DAV-2

Lecture - 6

Probability Distributions - 1

Casino Case Study

Random Variable.

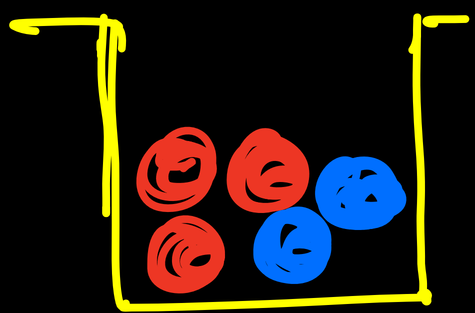
Expectation

Binomial Distribution.

Simulations.

Empirical Probability v/s Theoretical Prob.

Case-Study - Casino.



Experiment - 4 balls from the bag
"with replacement"

Independent A and B $P(A \cap B) = P(A)\,P(B)$.

If all 4 balls are red $\longrightarrow$ +150
If not all 4 balls are red $\rightarrow$ -10

Would you play the game?

P(B) ???
P(R) ???

Estimating Theoretical Prob.
by
Empirical Prob.

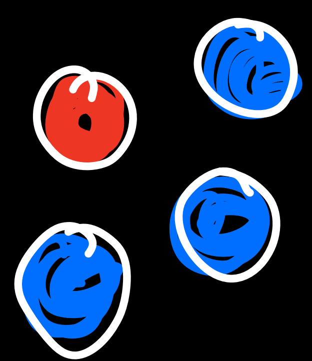
Random Variable. x → Randomly takes different values

$X \rightarrow 1$

$X \rightarrow 2$

$X \rightarrow 3$

$X = 4$ — constant



R B B B

B R B B

B B R B

B B B R

Outcomes.

Event → Getting 1 Red ball.

↓

Numbers — different values

Event1 → Getting 1 Red ball

Event2 → Getting 5 Red ball.

$X = \#$ of red balls.

0

1

2

3

4

... 5 ...

X = no. of red balls.

$P(X=0) \rightarrow$ all balls are blue.

$= B.B.B.B$

$= 2/5 . 2/5 . 2/5 . 2/5$ (with replacement)

$= (2/5)^4$

$P(X=1) =$ 1 ball is red

$= \underline{BRBB}$, RBBB, BBRB, BBBR.

$= \left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right) + \ldots + \ldots$

$= 4 \times \left(\frac{2}{5}\right)^{3} \left(\frac{3}{5}\right)$

B   R

3   1

$$E[X] = \sum X_i P(X_i)$$

$$= X_0 P(X=0) + X_1 P(X=1) + X_2 P(X=2) \ldots$$

$X_0 \downarrow$ Value of Random Variable

$P(X=0) \searrow$ Theoretical Probability.

"Expected" Average value.

Theoretically?

$I1 + I2 + I3$ - - - - - - I_{10000}

Avg. all prob.

$$\frac{\sum P(x=i)}{n}$$

$(X=0) \rightarrow$ __ times.
$(X=1) \rightarrow$ __ times
:
$(X=4) \rightarrow$ __ times.

$$\frac{\underbrace{[0, 0, 0, 0, 0, 0 \_]}_{x=0} + \underbrace{[1, 1, 1, 1, 1]}_{(x=1)} + \ldots \underbrace{[4, 4, 4, 4, 4]}_{x=4}}{10000}$$

$$= \frac{0 \times P(X=0)}{10000} + \frac{1 \times P(X=1)}{10000} + \ldots$$

$\hookrightarrow$ Probab. $P(X=0)$

Casino case study

A bag has 3 red and 2 blue balls.

You pick a ball, write its colour, and put it back in the bag. This is done 4 times in total.
If all 4 times, the red ball was drawn, you win Rs 150. In any other case, you lose Rs 10.
Would you play this game?

Let "X" denote the number of red balls when you draw 4 balls with replacement

Here, X is an example of what is called a "Random Variable"

What are all the outcomes?

0 red 1 red 2 red 3 red 4 red

weighted sum.

weights.

