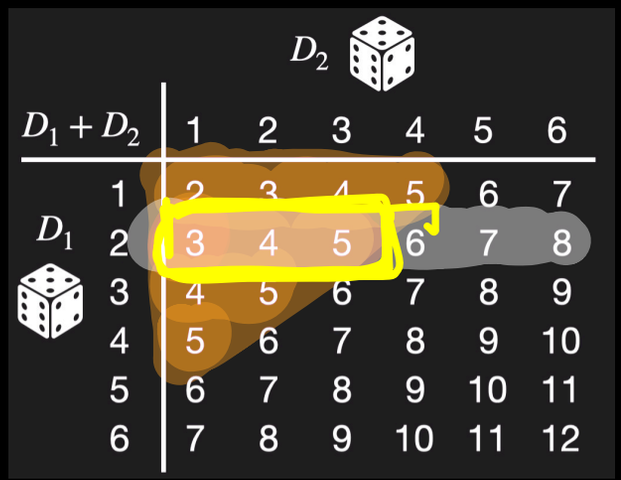
DAY-2

Bayes Theorem



$D_1 + D_2$	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Experiment: Rolling 2 dice

Samplespace: 36 outcomes.

$$\begin{bmatrix} 11 & 12 & 13 & 14 & 15 & 16 \\ 21 & 22 & 23 & 24 & 25 & 26 \\ \vdots & & \vdots & & & \\ 61 & 62 & 63 & 64 & 65 & 66 \end{bmatrix}$$

$$P(D1=2) = 6/36$$

$$P(D1+D2 \leq 5) = 10/36$$

$$P[(D1=2) \cap (D1+D2 \leq 5)] = 3/36$$

Someone rolled 2 dice for me and told me that sum of $D1+D2 \leq 5$.

$D_1 + D_2$	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

D_1 D_2

$$P(D1=2 \mid D1+D2 \leq 5) = \frac{3}{10} = \frac{3/36}{10/36}$$

given

$$\frac{P(D1=2 \cap D1+D2 \leq 5)}{P(D1+D2 \leq 5)}$$

Someone rolled 2 dice for me and told me that sum of $D1+D2 \leq 5$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Conditional Probability

$$\frac{X}{0.7} = \_\_\_ > X$$

$P(A \cap B) = 0.08$ 8%

Wearing Tendulkar.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.08}{0.12} =$$

Summarise

Condition Prob. $P[A|B] = \frac{P[A \cap B]}{P[B]}$

Multiplication Rule. $P[A \cap B] = P[A|B]\,P[B]$

Q Multiplication Rule $P(A \cap B) = P(A).P(B)$????

Only applicable when A and B are Independent.

$P(A \cap B) = P(A|B)\,P(B)$

$P(A \cap B) = P(A)\,P(B)$

$A \not\leftrightarrow B$

D1	D2
✓	✗

Century	Win: False	Win: True	
False	160	154	314
True	16	30	46
	176	184	360

$P(W) = 184/360$

$P(C) = 46/360$

$P(W \cap C) = 30/360$.

