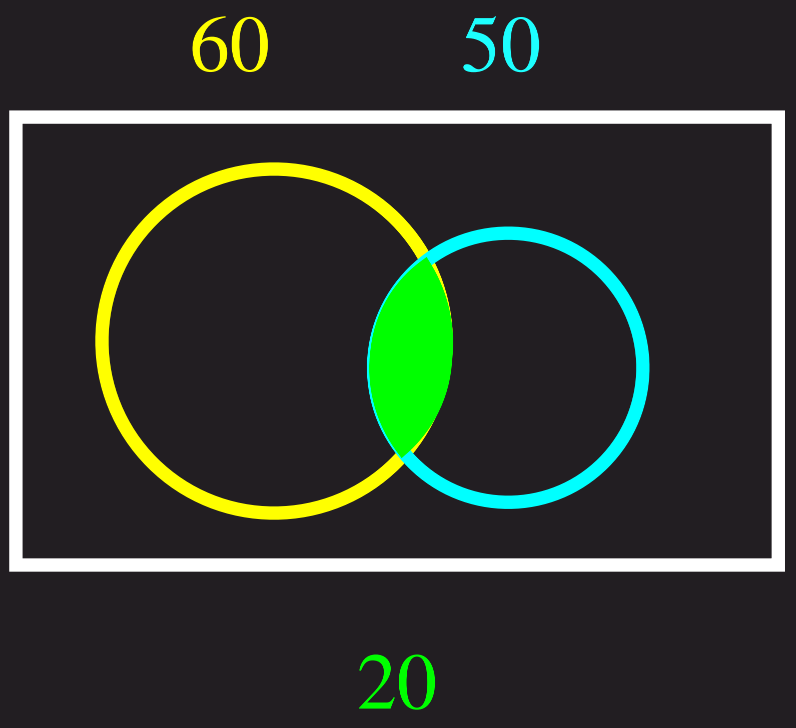
Among 100 students, 60 have taken the computer vision (CV) module, 50 have taken natural language processing (NLP). Also, it is seen that 20 have taken both CV and NLP. Given that a person has taken NLP, what is the probability that he has also taken CV?



A: Students who have taken CV

What is $P[A]$?

$$P[A] = \frac{60}{100}$$

What is $P[A \cap B]$?

$$P[A \cap B] = \frac{20}{100}$$

B: Students who have taken NLP

What is $P[B]$?

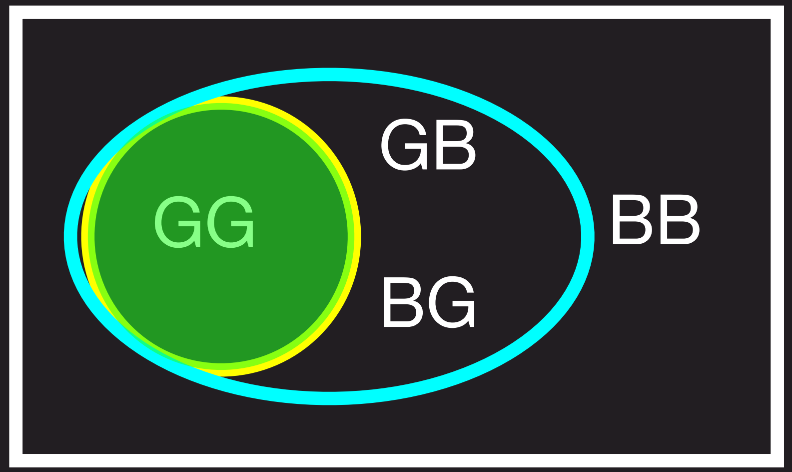
$$P[B] = \frac{50}{100}$$

What is $P[A \mid B]$?

$$P[A \mid B] = \frac{P[A \cap B]}{P[B]} = \frac{20/100}{50/100} = \frac{20}{50}$$

A family has 2 children, at least one of them is a girl. What is the probability that both are girls?

