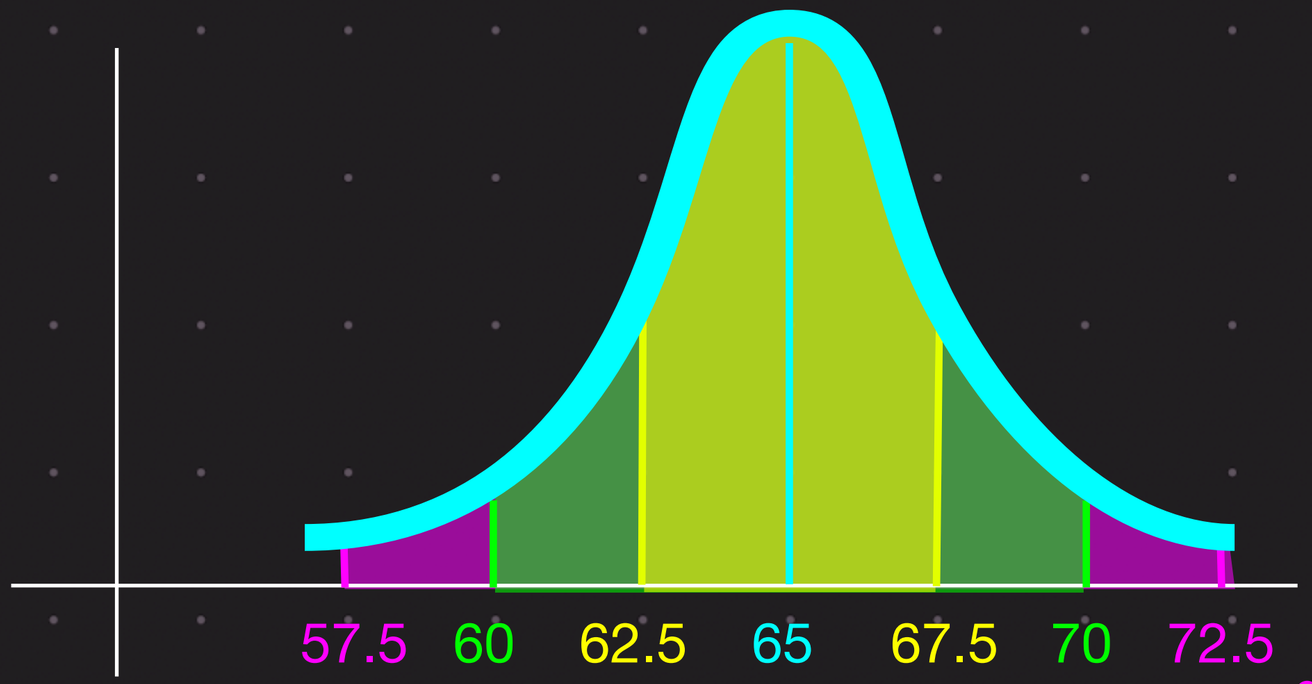
The height of people is Gaussian with mean 65 inches and standard deviation 2.5 inches



