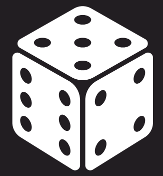
D_2 

D_1 

$D_1 + D_2$	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$\frac{3}{10}$$

$$P\left[D_1 = 2\right] = \frac{6}{36}$$

$$P\left[\, D_1 + D_2 \leq 5 \,\right] = \frac{10}{36}$$

$$P\left[\, (D_1 = 2) \bigcap (D_1 + D_2 \leq 5) \,\right] = \frac{3}{36}$$

$$P\left[\, D_1 = 2 \,\middle|\, D_1 + D_2 \leq 5 \,\right] = \frac{3}{10}$$

$$P\left[\, D_1 = 2 \,\middle|\, D_1 + D_2 \leq 5 \,\right] = \frac{P\left[\, (D_1 = 2) \bigcap (D_1 + D_2 \leq 5) \,\right]}{P\left[\, D_1 + D_2 \leq 5 \,\right]} = \frac{3/36}{10/36} = \frac{3}{10}$$

$$P[A \mid B] = \frac{P[A \cap B]}{P[B]} \qquad P[A \cap B] = P[A \mid B]\; P[B]$$

Conditional Probability Multiplication Rule

Conditional Probability $P[A \mid B] = \dfrac{P[A \cap B]}{P[B]}$

Multiplication Rule $P[A \cap B] = P[A \mid B] \; P[B]$

		Win False	Win True	
Century	False	160	154	314
	True	16	30	46
		176	184	360

$$P[W] = \frac{184}{360} \qquad P[C] = \frac{46}{360} \qquad P[W \cap C] = \frac{30}{360}$$

$$P[W|C] = \frac{30}{46}$$

$$P[C|W] = \frac{30}{184}$$

$$P[W|C] = \frac{P[W \cap C]}{P[C]} = \frac{30/360}{46/360} = \frac{30}{46}$$

$$P[C|W] = \frac{P[W \cap C]}{P[W]} = \frac{30/360}{184/360} = \frac{30}{184}$$

$$P[W|C] = \frac{P[W \cap C]}{P[C]}$$

$$P[C|W] = \frac{P[W \cap C]}{P[W]}$$

$$P[W \cap C] = P[W|C] \; P[C] = \frac{30}{46}\frac{46}{360} = \frac{30}{360}$$

$$P[W \cap C] = P[C|W] \; P[W] = \frac{30}{184}\frac{184}{360} = \frac{30}{360}$$

$$P[W|C] \; P[C] = P[C|W] \; P[W]$$

$$P[W|C] = \frac{P[C|W] \; P[W]}{P[C]}$$

$$P[B|A] = \frac{P[A|B] \; P[B]}{P[A]}$$

Bayes Theorem

Conditional Probability $P[A \mid B] = \dfrac{P[A \cap B]}{P[B]}$

Multiplication Rule $P[A \cap B] = P[A \mid B]\ P[B]$

Bayes Theorem $P[B \mid A] = \dfrac{P[A \mid B]\ P[B]}{P[A]}$

Among 30 faculty members in a department, 5 are females and 25 are males. 3 females and 12 males have a PhD

F	M	M	M	M	M
F	M	M	M	M	M
F	M	M	M	M	M
F	M	M	M	M	M
F	M	M	M	M	M

$$P[F] = \frac{5}{30} \quad P[M] = \frac{25}{30} \quad P\left[F \bigcap \text{phd}\right] = \frac{3}{30} \quad P\left[M \bigcap \text{phd}\right] = \frac{12}{30} \quad P[\text{phd}] = \frac{15}{30}$$

Among those who have done PhD, what fraction are female?

$$P\left[F \,\middle|\, \text{phd}\right] = \frac{3}{15} = \frac{3}{3+12}$$

$$P\left[F \,\middle|\, \text{phd}\right] = \frac{P\left[\text{phd} \,\middle|\, F\right] P[F]}{P[\text{phd}]} = \frac{\blacksquare}{\blacksquare + \blacksquare}$$

$$\blacksquare \longrightarrow P\left[\text{phd} \,\middle|\, F\right] P[F] \longrightarrow P\left[F \bigcap \text{phd}\right]$$

$$\blacksquare \longrightarrow P\left[\text{phd} \,\middle|\, M\right] P[M] \longrightarrow P\left[M \bigcap \text{phd}\right]$$