Sample space: {BB, BG, GB, GG}



A: Event that both are girls

As a subset of sample space, what is A?

$$A = \{GG\}$$

$$P[A] = \frac{1}{4}$$

B: Event that there is at least one girl

As a subset of sample space, what is B?

$$B = \{BG, GB, GG\}$$

$$P[B] = \frac{3}{4}$$

What is $A \cap B$? $A \cap B = \{GG\}$

$$P[A \cap B] = \frac{1}{4}$$

What is $P[A \mid B]$?

$$P[A \mid B] = \frac{P[A \cap B]}{P[B]} = \frac{1/4}{3/4} = \frac{1}{3}$$

```
import numpy as np
numerator = 0
denominator = 0
num_families = 10000
for family in range(num_families):
    x1 = np.random.choice([0, 1]) # First child 0--> boy, 1--> girl
    x2 = np.random.choice([0, 1]) # First child 0--> boy, 1--> girl
    y = x1 + x2 # y represents number of girls. y can be 0, 1, or 2
    if y >=1 : # atleast one girl child
        denominator += 1
        if y==2: # two girls
            numerator +=1

cond_prob = numerator / denominator
print(cond_prob)
```

A and B are said to be independent if $P[A \mid B] = P[A]$.

Is tossing a coin and throwing a dice independent?

$$S = \left\{ \begin{matrix} (H,\,1), (H,\,2), (H,\,3), (H,\,4), (H,\,5), (H,\,6) \\ (T,\,1), (T,\,2), (T,\,3), (T,\,4), (T,\,5), (T,\,6) \end{matrix} \right\}$$

Let A denote the event of getting a heads in coin toss,
and B denote the event of getting a 3 on dice

$$A = \{(H,\,1), (H,\,2), (H,\,3), (H,\,4), (H,\,5), (H,\,6)\}$$

$$P[A] = \frac{6}{12} = \frac{1}{2}$$

$$B = \{(H,\,3), (T,\,3)\}$$

$$P[B] = \frac{2}{12} = \frac{1}{6}$$

$$A \cap B = \{(H,\,3)\}$$

$$P[A \cap B] = \frac{1}{12}$$

$$P[A \mid B] = \frac{P[A \cap B]}{P[B]} = \frac{1/12}{2/12} = \frac{1}{2} = P[A]$$

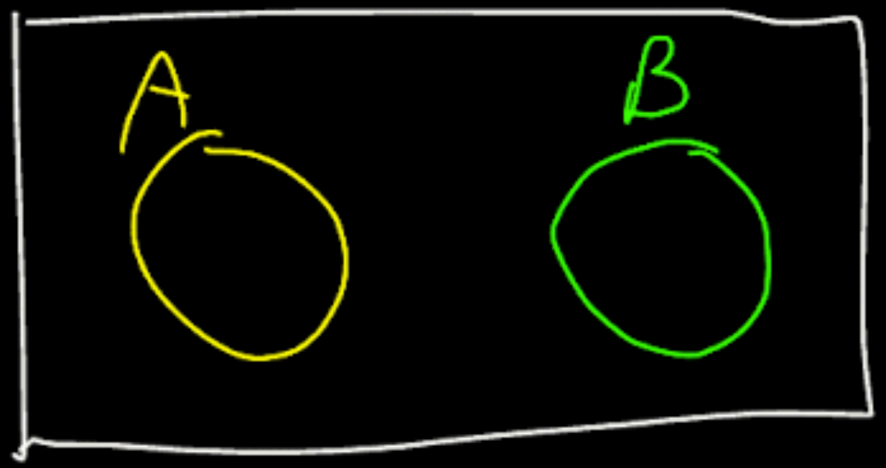
Independence

$$P[A \mid B] = P[A]$$

Claim: If A and B are mutually exclusive, then A and B are **not** independent

Proof: $A \cap B = \{\}$ $\quad P[A \cap B] = 0$

$$P[A \mid B] = \frac{P[A \cap B]}{P[B]} = 0 \neq P(A)$$



Conditional Probability $P[A \mid B] = \dfrac{P[A \cap B]}{P[B]}$

Multiplication Rule $P[A \cap B] = P[A \mid B]\ P[B]$

Bayes Theorem $P[B \mid A] = \dfrac{P[A \mid B]\ P[B]}{P[A]}$

Law of Total probability $P[B] = P[B \mid A]\ P[A] + P[B \mid A^c]\ P[A^c]$

$$P[B] = P[B \cap A] + P[B \cap A^c]$$

Independence $P[A \mid B] = P[A]$

$$P[A \cap B] = P[A]\ P[B]$$

Suppose 5 percent of men and 0.25 percent of the women are color-blind. A random color-blind person is chosen. What is the probability of this person being male? Assume there are equal number of men and women overall.