$\mu = 65$

$\sigma = 2.5$

Fraction of people whose height is between 62.5 and 67.5 is 68%

$P[62.5 < X < 67.5] = 0.68$

$P[\mu - \sigma < X < \mu + \sigma] = 0.68$

Fraction of people whose height is between 60 and 70 is 95%

$P[60 < X < 70] = 0.95$

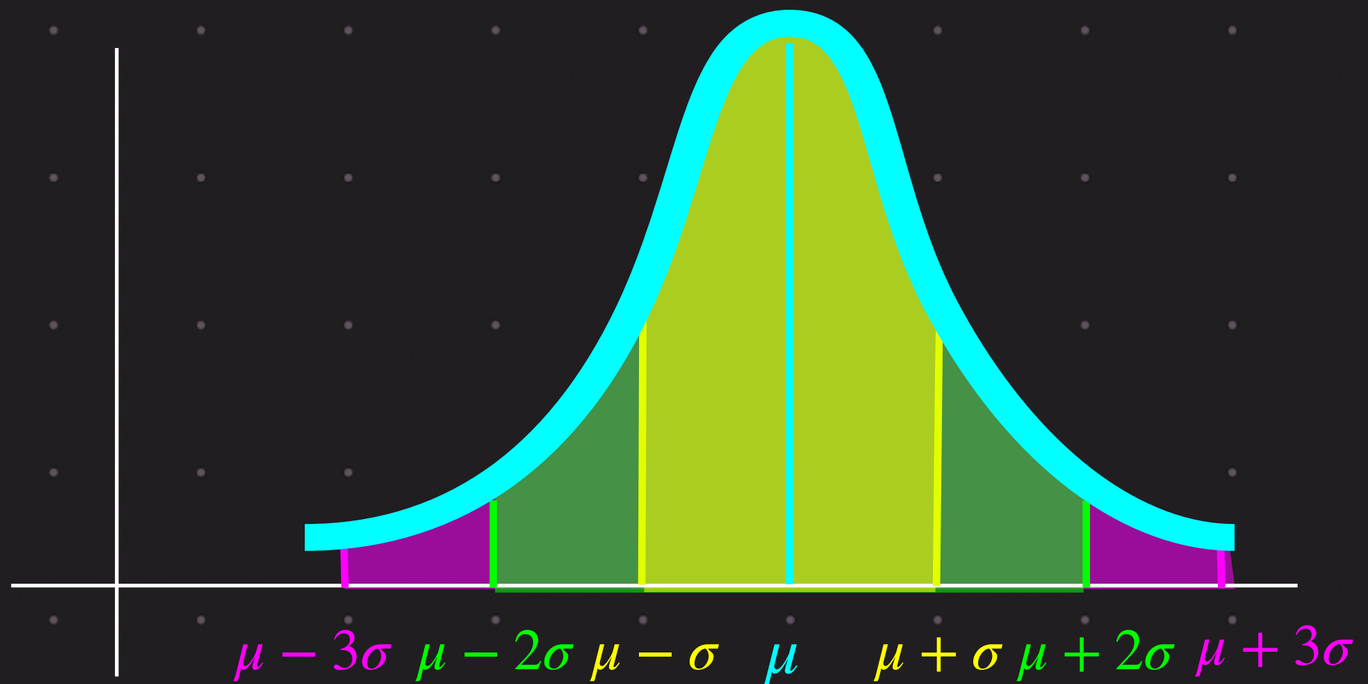
$P[\mu - 2\sigma < X < \mu + 2\sigma] = 0.95$

Fraction of people whose height is between 57.5 and 72.5 is 99.7%

$P[57.5 < X < 72.5] = 0.997$

$P[\mu - 3\sigma < X < \mu + 3\sigma] = 0.997$

Gaussian Empirical Rule or 68/95/99 Rule



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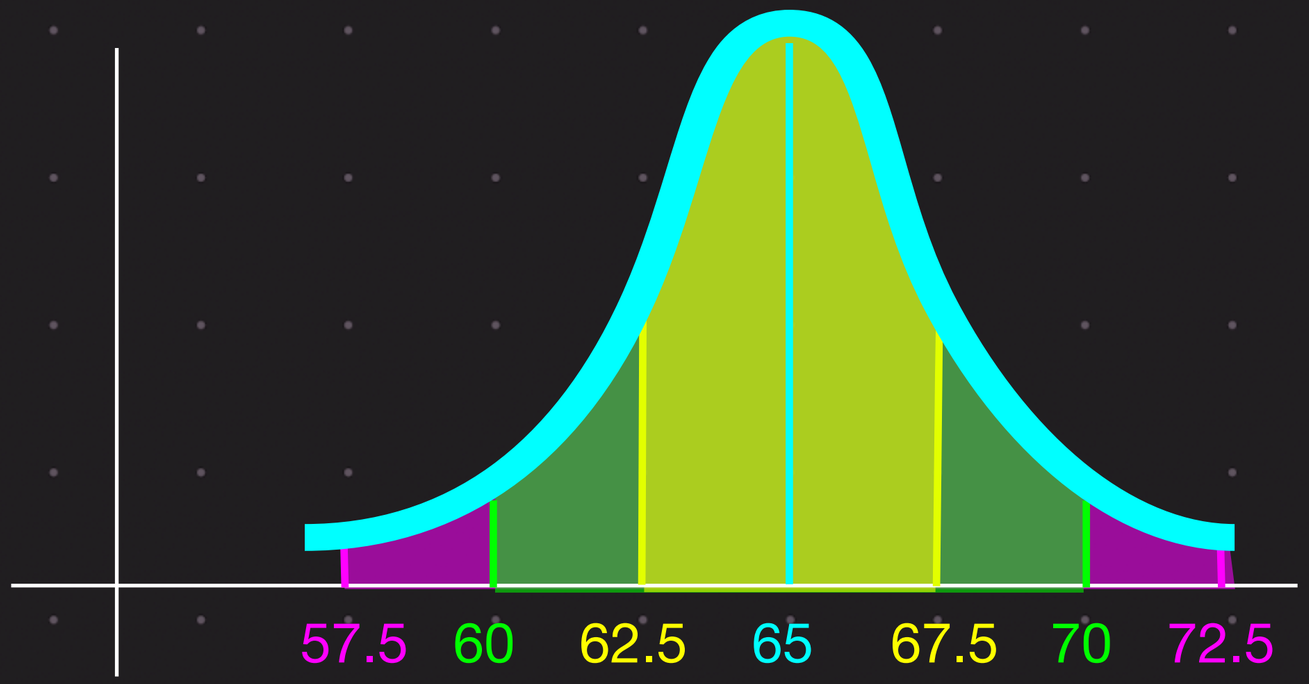