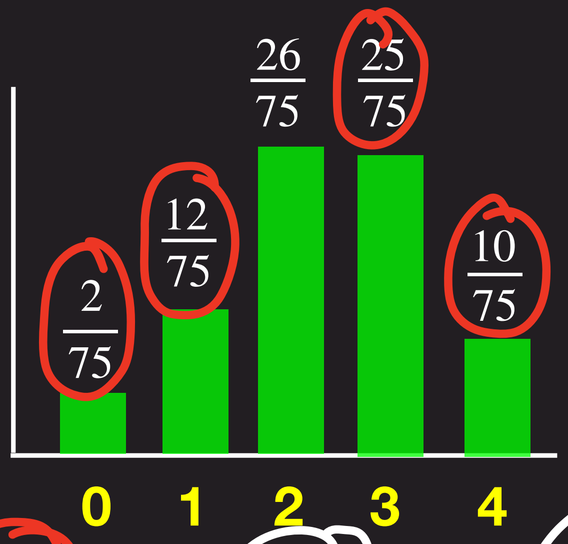
$\sum x\, p(x)$

Random Variable

Empirical approach: Estimate probability using data

Data from 75 people

$X = 0$	2 people
$X = 1$	12 people
$X = 2$	26 people
$X = 3$	25 people
$X = 4$	10 people



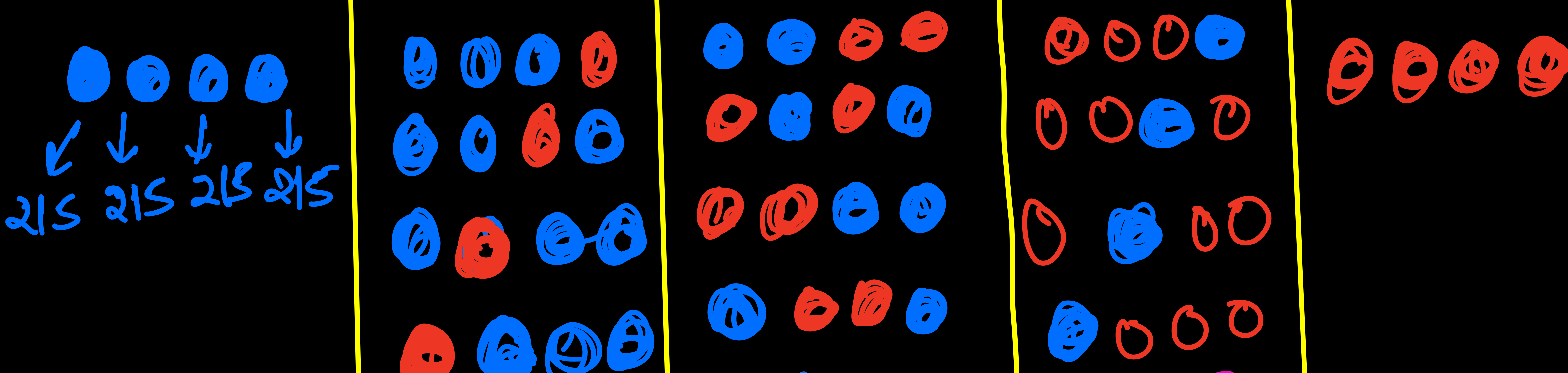
X	$P[X]$	$E[X]$
0	$\frac{2}{75}$	$(0)\cdot\left(\frac{2}{75}\right)$ +
1	$\frac{12}{75}$	$(1)\left(\frac{12}{75}\right)$ +
2	$\frac{26}{75}$	$(2)\left(\frac{26}{75}\right)$ +
3	$\frac{25}{75}$	$(3)\left(\frac{25}{75}\right)$ +
4	$\frac{10}{75}$	$(4)\left(\frac{10}{75}\right)$

Expectation of X is the weighted average of the values that X takes, with the weights being the probabilities