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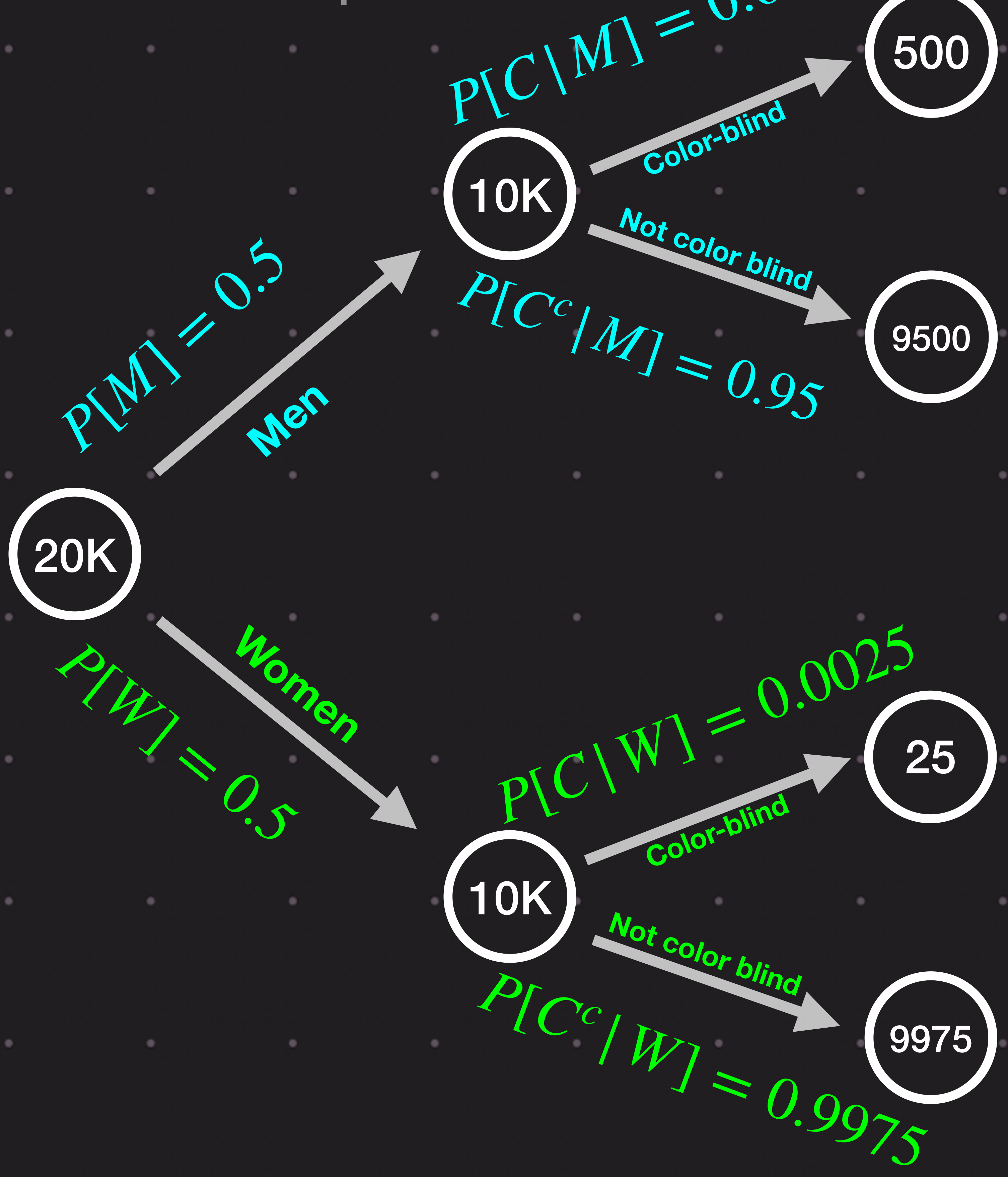