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What is the fraction of people whose height is between 60 and 72.5?

Between 60 and 65? $\frac{95}{2} = 47.5$

Between 65 and 72.5? $\frac{99.7}{2} = 49.85$

Totally, 47.5 + 49.85 = 97.35

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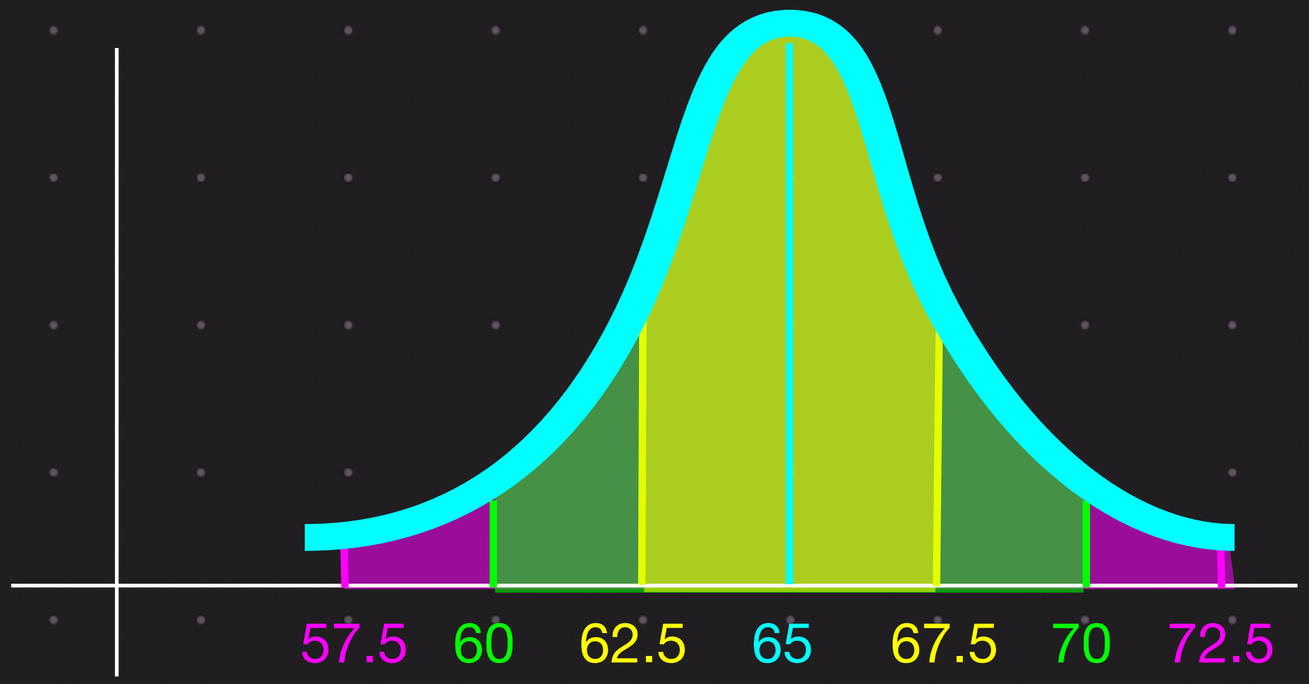

