128 -0.465286  0.354011 1.198061
Dim. 2    0.592420 -1.122783 -2.274122 0.556419
```

The resulting `ca` object can be plotted just by running the `plot()` method on the `ca` object, giving the result in Figure 4. `plot.ca()` does not allow labels for dimensions; these can be added with `title()`. It can be seen that most of the association is accounted for by the ordering of both hair color and eye color along Dimension 1, a dark to light dimension.

```
> plot(ca(HairEye), main="Hair Color and Eye Color")
> title(xlab="Dim 1", ylab="Dim 2")
```

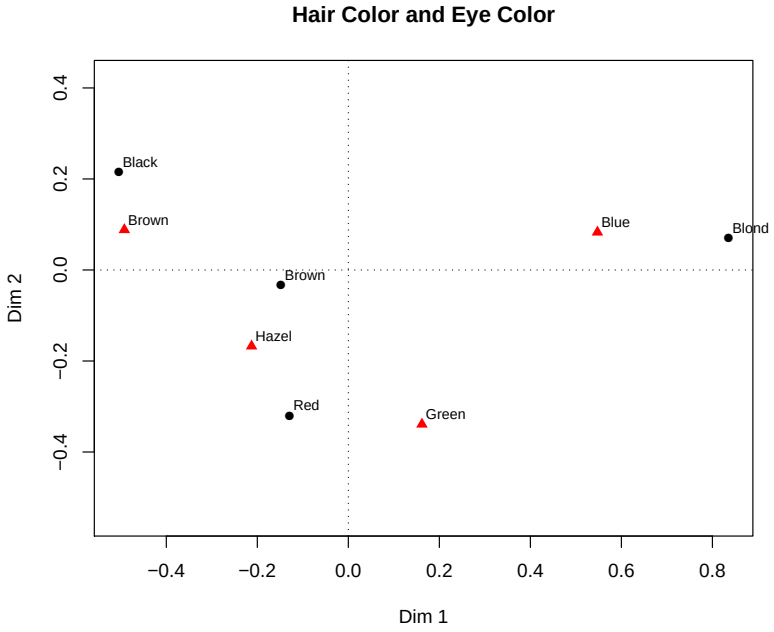


Figure 4: Correspondence analysis plot for the `HairEye` data.

4. Loglinear Models

You can use the `loglm()` function in the **MASS** package to fit log-linear models. Equivalent models can also be fit (from a different perspective) as generalized linear models with the `glm()` function using the `family='poisson'` argument, and the **gnm** package provides a wider range of generalized *nonlinear* models, particularly for testing structured associations. The visualization methods for these models are most advanced for models fit using `loglm()`, so this approach is emphasized here.

Assume we have a 3-way contingency table based on variables A, B, and C. The possible different forms of loglinear models for a 3-way table are shown in Table 2. The **Model formula** column

shows how to express each model in R. In the **Interpretation** column, the symbol "⊥" is to be read as "is independent of," and "|" means "conditional on," or "adjusting for," or just "given".

Table 2: Log-linear Models for Three-Way Tables

Model	Model formula	Symbol	Interpretation
Mutual independence	`~A + B + C`	$[A][B][C]$	$A \perp B \perp C$
Joint independence	`~A*B + C`	$[AB][C]$	$(A\,B) \perp C$
Conditional independence	`~(A+B)*C`	$[AC][BC]$	$(A \perp B)\,\vert\,C$
All two-way associations	`~A*B + A*C + B*C`	$[AB][AC][BC]$	homogeneous association
Saturated model	`~A*B*C`	$[ABC]$	3-way association

For example, the formula `~A + B + C` specifies the model of mutual independence with no associations among the three factors. In standard notation for the expected frequencies m_{ijk}, this corresponds to

$$\log(m_{ijk}) = \mu + \lambda_i^A + \lambda_j^B + \lambda_k^C \equiv \texttt{A + B + C}$$

The parameters λ_i^A, λ_j^B and λ_k^C pertain to the differences among the one-way marginal frequencies for the factors A, B and C.

Similarly, the model of joint independence allows an association between A and B, but specifies that C is independent of both of these and their combinations,

$$\log(m_{ijk}) = \mu + \lambda_i^A + \lambda_j^B + \lambda_k^C + \lambda_{ij}^{AB} \equiv \texttt{A * B + C}$$

In the literature or text books, you will often find these models expressed in shorthand symbolic notation, using brackets, `[ ]` to enclose the *high-order terms* in the model. Thus, the joint independence model can be denoted `[AB][C]`, as shown in the Symbol column in Table 2.

4.1. Fitting with `loglm()`

For example, we can fit the model of mutual independence among hair color, eye color and sex in `HairEyeColor` as