$$\frac{500}{500 + 25} = 0.9523$$

$$P[M \mid C] = \frac{P[C \mid M]P[M]}{P[C \mid M]P[M] + P[C \mid W]P[W]}$$

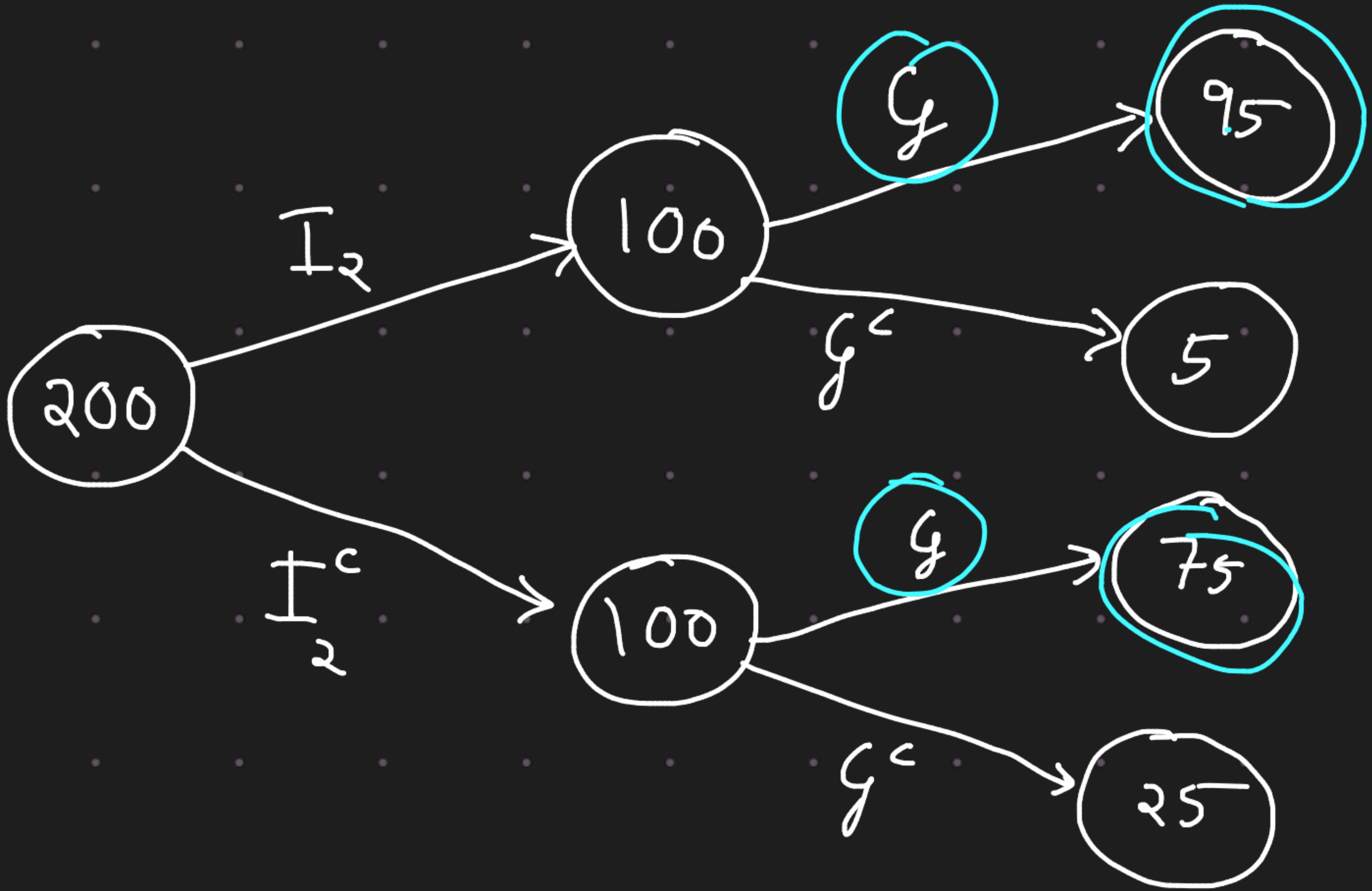
$$= \frac{\frac{500}{10K}\,\frac{10K}{20K}}{\frac{500}{10K}\,\frac{10K}{20K} + \frac{25}{10K}\,\frac{10K}{20K}}$$

50% of the people who gave the first round were called for the second round

95% of the people who got invited for the second round felt that they had a good first round

75% of the people who did not get invited for the second round also felt that they had a good first round