$P(C|W) = 30/184$

$$P(W|C) = \frac{P(W \cap C)}{P(C)}$$

$$= \frac{30/360}{46/360}$$

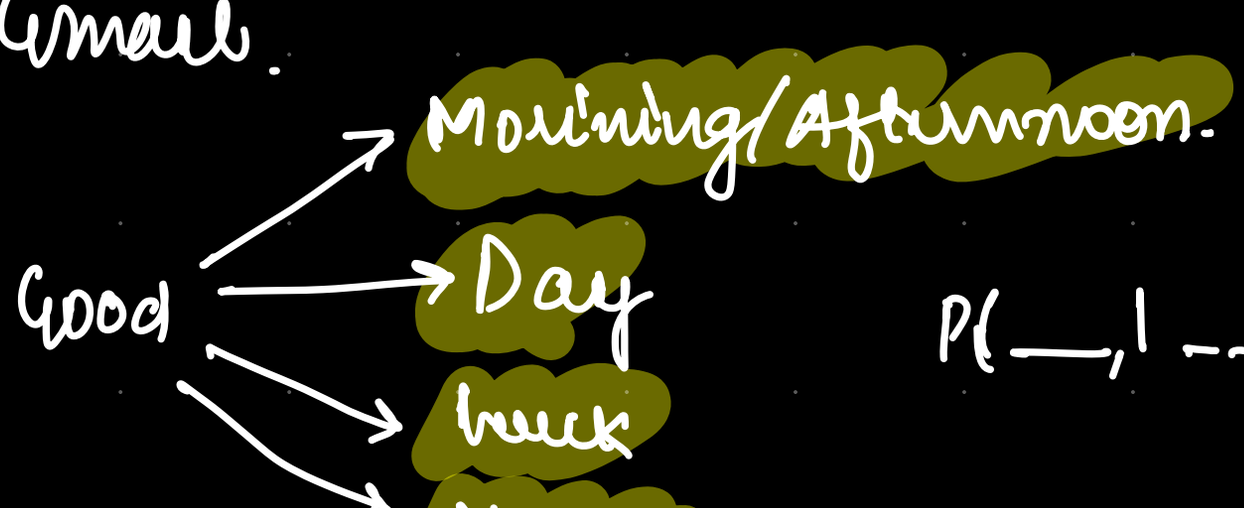
$$= 30/46$$

$$P(C|W) = \frac{P(W \cap C)}{P(W)}$$

$$= \frac{30/360}{184/360}$$

$$= \frac{30}{18}$$

Email.



P(___ | Email Body: ___)

P(Morng. | Good) ↑

P(Day | Good) ↑

P(News | Good) ↑

P(life. | Good) ✗

$$P(W|C) = \frac{P(W \wedge C)}{P(C)} \qquad P(C|W) = \frac{P(W \wedge C)}{P(W)}$$

$$P(W \wedge C) = P(W|C) \times P(C) \qquad P(W \wedge C) = P(C|W)\,P(W)$$

$$P(A|B)\,P(B) = P(B|A)\,P(A) = P(A \wedge B)$$

BAYES THEOREM.

Conditional Probability $P(A|B) = \frac{P(A \cap B)}{P(B)}$

Multiplication Rule. $P(A|B)P(B) = P(A \cap B)$

$P(B|A)P(A)$

Bayes Theorem. $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$

30 faculty members, 5F, 25M, 3F and 12M → Phd.

$P(F) = 5/30 \quad P(M) = 25/30$

$P(F \cap PHd) = 3/30$

$P(M \cap Phd) = 12/30$

Among those who have a Phd, what % are female?

$$\frac{3}{3+12}$$

$$P(F|Phd) = \frac{P(F \cap Phd)}{P(Phd)} = \frac{3/30}{3/30 + 12/30}$$

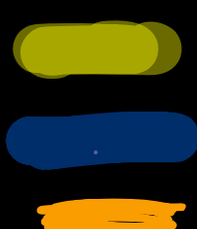
$$P(Phd) = P(Phd \cap M) + P(Phd \cap F)$$

$$= 12/20 + 3/20$$

$$= 15/30 .$$

Phd ∧ M ① Phd ∧ F ②

→ Disjoint. → A + B



M F

Phd.