Theoretically

$$E[X] = (0)\left(\frac{2}{75}\right) + (1)\left(\frac{12}{75}\right) + (2)\left(\frac{26}{75}\right) + (3)\left(\frac{25}{75}\right) + (4)\left(\frac{10}{75}\right) = 2.38$$

0R	1R	2R	3R	4R



0R	1R	2R	3R	4R
$4-0$	$4-1$	$4-2$	$4-3$	
$\left(\frac{2}{5}\right)^{4}\left(\frac{3}{5}\right)^{0}$	$\left(\frac{2}{5}\right)^{3}\left(\frac{3}{5}\right)^{1}$	$\left(\frac{2}{5}\right)^{2}\left(\frac{3}{5}\right)^{2}$	$\left(\frac{2}{5}\right)^{1}\left(\frac{3}{5}\right)^{3}$	$\left(\frac{2}{5}\right)^{0}\left(\frac{3}{5}\right)^{4}$
1	4	6	4	1
${}^4C_0$	${}^4C_1$	${}^4C_2$	${}^4C_3$	${}^4C_4$

$$\boxed{P(X=k) = {}^4C_k \left(\frac{2}{5}\right)^{N-k}\left(\frac{3}{5}\right)^{k}}$$

$\left(\frac{2}{5}\right)$: $(1-p)$ $\quad$ $\left(\frac{3}{5}\right)$: (p)

R R
_ _ _ _

2R ${}^4C_2$

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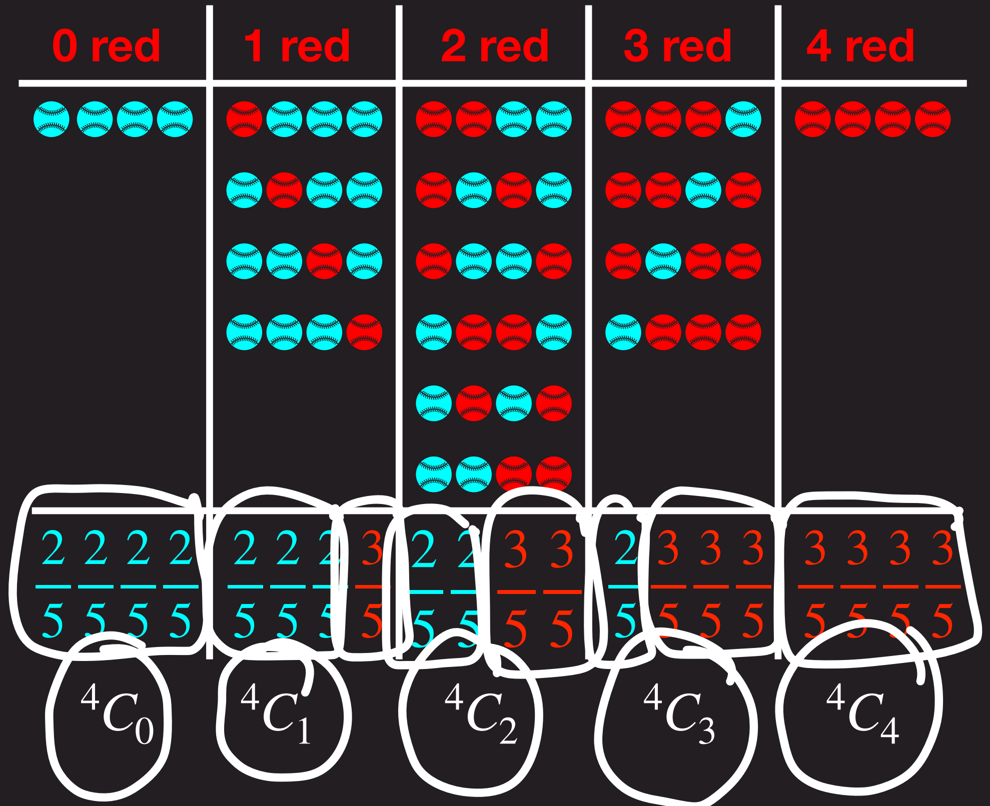
Let "X" denote the number of red balls when you draw 4 balls with replacement

Here, X is an example of what is called a "Random Variable"

What are all the outcomes?