$$P[\text{phd}] = P\left[\text{phd} \,\middle|\, F\right] P[F] + P\left[\text{phd} \,\middle|\, M\right] P[M]$$

$$\frac{3}{5}\,\frac{5}{30} + \frac{12}{25}\,\frac{25}{30} = \frac{3}{30} + \frac{12}{30} = \frac{15}{30}$$

$$P[\text{phd}] = P\left[F \bigcap \text{phd}\right] + P\left[M \bigcap \text{phd}\right]$$

$$P[B] = P[B \mid A]\, P[A] + P[B \mid A^c]\, P[A^c]$$

$$P[B] = P[B \cap A] + P[B \cap A^c]$$

Law of Total probability

Conditional Probability $P[A \mid B] = \dfrac{P[A \cap B]}{P[B]}$

Multiplication Rule $P[A \cap B] = P[A \mid B]\ P[B]$

Bayes Theorem $P[B \mid A] = \dfrac{P[A \mid B]\ P[B]}{P[A]}$

Law of Total probability $P[B] = P[B \mid A]\ P[A] + P[B \mid A^c]\ P[A^c]$

$$P[B] = P[B \cap A] + P[B \cap A^c]$$

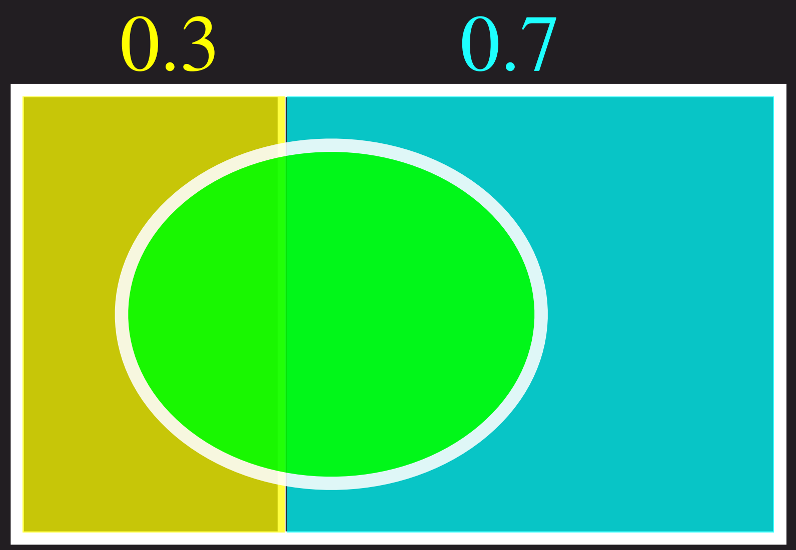
In a university, 30% of faculty members are females. Of the female faculty members, 60% have a PHD. Of the male faculty members, 40% have a PHD.

What is the probability that a randomly chosen faculty member is a female and has PHD?
What is the probability that a randomly chosen faculty member is a male and has PHD?
What is the probability that a randomly chosen faculty member has a PHD?

What is the probability that a randomly chosen PHD holder is female?

$P[F] = 0.3 \quad P[\text{ phd } | F] = 0.6 \qquad P[\text{ phd } \cap F] = P[\text{ phd } | F]\ P[F] = 0.6 * 0.3 = 0.18$

$P[M] = 0.7 \quad P[\text{ phd } | M] = 0.4 \qquad P[\text{ phd } \cap M] = P[\text{ phd } | M]\ P[M] = 0.4 * 0.7 = 0.28$



$$P[\text{ phd }] = P[\text{ phd } \cap F] + P[\text{ phd } \cap M] = 0.18 + 0.28 = 0.46$$

$$P[\text{ phd }] = P[\text{ phd } | F]\ P[F] + P[\text{ phd } | M]\ P[M] = 0.6 * 0.3 + 0.4 * 0.7 = 0.46$$