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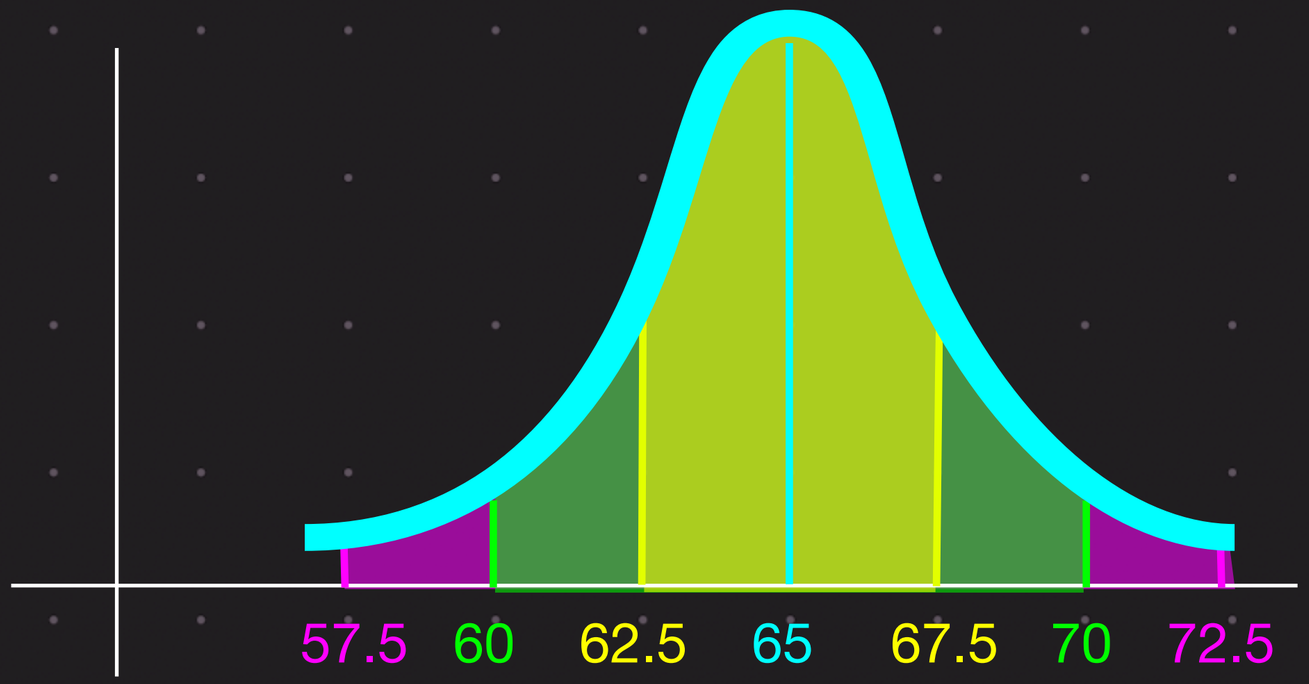
What fraction of people are shorter than 67.5?

What fraction of people are shorter 65? 50%

What fraction of people are in between 65 and 67.5? 68/2 = 34%

Totally 50 + 34 = 84% $P[X < 67.5] = P[X < 65] + P[65 < X < 67.5] = 0.5 + 0.34 = 0.84$

The height of people is Gaussian with mean 65 inches and standard deviation 2.5 inches



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What fraction of people are shorter than 69.1?