$$P(B) = P(B \cap A) + P(B \cap A^c)$$

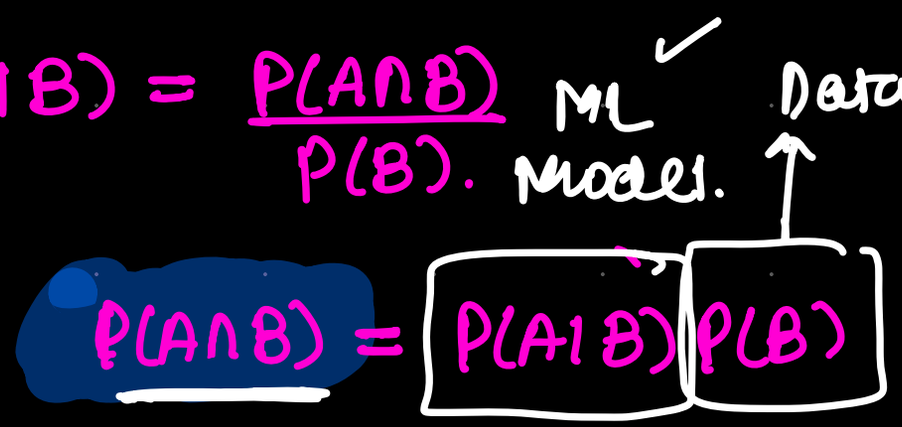
law of total probability.

$A \rightarrow M$
$A^c \rightarrow F$

Exclusively.

Conditional Prob. $P(A|B) = \frac{P(A \cap B)}{P(B)}$

ML ✓ Data ✓ Models.



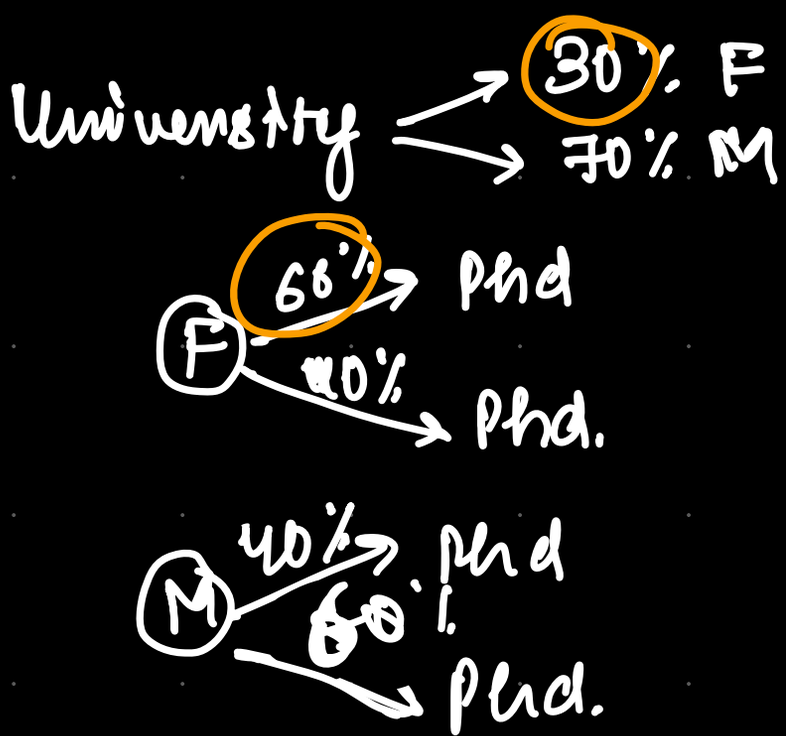
Multiplication Rule - $P(A \cap B) = P(A|B)\,P(B)$