Theoretical approach: Compute probability using rules

What is the probability of 1 red ball in 1 pick?

P[🔴] = 3/5

What is the probability of 1 blue ball in 1 pick?

P[🔵] = 2/5

What is the probability of 2 red balls in 2 picks?

P[🔴🔴] = (3/5)(3/5)

What is the probability of 1 red ball in first pick and 1 blue ball in second?

P[🔴🔵] = (3/5)(2/5)

What is the probability of 1 blue ball in first pick and 1 red ball in second?

P[🔵🔴] = (2/5)(3/5)

P[🔴🔴🔴🔵] = (3/5)(3/5)(3/5)(2/5)

P[🔵🔴🔴🔴] = (3/5)(3/5)(3/5)(2/5)

P[🔵🔵🔵🔵] = (2/5)(2/5)(2/5)(2/5)

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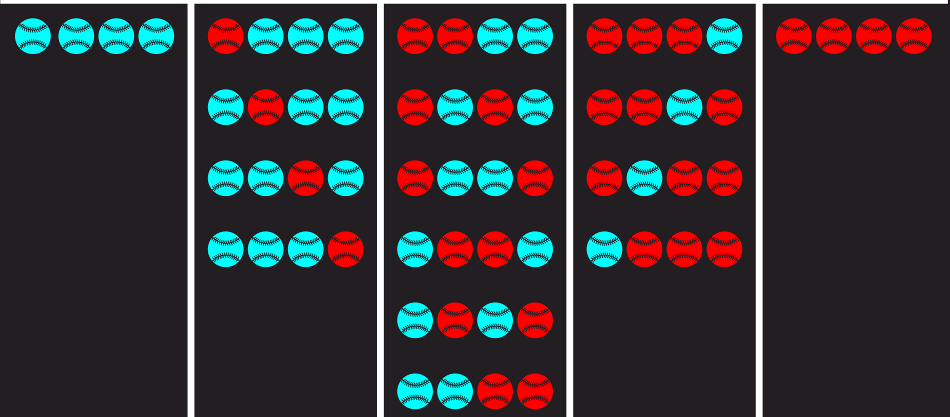
What are all the outcomes?

0 red	1 red	2 red	3 red	4 red



0 red	1 red	2 red	3 red	4 red
$\frac{2}{5}\frac{2}{5}\frac{2}{5}\frac{2}{5}$	$\frac{2}{5}\frac{2}{5}\frac{2}{5}\frac{3}{5}$	$\frac{2}{5}\frac{2}{5}\frac{3}{5}\frac{3}{5}$	$\frac{2}{5}\frac{3}{5}\frac{3}{5}\frac{3}{5}$	$\frac{3}{5}\frac{3}{5}\frac{3}{5}\frac{3}{5}$
${}^4C_0$	${}^4C_1$	${}^4C_2$	${}^4C_3$	${}^4C_4$

Theoretical approach: Compute probability using rules

X	Number of outcomes	Probability per outcome	$P[X]$	Code
0	${}^4C_0$	$\left(\frac{2}{5}\right)^4$	${}^4C_0\left(\frac{2}{5}\right)^4$	`binom.pmf(k=0, n=4, p=3/5)`
1	${}^4C_1$	$\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^1$	${}^4C_1\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^1$	`binom.pmf(k=1, n=4, p=3/5)`
2	${}^4C_2$	$\left(\frac{2}{5}\right)^2\left(\frac{3}{5}\right)^2$	${}^4C_2\left(\frac{2}{5}\right)^2\left(\frac{3}{5}\right)^2$	`binom.pmf(k=2, n=4, p=3/5)`
3	${}^4C_3$	$\left(\frac{2}{5}\right)^1\left(\frac{3}{5}\right)^3$	${}^4C_3\left(\frac{2}{5}\right)^1\left(\frac{3}{5}\right)^3$	`binom.pmf(k=3, n=4, p=3/5)`
4	${}^4C_4$	$\left(\frac{3}{5}\right)^4$	${}^4C_4\left(\frac{3}{5}\right)^4$	`binom.pmf(k=4, n=4, p=3/5)`

$E[X] = 2.4$

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Would you play this game?