How many σ (std devs) away from 65 is this number?

$$65 + z\ (2.5) = 69.1$$

$$z = \frac{(69.1 - 65)}{2.5} = 1.64$$

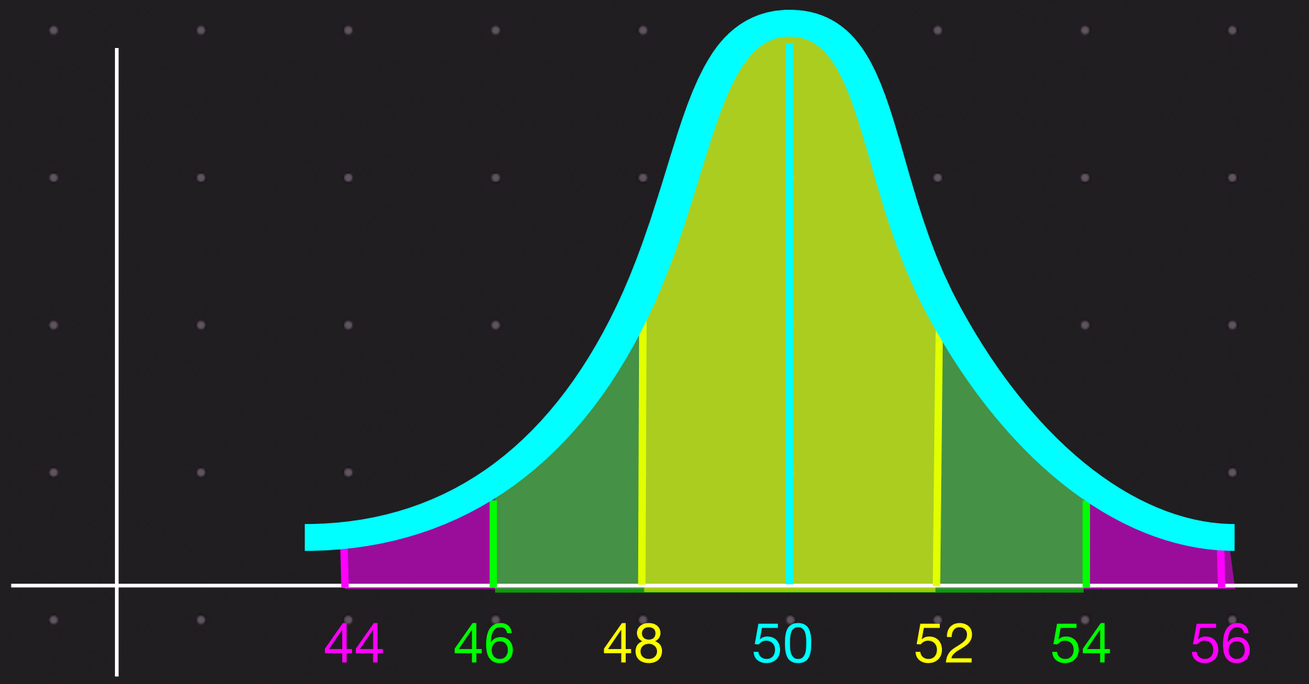
To find this probability, we use the Z-table 94.9%

Z-Score

$$z = \frac{(x - \mu)}{\sigma}$$

```
from scipy.stats import norm
norm.cdf(1.64)
```

