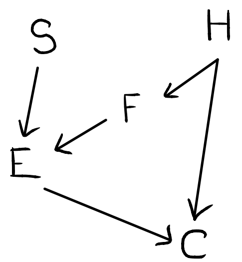
Question 1

(a)



b) $P(F \mid H)$

	F = 1	F = 0
H = 0	y	$1-y$
H = 1	x	$1-x$

c) $P(E \mid F, S)$

		E = 0	E = 1
S = 0	F = 0	1	0
	F = 1	0	1
S = 1	F = 0	0.5	0.5
	F = 1	0	1

e H c f s

d) $$P(F \mid C) = \frac{P(F, C)}{P(C)} = \frac{P(F, C)}{\sum_F P(F = f, C)}$$

$$P(F, C) = \sum_{e,h,s} P(C \mid H = h, E = e)\, P(E = e, F, S = s)\, P(S = s)\, P(F \mid H = h)$$

We are looking for $P(F = 1 \mid C = 1)$

$$P(F=1 \mid C=1) = \frac{\sum_{e,h,s} P(C=1 \mid H=h, E=e)\, P(E=e, F=1, S=s)\, P(S=s)\, P(F=1, H=h)}{\sum_{e,h,s,f} P(C=1 \mid H=h, E=e)\, P(E=e, F=f, S=s)\, P(S=s)\, P(F=f, H=h)}$$

Question 2

Compute the following terms using basic axioms of probability and the conditional independence properties encoded in the above graph.

a. P(a, ¬r)

b. P(b, a)

a. $P(a, \neg r)$

$$P(a, \neg r) = P(\neg r) \sum_b P(a \mid b) P(b)$$
$$= P(\neg r) \left[P(a \mid b) P(b) + P(a \mid \neg b) P(\neg b) \right]$$
$$= 0.8 \left[0.65 \cdot 0.4 + 0.25 \cdot 0.6 \right] = 0.328$$

b. $P(b, a)$

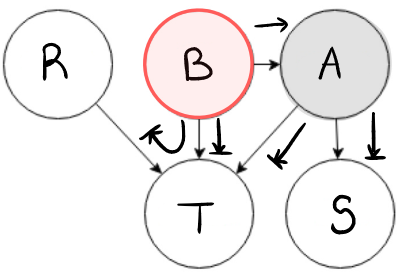
$$P(b, a) = P(a \mid b) P(b) = 0.65 \cdot 0.4 = 0.26$$

For the query P(b|a):

c. Use Bayes Ball to determine the set of nodes that can be pruned from the graph.

d. Compute P(b|a) using the simplifications determined in Part c.

c. Apply Baye's Ball Algorithm



$$B \perp \{R, T, S\} \mid A$$

d.

$$P(b \mid a) = \frac{P(a \mid b) P(b)}{P(a)} = \frac{P(a \mid b) P(b)}{\sum_b P(b, a)} = \frac{P(a \mid b) P(b)}{P(b) P(a \mid b) + P(\neg b) P(a \mid \neg b)}$$

$$\frac{0.65 \cdot 0.4}{0.4 \cdot 0.65 + 0.6 \cdot 0.25} = \frac{0.26}{0.41} \approx 0.63$$

Question 3

$P(T \mid B)$ with the following order : $B = b, R, A, S, T$

List of factors : $P(R)$, $P(b)$, $P(T \mid R, B, A)$, $P(A \mid b)$, $P(S \mid A)$

ordering : b, R, A, S, T

Marginalizing :

- eliminate b

$$m_b(T,R,A) = \sum_b P(b)\, P(T \mid R, A, b)\, P(A \mid b)$$

List : $P(R)$, $P(S \mid A)$, $m_b(T,R,A)$

- eliminate R

$$m_R(T,A) = \sum_r m_b(T,r,A)\, P(r)$$

List : $P(S \mid A)$ $m_R(T,A)$

- eliminate A

$$m_A(S,T) = \sum_a P(S \mid a)\, m_R(T,a)$$

List : $m_a(S,T)$

- eliminate S

$$m_S(T) = \sum_S m_a(S,T)$$

List : $m_S(T)$

- Eliminate T

$$m_t = \sum_T m_S(t)$$

List : m_t

Now, $$P(T \mid B) = \frac{1}{P(B)} \sum_{R,A,S} P(R)\, P(b)\, P(T \mid A, b, R)\, P(A \mid b)\, P(S \mid A)$$