```
> library(MASS)
> ## Independence model of hair and eye color and sex.
> hec.1 <- loglm(~Hair+Eye+Sex, data=HairEyeColor)
> hec.1

Call:
loglm(formula = ~Hair + Eye + Sex, data = HairEyeColor)

Statistics:
                     X^2 df P(> X^2)
Likelihood Ratio 166.3001 24        0
Pearson          164.9247 24        0
```

Similarly, the models of conditional independence and joint independence are specified as

```
> ## Conditional independence
> hec.2 <- loglm(~(Hair + Eye) * Sex, data=HairEyeColor)
> hec.2
```

```
Call:
loglm(formula = ~(Hair + Eye) * Sex, data = HairEyeColor)

Statistics:
                      X^2 df P(> X^2)
Likelihood Ratio 156.6779 18        0
Pearson          147.9440 18        0

> ## Joint independence model.
> hec.3 <- loglm(~Hair*Eye + Sex, data=HairEyeColor)
> hec.3

Call:
loglm(formula = ~Hair * Eye + Sex, data = HairEyeColor)

Statistics:
                      X^2 df  P(> X^2)
Likelihood Ratio 19.85656 15 0.1775045
Pearson          19.56712 15 0.1891745
```

Note that printing the model gives a brief summary of the goodness of fit. A set of models can be compared using the `anova()` function.

```
> anova(hec.1, hec.2, hec.3)

LR tests for hierarchical log-linear models

Model 1:
 ~Hair + Eye + Sex
Model 2:
 ~(Hair + Eye) * Sex
Model 3:
 ~Hair * Eye + Sex