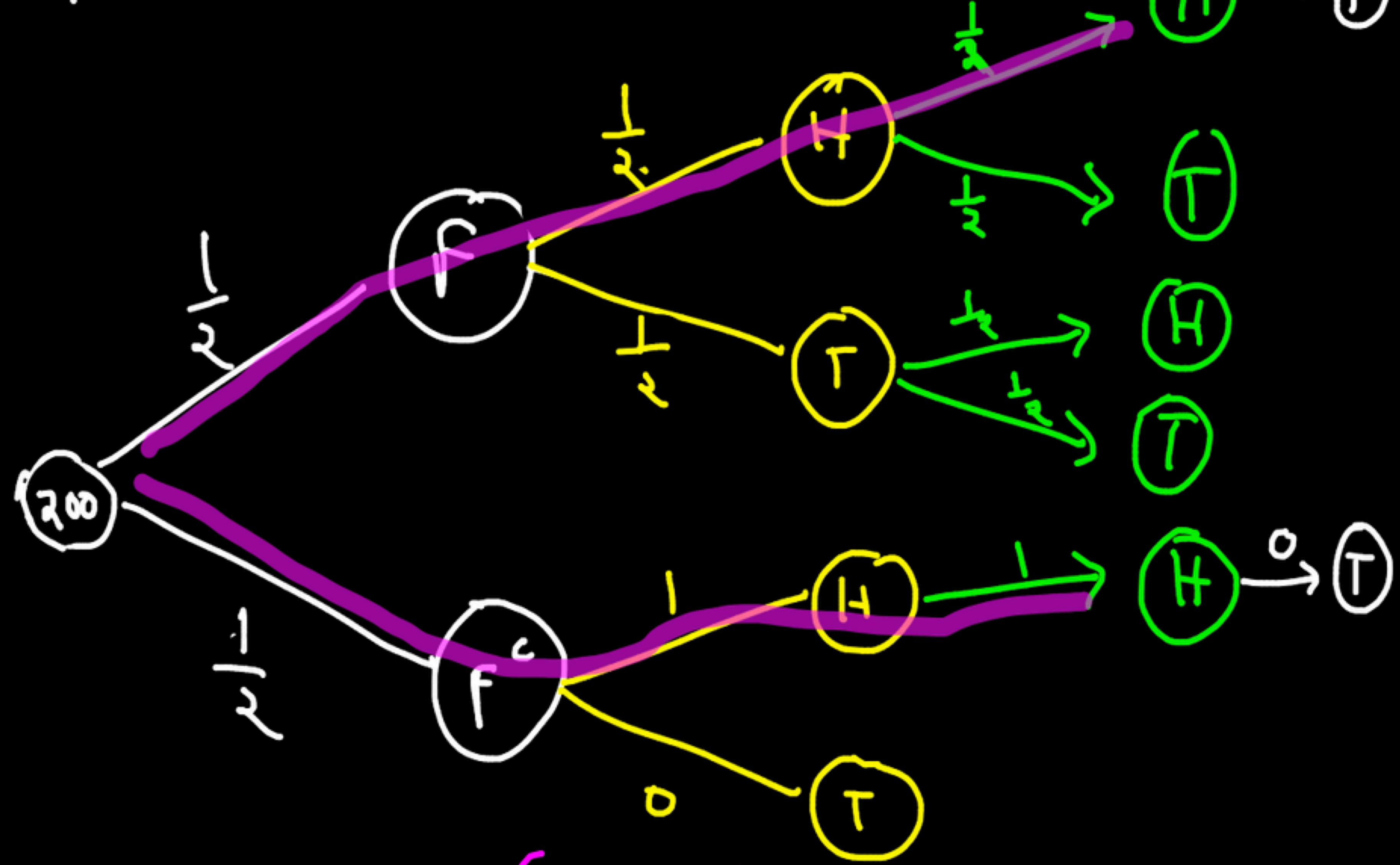
Given that a person felt good about the first round, what is the probability that he cleared the first round?



$$\frac{95}{95+75}$$

A gambler has in his pocket a fair coin and a two-headed coin. He selects one of the coins at random, and he flips it twice. It shows heads both the times. What is the probability that it is the fair coin?

Fair , Biased (2-headed)



$P[HH|F] = \frac{1}{2} \cdot \frac{1}{2}$

$P[HH|F^c] = (1)(1)$

$$P[F|HH] = \frac{P[HH|F]}{P[HH|F]\,P[F] + P[HH|F^c]\,P[F^c]} = \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + (1)(1)\left(\frac{1}{2}\right)}$$