Balls produced by manufacturer have mean 50 mm and std dev 2 mm

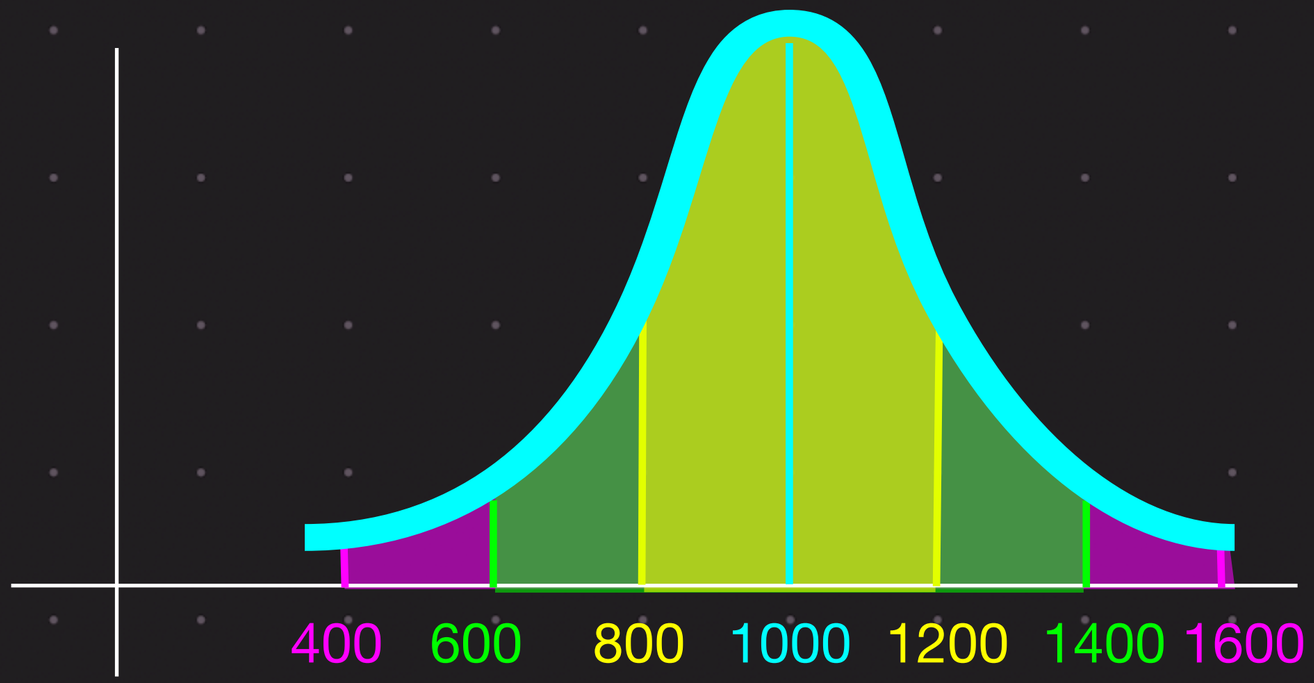


What fraction of balls are smaller than 53 mm?

$$z = \frac{(53 - 50)}{2} = 1.5$$

From Z-table, we see that the answer is 93.32%

A retail outlet sells around 1000 toothpastes a week, with std dev = 200.
If the on-hand inventory is 1300, what is the need for replenishment within the week?



Let X denote the weekly sales. The questions asks for the probability that X > 1300

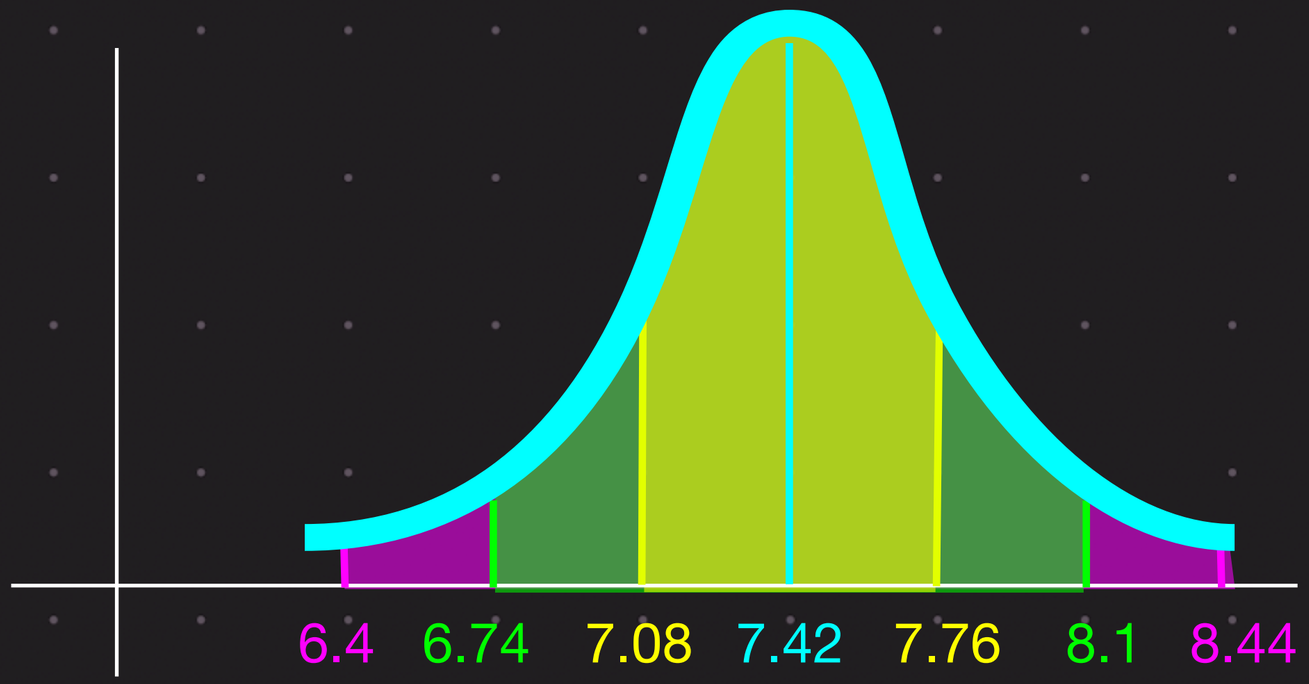
What is the Z-score of 1300?