We get $P(T \mid B = 1)$ with excel $= 0.5811$

R	B	A	T	S	P(b)	P(A|b)	P(R)	P(S|A)	P(T|R,b,A)	m_b(T,R,A)	m_r(T,A)	m_a(S,T)	m_s(T)	p(T=1|b)	p(T=0|b)
1	1	1	1	1	0.4	0.65	0.2	0.92	0.95	0.247	0.0494	0.045448	0.045448	0.045448	0
1	1	1	1	0	0.4	0.65	0.2	0.08	0.95	0.247	0.0494	0.003952	0.003952	0.003952	0
1	1	1	0	1	0.4	0.65	0.2	0.92	0.05	0.013	0.0026	0.002392	0.002392	0	0.00239
1	1	1	0	0	0.4	0.65	0.2	0.08	0.05	0.013	0.0026	0.000208	0.000208	0	0.00021
1	1	0	1	1	0.4	0.35	0.2	0.3	0.88	0.1232	0.02464	0.007392	0.007392	0.007392	0
1	1	0	1	0	0.4	0.35	0.2	0.7	0.88	0.1232	0.02464	0.017248	0.017248	0.017248	0
1	1	0	0	1	0.4	0.35	0.2	0.3	0.12	0.0168	0.00336	0.001008	0.001008	0	0.00101
1	1	0	0	0	0.4	0.35	0.2	0.7	0.12	0.0168	0.00336	0.002352	0.002352	0	0.00235
0	1	1	1	1	0.4	0.65	0.8	0.92	0.6	0.156	0.1248	0.114816	0.114816	0.114816	0
0	1	1	1	0	0.4	0.65	0.8	0.08	0.6	0.156	0.1248	0.009984	0.009984	0.009984	0
0	1	1	0	1	0.4	0.65	0.8	0.92	0.4	0.104	0.0832	0.076544	0.076544	0	0.07654
0	1	1	0	0	0.4	0.65	0.8	0.08	0.4	0.104	0.0832	0.006656	0.006656	0	0.00666
0	1	0	1	1	0.4	0.35	0.8	0.3	0.3	0.042	0.0336	0.01008	0.01008	0.01008	0
0	1	0	1	0	0.4	0.35	0.8	0.7	0.3	0.042	0.0336	0.02352	0.02352	0.02352	0
0	1	0	0	1	0.4	0.35	0.8	0.3	0.7	0.098	0.0784	0.02352	0.02352	0	0.02352
0	1	0	0	0	0.4	0.35	0.8	0.7	0.7	0.098	0.0784	0.05488	0.05488	0	0.05488
					0.4					1.6	0.8	0.4	0.4	***0.23244***	***0.1676***
													Divide by P(b)	0.5811	0.4189
														T=1	T=0

Question 4

i)	ii)	iii)
$\theta_a = P(a)$	46 / 129	47 / 131
$\theta_{b\mid a} = P(b \mid a)$	26 / 46	28 / 48
$\theta_{b\mid \neg a} = P(b \mid \neg a)$	35 / 83	37 / 85
$\theta_{c\mid a} = P(c \mid a)$	25 / 46	27 / 48
$\theta_{c\mid \neg a} = P(c \mid \neg a)$	40 / 83	41 / 85
$\theta_{d\mid c,b} = P(d \mid c, b)$	14 / 36	15 / 38
$\theta_{d\mid \neg c,b} = P(d \mid \neg c, b)$	11 / 25	12 / 27
$\theta_{d\mid c,\neg b} = P(d \mid c, \neg b)$	3 / 29	4 / 31
$\theta_{d\mid \neg c,\neg b} = P(d \mid \neg c, \neg b)$	5 / 39	6 / 41