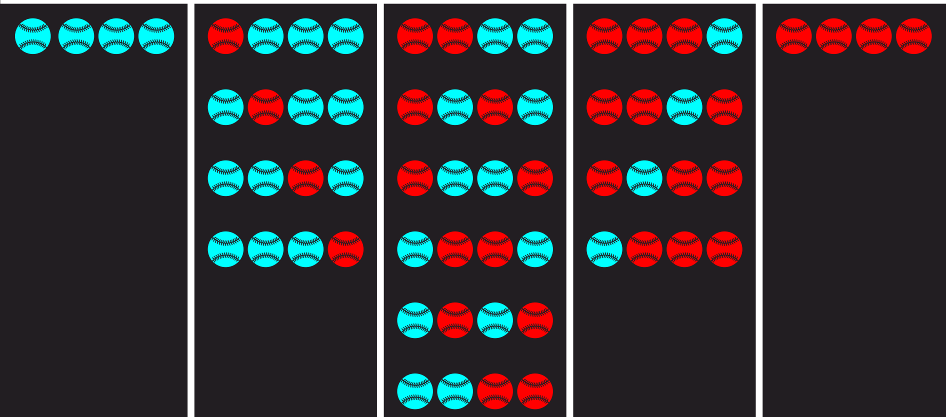
Let "X" denote the number of red balls when you draw 4 balls with replacement

Here, X is an example of what is called a "Random Variable"

Let "Y" be the amount won. This is also another example of a random variable

What are all the outcomes?

0 red	1 red	2 red	3 red	4 red



0 red	1 red	2 red	3 red	4 red
$\frac{2}{5}\frac{2}{5}\frac{2}{5}\frac{2}{5}$	$\frac{2}{5}\frac{2}{5}\frac{2}{5}\frac{3}{5}$	$\frac{2}{5}\frac{2}{5}\frac{3}{5}\frac{3}{5}$	$\frac{2}{5}\frac{3}{5}\frac{3}{5}\frac{3}{5}$	$\frac{3}{5}\frac{3}{5}\frac{3}{5}\frac{3}{5}$
${}^4C_0$	${}^4C_1$	${}^4C_2$	${}^4C_3$	${}^4C_4$

What are all the outcomes for "Y"?

"$Y = 150$" If we get 4 red balls

"$Y = -10$" Otherwise

Y	$P[Y]$	
150	${}^4C_4\left(\frac{3}{5}\right)^4$	0.1296
-10	${}^4C_0\left(\frac{2}{5}\right)^4 + {}^4C_1\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^1 + {}^4C_2\left(\frac{2}{5}\right)^2\left(\frac{3}{5}\right)^2 + {}^4C_3\left(\frac{2}{5}\right)^1\left(\frac{3}{5}\right)^3$	0.8704

$$E[Y] = (150)(0.1296) + (-10) * (0.8704) = 10.736$$

Binomial Distribution

If X is random variable that follows the Binomial distributions with parameters "n" and "p", then

$$P[X = k] = {}^{n}C_{k}\ p^{k}\ (1 - p)^{(n-k)}$$

Event

Expected value

Toss a coin → H → win

→ T

1/2

Random variable X

1 0 1 , 2 0 0 . . . . . ∞

0.5

$E[X] = \sum X \cdot P(X)$,

$P(C=1) \rightarrow$

$P(C=0) \rightarrow$

$$P(X=180) = {}^{200}C_{180}\, P(C=1)^{180}\, P(C=0)^{20}$$

$$\begin{tikzcd}
 & P \arrow[dl] \arrow[dr] & \\
1 & & 0
\end{tikzcd}$$

0.96 0.04

X

Y

$P(X) = 0.$

$P(Y) = 1 - P(X)$

Binomial.

200

$P(X = K)$

$P(X = 180)$