           Deviance df Delta(Dev) Delta(df) P(> Delta(Dev)
Model 1   166.30014 24
Model 2   156.67789 18    9.62225         6        0.14149
Model 3    19.85656 15  136.82133         3        0.00000
Saturated   0.00000  0   19.85656        15        0.17750
```

5. Mosaic plots

Mosaic plots provide an ideal method both for visualizing contingency tables and for visualizing the fit— or more importantly— lack of fit of a loglinear model. For a two-way table, `mosaic()` fits a model of independence, $[A][B]$ or `~A+B` as an R formula. For n-way tables, `mosaic()` can fit any loglinear model, and can also be used to plot a model fit with `loglm()`. See Friendly (1994, 1999) for the statistical ideas behind these uses of mosaic displays in connection with loglinear models.

The essential idea is to recursively sub-divide a unit square into rectangular "tiles" for the cells of the table, such that the are area of each tile is proportional to the cell frequency. For a given

loglinear model, the tiles can then be shaded in various ways to reflect the residuals (lack of fit) for a given model. The pattern of residuals can then be used to suggest a better model or understand *where* a given model fits or does not fit.

`mosaic()` provides a wide range of options for the directions of splitting, the specification of shading, labeling, spacing, legend and many other details. It is actually implemented as a special case of a more general class of displays for n-way tables called `strucplot`, including sieve diagrams, association plots, double-decker plots as well as mosaic plots. For details, see `help(strucplot)` and the "See also" links, and also Meyer, Zeileis, and Hornik (2006), which is available as an R vignette via `vignette("strucplot", package="vcd")`.

Figure 1, showing the association between `Treatment` and `Improved` was produced with the following call to `mosaic()`.

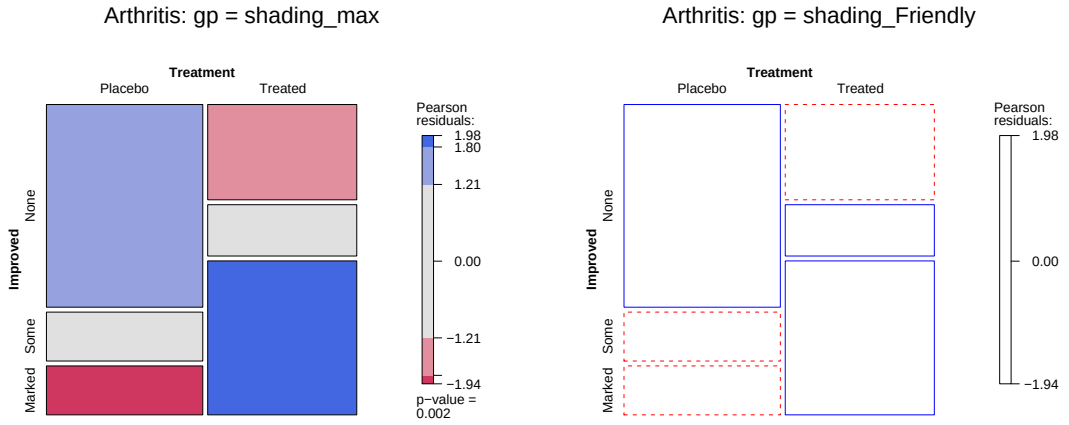
```
> mosaic(art, gp = shading_max, split_vertical = TRUE,
+        main="Arthritis: [Treatment] [Improved]")
```

Note that the residuals for the independence model were not large (as shown in the legend), yet the association between `Treatment` and `Improved` is highly significant.

```
> summary(art)

Call: xtabs(formula = ~Treatment + Improved, data = Arthritis)
Number of cases in table: 84
Number of factors: 2
Test for independence of all factors:
        Chisq = 13.055, df = 2, p-value = 0.001463
```

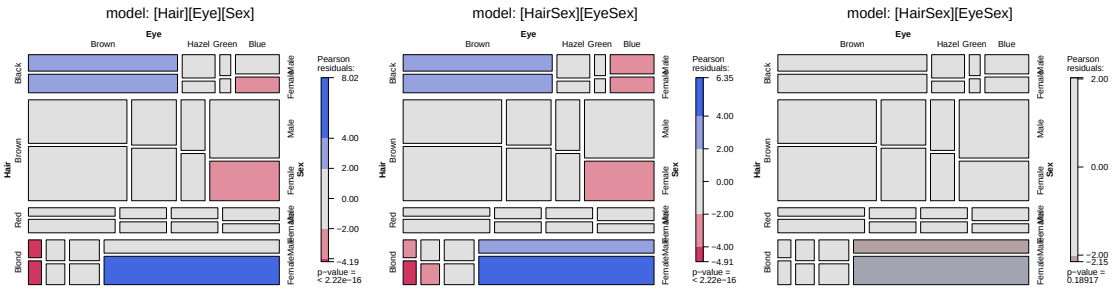
In contrast, one of the other shading schemes, from Friendly (1994) (use: `gp = shading_Friendly`), uses fixed cutoffs of $\pm 2, \pm 4$, to shade cells which are *individually* significant at approximately $\alpha = 0.5$ and $\alpha = 0.001$ levels, respectively. The right panel below uses `gp = shading_Friendly`.



5.1. Mosaics for loglinear models

When you have fit a loglinear model using `loglm()`, and saved the result (as a `loglm` object) the simplest way to display the results is to use the `plot()` method for the `loglm` object. Calling `mosaic(loglm.object)` has the same result. In Section 4.1 above, we fit several different models to the `HairEyeColor` data. We can produce mosaic displays of each just by plotting them:

