

Spacecraft landing

For the first version of our implementation we assume that aerodynamic and gravitational forces of bodies other than the Moon (Planet) are negligible and that the lateral motion can be ignored.

Accordingly, the descent trajectory is vertical, and the thrust vector is tangent to the trajectory.

We also assume that the spacecraft is near the Moon (Planet) so we can assume that the gravity is a constant, in the case of the Moon $g = 1,63$. To keep it simple we use a constant relative velocity of the exhausted gases relative to our spacecraft and the mass rate $m'(t)$ is constrained by $-\mu \leq m'(t) \leq 0$, where μ is a constant and gives the maximum rate of change of mass due to burning the fuel.

Notation:

- t is time
- $m(t)$ is the mass of the spacecraft, which varies as fuel is burned.
- $m'(t)$ is the rate of change of mass, constrained by $-\mu \leq m'(t) \leq 0$
- $g = 1,63$, the gravitational constant near the Moon
- k is a constant, the relative velocity of the exhausted gases with respect to the spacecraft
- $T(t) = -km'(t)$, the thrust
- $h(t)$ is the height, with $h(t) \geq 0$
- $v(t) = h'(t)$, the velocity of the spacecraft
- $u(t) = m'(t)$ the control function

The descent trajectory of our spacecraft is vertical and the thrust vector is perpendicular to the ground.

Equations of Motion

According to Newton's second law

$$m(t)h''(t) = -gm(t) + T(t) = -gm(t) - km'(t)$$

With our Notations:

$$h'(t) = v(t)$$

$$v'(t) = -g - k u(t) / m(t)$$

$$m'(t) = u(t)$$