$$z = \frac{1300 - 1000}{200} = 1.5$$

From Z-table, we see that $P[X \leq 1300] = 0.933$

$$P[X > 1300] = 1 - 0.933 = 0.067$$

Skaters take a mean of 7.42 seconds and std dev of 0.34 seconds for 500 meters.
What should his speed be such that he is faster than 95% of his competitors?



Unlike earlier examples, here the fraction is given, and we have to find Z-score

Let us use the Z-table We need the Z-score of the area corresponding to 0.05

From Z-table, z-score is -1.65

$$z = \frac{(x - \mu)}{\sigma} \qquad x = \sigma\, z + \mu = (0.34)\ (-1.65) + 7.42 \quad = 6.859$$

You toss a coin two times. Suppose you get 1 Rs for every Heads

Q1) What are the possible amounts that you will receive out of this?

Q2) What is the probability of getting 2 Rs?

Q3) What is the probability of getting 1 Rs?

Q4) What is the probability of getting 0 Rs?

Q5) What is the expected amount you will get?

Sample space

$$S = \{\ HH, HT, TH, TT\ \}$$

Let "X" denote the number of heads

$$X = 0 \longrightarrow \{TT\}$$

$$X = 1 \longrightarrow \{HT, TH\}$$

$$X = 2 \longrightarrow \{HH\}$$

$$E[X] = (0)\left(\frac{1}{4}\right) + (1)\left(\frac{1}{2}\right) + (2)\left(\frac{1}{4}\right) = 1$$

X	$P[X]$	Binomial
0	$\frac{1}{4}$	${}^2C_0\left(\frac{1}{2}\right)^2$
1	$\frac{1}{2}$	${}^2C_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$
2	$\frac{1}{4}$	${}^2C_2\left(\frac{1}{2}\right)^2$

You toss two dice.

If both dice are 6, you get 2 Rs

Else if one dice is 6, and another is not 6, then you get 1 Rs

Else, you get 0 Rs

Q 1) What is the probability of getting 0 Rs?

Q 2) What is the probability of getting 1 Rs?

Q 3) What is the probability of getting 2 Rs?

Q 4) What is the expected amount?

# of 6	1	2	3	4	5	6
1	0	0	0	0	0	1
2	0	0	0	0	0	1
3	0	0	0	0	0	1
4	0	0	0	0	0	1
5	0	0	0	0	0	1
6	1	1	1	1	1	2

D_2

D_1

$$\frac{5 * 5}{36}$$

$$\frac{5 * 1 + 1 * 5}{36}$$

$$\frac{1 * 1}{36}$$

X	$P(X)$	
0	${}^2C_0 \left(\frac{5}{6}\right)^2$	$\left(\frac{5}{6}\right)^2$
1	${}^2C_1 \left(\frac{1}{6}\right)\left(\frac{5}{6}\right)$	$2\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)$
2	${}^2C_2 \left(\frac{1}{6}\right)^2$	$\left(\frac{1}{6}\right)^2$