```
> # mosaic plots, using plot.loglm() method
> plot(hec.1, main="model: [Hair][Eye][Sex]")
> plot(hec.2, main="model: [HairSex][EyeSex]")
> plot(hec.3, main="model: [HairEye][Sex]")
```



Alternatively, you can supply the model formula to `mosaic()` with the `expected` argument. This is passed to `loglm()`, which fits the model, and returns residuals used for shading in the plot.

For example, here we examine the `TV2` constructed in Section 2.6 above. The goal is to see how Network choice depends on (varies with) Day and Time. To do this:

- We fit a model of joint independence of `Network` on the combinations of `Day` and `Time`, with the model formula `~Day:Time + Network`.
- To make the display more easily read, we place `Day` and `Time` on the vertical axis and `Network` on the horizontal,
- The `Time` values overlap on the right vertical axis, so we use `level()` to abbreviate them. `mosaic()` also supports a more sophisticated set of labeling functions. Instead of changing the data table, we could have used `labeling_args = list(abbreviate = c(Time = 2))` for a similar effect.

The following call to `mosaic()` produces Figure 5

```
> dimnames(TV2)$Time <- c("8", "9", "10")     # re-level for mosaic display
> mosaic(~ Day + Network + Time, data=TV2, expected=~Day:Time + Network,
+          legend=FALSE, gp=shading_Friendly)
```

From this, it is easy to read from the display how network choice varies with day and time. For example, CBS dominates in all time slots on Monday; ABC and NBC dominate on Tuesday, particularly in the later time slots; Thursday is an NBC day, while on Friday, ABC gets the greatest share.

In interpreting this mosaic and other plots, it is important to understand that associations included in the model—here, that between day and time—are *not* shown in the shading of the cells, because they have been fitted (taken into account) in the loglinear model.

For comparison, you might want to try fitting the model of homogeneous association. This allows all pairs of factors to be associated, but asserts that each pairwise association is the same across the levels of the remaining factor. The resulting plot displays the contributions to a 3-way association, but is not shown here.