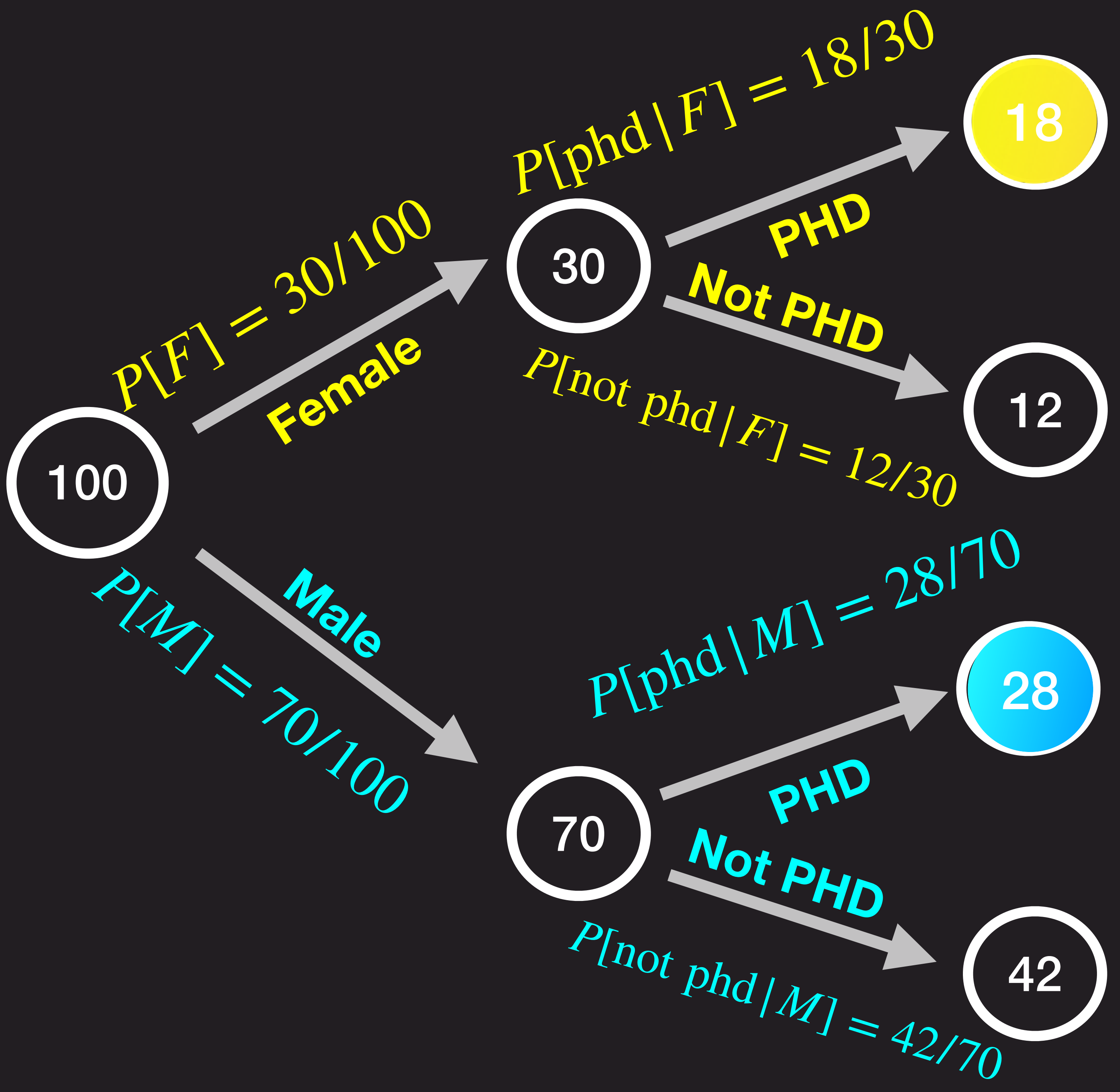
$$P[F | \text{ phd }] = \frac{P[F \cap \text{ phd }]}{P[\text{ phd }]} = \frac{0.18}{0.46} = 0.39$$

$$P[F | \text{ phd }] = \frac{P[\text{ phd } | F]\ P[F]}{P[\text{ phd }]} = \frac{P[\text{ phd } | F]\ P[F]}{P[\text{ phd } | F]\ P[F] + P[\text{ phd } | M]\ P[M]} = \frac{0.6 * 0.3}{0.6 * 0.3 + 0.4 * 0.7} = 0.39$$

$P[F] = 0.3 \quad P[\text{ phd } | F] = 0.6$

$P[M] = 0.7 \quad P[\text{ phd } | M] = 0.4$

$$P[F | \text{ phd }] = \frac{P[\text{ phd } | F]\ P[F]}{P[\text{ phd } | F]\ P[F] + P[\text{ phd } | M]\ P[M]}$$



$$\frac{18}{18 + 28} = 0.39$$