Math 42 Assignment 2

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Instructions:

- This assignment is about mathematical modeling of happiness in marriages. A document is posted on the Canvas account of the course. The mathematical modeling of marriage is disused in Chapter 10 of the document. Please read the chapter and complete the assignments:

  Exercise 10.10
  1. Exercise 10.10.1
  2. Exercise 10.10.2
  3. Exercise 10.10.3

  Type your solutions in a latex file.

Solution

1. Exercise 10.10.1

   Part (a) answer: Based on given information, we know the marriage could be stable since $0 < a_1/r_1 < 1$ and $0 < a_2/r_2 < 0$ where a_1 measures how easily the husband is influenced by the wife's emotions, and a_2 is the corresponding quantity for the wife.
   To find the long-term equilibrium state for a couple, we can set up (10.3) and (10.4) equations to zero
   $r_1(x_0 - x') + a_1 y' = 0$ and $r_2(y_0 - y') + a_2 x' = 0$
   Then we have $x' = (x_0 + \frac{a_1}{r_1} y_0)/(1 - \frac{a_1 a_2}{r_1 r_2})$ and $y' = (y_0 + \frac{a_2}{r_2} x_0)/(1 - \frac{a_1 a_2}{r_1 r_2})$
   Thus, we can conclude this equilibrium is stable if $0 < (a_1 a_2)/(r_1 r_2) < 1$. The marriage is heading in stable direction since $0 < a_1/r_1 < 1$, $0 < a_2/r_2 < 1$ implies $0 < (a_1 a_2)/(r_1 r_2) < 1$.

   Part (b)(i) answer: I can state if each of the spouses is naturally happy $(x_0 > 0, y_0 > 0)$, the marriage is "regulated". Because their happiness is enhanced by the positivity of their spouse by showing these equations.

For husband, Single: $x_0 \Rightarrow$ married: $x_0 + \frac{a_1}{r_1} y_0$

For Wife, Single: $y_0 \Rightarrow$ married: $y_0 + \frac{a_2}{r_2} x_0$

So, the sums $(x_0 + \frac{a_1}{r_1} y_0)$ and $(y_0 + \frac{a_2}{r_2} x_0)$ are both magnified by the factor $1/(1 - \frac{a_1 a_2}{r_1 r_2}) > 1$ in marriage.

This brings the result, $x' = (x_0 + \frac{a_1}{r_1} y_0)/(1 - \frac{a_1 a_2}{r_1 r_2}) >> x_0$ and $y' = (y_0 + \frac{a_2}{r_2} x_0)/(1 - \frac{a_1 a_2}{r_1 r_2}) >> y_0$

Thus, I can say that the spouses are both much happier in marriage than if they were single.

Part (b)(ii) answer: If $x_0 < 0$ and $y_0 < 0$ which mean both spouse have negative states, the marriage is "unregulated". Because their unhappiness is enhanced by the negativity of their spouse by showing these equations. Based on the equations I mentioned in part(b)(i),

$x' = (x_0 + \frac{a_1}{r_1} y_0)/(1 - \frac{a_1 a_2}{r_1 r_2}) << x_0$ and $y' = (y_0 + \frac{a_2}{r_2} x_0)/(1 - \frac{a_1 a_2}{r_1 r_2}) << y_0$

Thus, I can say that the spouses are both much unhappier in marriage than if they were single.

2. Exercise 10.10.2

Answer: Both models (validating and arms race) are based on the relationship of both parties an additive impact on each other. However, even though both models' equations (structure) are exactly the same, the type of results face differently: Happiness in validating marriage = War potential between two nations, when both parameters x_0 and y_0 are greater than 0. It shows that the same characteristic doe not work in favor of two nations in the arms race model. When we set $x_0 > 0$ and $y_0 > 0$ for two nations, this indicates that both countries have a positive "war potential" which means they could actively prepare or build up to the enemy armament.

3. Exercise 10.10.3

Part (a) answer: Based on the given information, a Hostile-detached couple has a husband who is naturally very negative ($x_0 << 0$) and a wife who is also negative ($y_0 < 0$). The influence as a result of marriage is

For husband, Single: $x_0 \Rightarrow$ married: $x_0 + \frac{a_1}{r_1} y_0$

For Wife, Single: $y_0 \Rightarrow$ married: $y_0 + \frac{a_2}{r_2} x_0$

For the Hostile-detached marriage, the sum of between $(x_0 + \frac{a_1}{r_1} y_0)$ and $(y_0 + \frac{a_2}{r_2} x_0)$ will be negative since both spouses are negative, and both are magnified by the factor $1/(1 - \frac{a_1 a_2}{r_1 r_2}) > 1$. So, when both sums are negative, the equilibrium solution for this marriage

$x' = (x_0 + \frac{a_1}{r_1} y_0)/(1 - \frac{a_1 a_2}{r_1 r_2}) < x_0 < 0$ and $y' = (y_0 + \frac{a_2}{r_2} x_0)/(1 - \frac{a_1 a_2}{r_1 r_2}) < y_0 < 0$

Thus, I can conclude the couple will be even more unhappy in a Hostile-detached marriage at equilibrium.

Park (b) answer: Based on between hostile-volatile and validating marriage cases, we can say that the hostile-volatile model is almost similar to the validating marriage model with negative initial (x_0, y_0) values. Moreover, the hostile-volatile model is almost similar to the validating marriage model if both initial (x_0, y_0) are positive.

This could be a resulted by the fact that in both marriage cases couples interact with each other. But, there will be one difference that could occur, caused by the characteristic of the wife in the hostile-detached since she is actively arguing and fighting in the relationship. So, we can estimate that the a_1 is greater in the Hostile-detached model than Validating model.

In conclusion, if $x_0 > 0$ and $y_0 > 0$ for both marriage models, the husband in the hostile-detached marriage will be happier than the husband in the validating marriage given all else equal.