```
> mosaic(~ Day + Network + Time, data=TV2,
+          expected=~Day:Time + Day:Network + Time:Network,
+          legend=FALSE, gp=shading_Friendly)
```

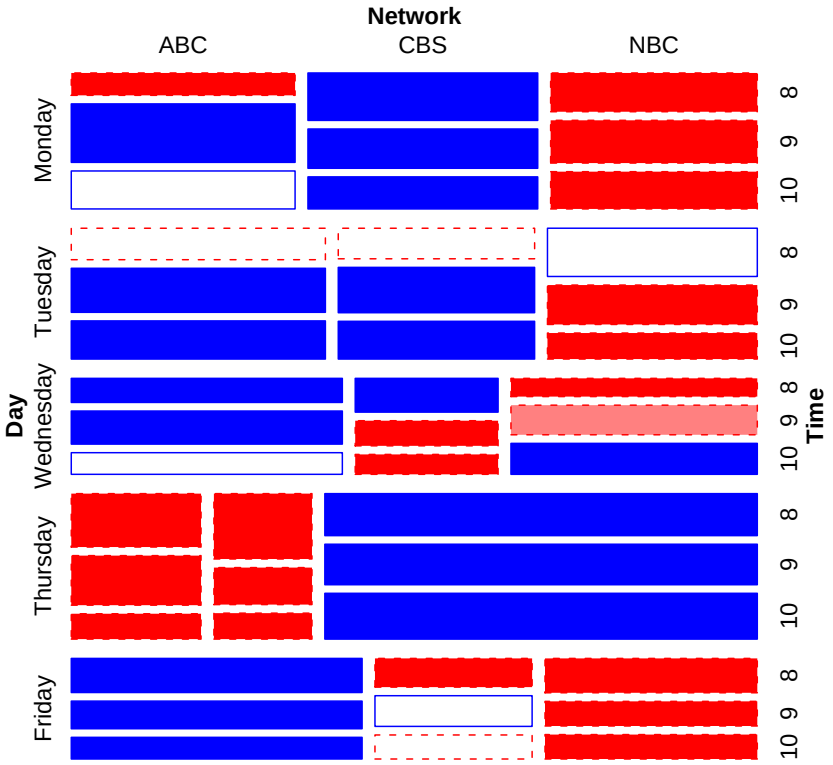


Figure 5: Mosaic plot for the `TV` data showing model of joint independence, `Day:Time + Network` .

6. Continuous predictors: spine and conditional density plots

When continuous predictors are available—and potentially important— in explaining a categorical outcome, models for that outcome include: logistic regression (binary response), the proportional odds model (ordered polytomous response), multinomial (generalized) logistic regression. Many of these are special cases of the generalized linear model using the `"poisson"` or `"binomial"` family and their relatives.

I don't go into fitting such models here, but I would be remiss not to illustrate some visualizations in **vcd** that are helpful here. The first of these is the spine plot or spinogram (Hummel 1996) (produced with `spine()`). These are special cases of mosaic plots with specific spacing and shading to show how a categorical response varies with a continuous or categorical predictor.

They are also a generalization of stacked bar plots where not the heights but the *widths* of the bars corresponds to the relative frequencies of `x`. The heights of the bars then correspond to the conditional relative frequencies of y in every `x` group.

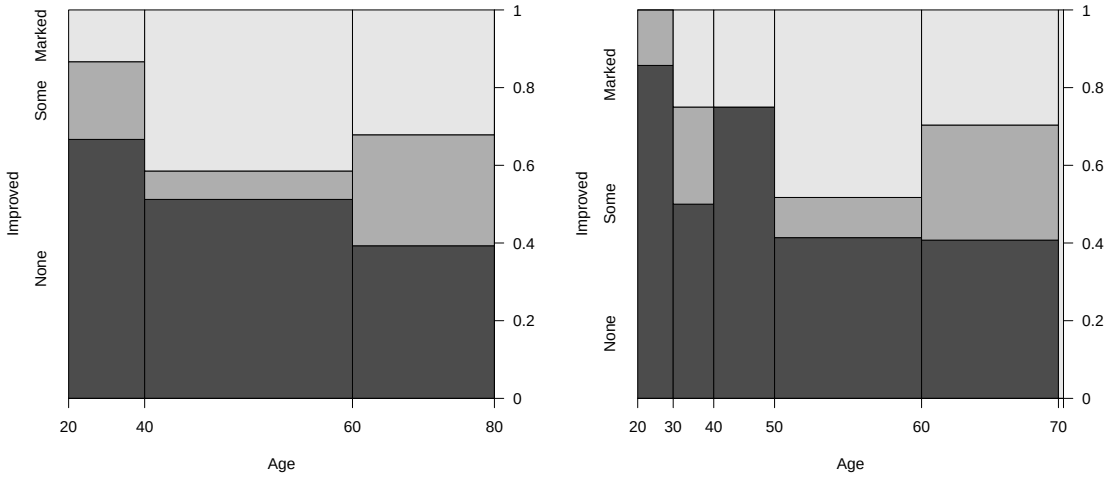
For the `Arthritis` data, we can see how `Improved` varies with `Age` as follows. `spine()` takes a formula of the form `y ~ x` with a single dependent factor and a single explanatory variable `x` (a numeric variable or a factor). The range of a numeric variable`x` is divided into intervals based on the `breaks` argument, and stacked bars are drawn to show the distribution of `y` as `x` varies. As shown below, the discrete table that is visualized is returned by the function.

