Václav Luňák

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1

$$\int \frac{x^{17}-5}{x-1}dx = \int (x^{16}+x^{15}+x^{14}+x^{13}+x^{12}+x^{11}+x^{10}+x^9+x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1-\frac{4}{x-1})dx =$$

$$= \frac{x^{17}}{17}+\frac{x^{16}}{16}+\frac{x^{15}}{15}+\frac{x^{14}}{14}+\frac{x^{13}}{13}+\frac{x^{12}}{12}+\frac{x^{11}}{11}+\frac{x^{10}}{10}+\frac{x^9}{9}+\frac{x^8}{8}+\frac{x^7}{7}+\frac{x^6}{6}+\frac{x^5}{5}+\frac{x^4}{4}+\frac{x^3}{3}+\frac{x^2}{2}+x-4\int \frac{1}{x-1}dx =$$

$$= \sum_{k=1}^{17} \frac{x^k}{k} - 4\log|x-1| + c, c \in R$$

na $(-\infty, 1)$ a na $(1, \infty)$

2

$$\int \log(x+\sqrt{1+x^2})dx =$$

[per partes: $u = \log(x+\sqrt{1+x^2})$, $v' = 1$]

$$= x\log(x+\sqrt{1+x^2}) - \int x\frac{1+\frac{x}{\sqrt{1+x^2}}}{x+\sqrt{1+x^2}}dx = x\log(x+\sqrt{1+x^2}) - \int \frac{x}{\sqrt{1+x^2}}dx =$$

[substituce: $t = 1+x^2$, $dt = 2xdx$]

$$= x\log(x+\sqrt{1+x^2}) - \frac{1}{2}\int \frac{1}{\sqrt{t}}dt = x\log(x+\sqrt{1+x^2}) - \frac{1}{2}\int t^{-\frac{1}{2}}dt = x\log(x+\sqrt{1+x^2}) - \frac{1}{2}\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c =$$

$$= x\log(x+\sqrt{1+x^2}) - t^{\frac{1}{2}} + c = x\log(x+\sqrt{1+x^2}) - \sqrt{1+x^2} + c, c \in R$$

na R (logaritmus je vždy definován, protože má všude kladný argument)

3

$$\int \frac{\cos^2 x}{\sin x(1-\cos x)}dx =$$

[substituce: $t = \tan\frac{x}{2}$, $dx = \frac{2}{1+t^2}dt$, $\sin x = \frac{2t}{1+t^2}$, $\cos x = \frac{1-t^2}{1+t^2}$]

$$= \int \frac{(\frac{1-t^2}{1+t^2})^2}{\frac{2t}{1+t^2}(1-\frac{1-t^2}{1+t^2})} \cdot \frac{2}{1+t^2}dt = \int \frac{(1-t^2)^2}{t(1+t^2)\cdot(1+t^2-1+t^2)}dt = \frac{1}{2}\int \frac{(1-t^2)^2}{(1+t^2)t^3}dt = \frac{1}{2}\int \frac{1-2t^2+t^4}{(1+t^2)t^3}dt =$$

$$= \frac{1}{2}\int (\frac{A}{t}+\frac{B}{t^2}+\frac{C}{t^3}+\frac{Dt+E}{1+t^2})dt = \frac{1}{2}\int (\frac{-3}{t}+\frac{0}{t^2}+\frac{1}{t^3}+\frac{4t+0}{1+t^2})dt = \int \frac{2t}{1+t^2}dt + \frac{1}{2}\int t^{-3}dt - \frac{3}{2}\int \frac{1}{t}dt =$$

[první integrál substitucí za $s = 1+t^2$, druhé dva přímo]

$$= \log|1+t^2| + \frac{t^{-2}}{-4} - \frac{3}{2}\log|t| + c = \log|1+\tan^2(\frac{x}{2})| - \frac{1}{4\tan^2(\frac{x}{2})} - \frac{3}{2}\log|\tan(\frac{x}{2})| + c, c \in R$$

na $\bigcup_{k\in Z}(k\pi, (k+1)\pi)$, jelikož $\tan\frac{x}{2}$ se nesmí rovnat nule.