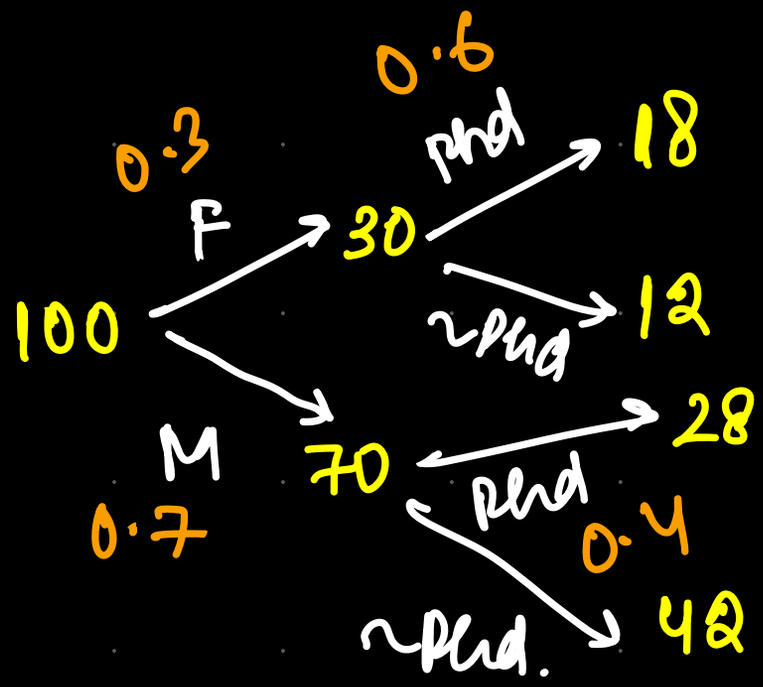
Bayes Theorem - $P(B|A) = \frac{P(A|B)\,P(B)}{P(A)}$

Law of total probability

$$P(B) = P(B \cap A) + P(B \cap A^c)$$

$$= P(B|A)P(A) + P(B|A^c)P(A^c)$$



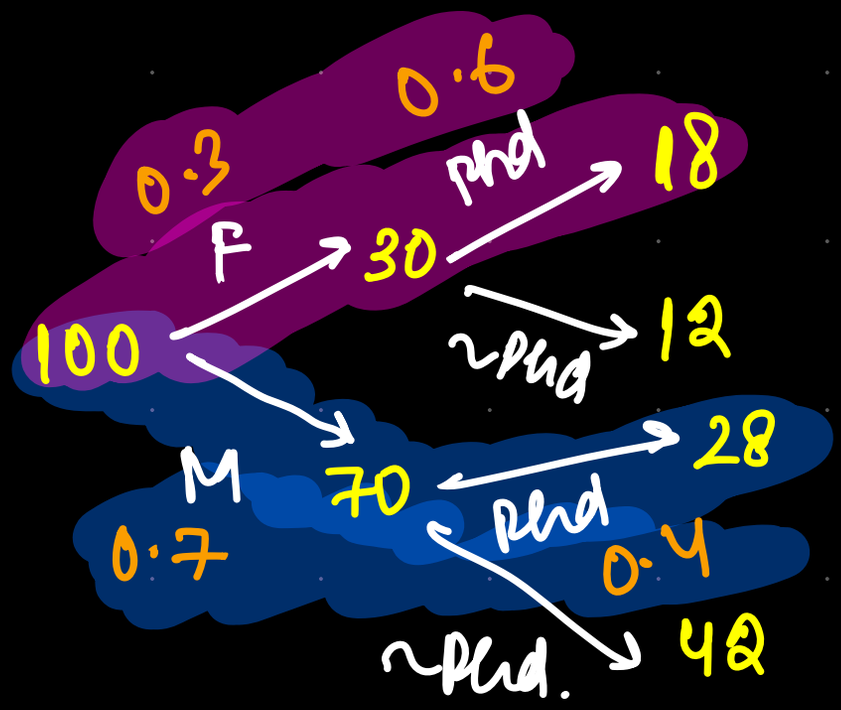


Q What is the P that a randomly chosen member is a F and has a Phd.

$$P(F \cap Phd) = P(F)\,P(Phd \mid F)$$
$$= 0.3 \times 0.6$$
$$= 0.18$$

Q P(Phd) ???

$$\frac{18}{100} + \frac{28}{100}$$



$P(Phd \wedge F) + P(Phd \wedge M) = P(Phd)$

$= 0.18 + 0.28 = 0.3 \times 0.6 + 0.7 \times 0.4$

$= 0.46 = P(F)P(Phd|F) + P(M)P(Phd|M)$

Q What is P that a randomly chosen Phd holder is a female?

P(F | Phd)