```
> (spine(Improved ~ Age, data = Arthritis, breaks = 3))

         Improved
Age       None Some Marked
  [20,40]   10    3      2
  (40,60]   21    3     17
  (60,80]   11    8      9

> (spine(Improved ~ Age, data = Arthritis, breaks = "Scott"))

         Improved
Age       None Some Marked
  [20,30]    6    1      0
  (30,40]    4    2      2
  (40,50]    9    0      3
  (50,60]   12    3     14
  (60,70]   11    8      8
  (70,80]    0    0      1
```



The conditional density plot (Hofmann and Theus 2005) is a further generalization. This visualization technique is similar to spinograms, but uses a smoothing approach rather than discretizing the explanatory variable. As well, it uses the original `x` axis and not a distorted one.

In such plots, it is useful to also see the distribution of the observations across the horizontal axis, e.g., with a `rug()` plot. Figure 6 uses `cdplot()` from the **graphics** package rather than `cd_plot()` from **vcd**, and is produced with

```
> cdplot(Improved ~ Age, data = Arthritis)
> with(Arthritis, rug(jitter(Age), col="white", quiet=TRUE))
```

From Figure 6 it can be easily seen that the proportion of patients reporting Some or Marked improvement increases with Age, but there are some peculiar bumps in the distribution. These may be real or artifactual, but they would be hard to see with most other visualization methods.

```
> cdplot(Improved ~ Age, data = Arthritis)
> with(Arthritis, rug(jitter(Age), col="white", quiet=TRUE))
```

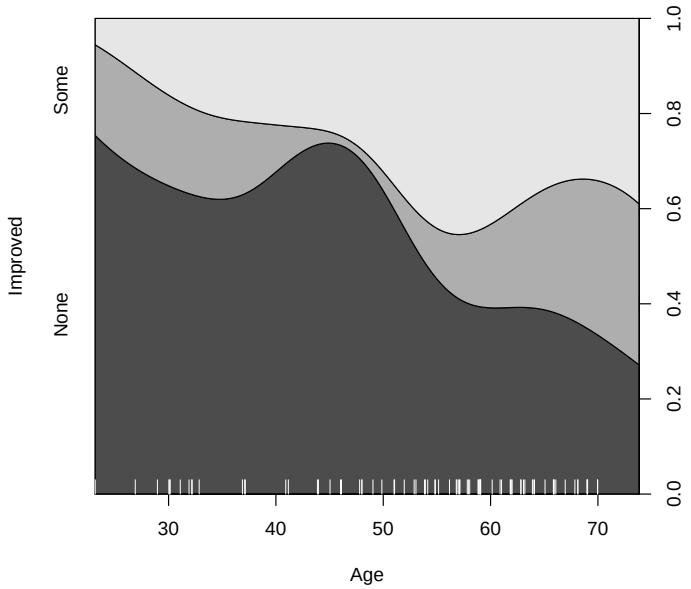


Figure 6: Conditional density plot for the `Arthritis` data showing the variation of Improved with Age.

When we switch from non-parametric data exploration to parametric statistical models, such effects are easily missed.

References

Agresti A (2002). *Categorical Data Analysis*. John Wiley & Sons, Hoboken, New Jersey, 2nd edition.

Cohen J (1960). "A coefficient of agreement for nominal scales." *Educational and Psychological Measurement*, **20**, 37–46.

Friendly M (1994). "Mosaic Displays for Multi-Way Contingency Tables." *Journal of the American Statistical Association*, **89**, 190–200.

Friendly M (1999). "Extending Mosaic Displays: Marginal, Conditional, and Partial Views of Categorical Data." *Journal of Computational and Graphical Statistics*, **8**(3), 373–395.

Friendly M (2000). *Visualizing Categorical Data*. SAS Insitute, Carey, NC. URL http://www.math.yorku.ca/SCS/vcd/.

Hartigan JA, Kleiner B (1984). "A Mosaic of Television Ratings." *The American Statistician*, **38**, 32–35.

Hofmann H, Theus M (2005). "Interactive Graphics for Visualizing Conditional Distributions." Unpublished Manuscript.

Hummel J (1996). "Linked Bar Charts: Analysing Categorical Data Graphically." *Computational Statistics*, **11**, 23–33.

Meyer D, Zeileis A, Hornik K (2006). "The Strucplot Framework: Visualizing Multi-way Contingency Tables with **vcd**." *Journal of Statistical Software*, **17**(3), 1–48. URL http://www.jstatsoft.org/v17/i03/.