IE 525 Part II HW1 Name_____ Score_____

Submit a pdf file on www.gradescope.com before the due date. Append your codes. Your homework must be finished independently.

(10 points) Consider a European vanilla call option on a stock index with strike price $K = 1870$ and maturity $T = 1/52$ (one week). Assume that the index follows a geometric Brownian motion in the risk neutral world:

$$S_T = S_0 \exp\left(\left(r - q - \frac{1}{2}\sigma^2\right)T + \sigma B_T\right).$$

The current value of the index is $S_0 = 1868.99$. The risk free interest rate is $r = 0.3866\%$. The dividend yield of the stock index is $q = 2.32\%$. The volatility is $\sigma = 29.79\%$.

1. (1 points) Compute the call price using the Black-Scholes formula.
2. (4 points) Write a C++ program to compute the call price using Monte Carlo simulation. Your program should output the **sample size, the estimated price, the estimated standard error, the 95% confidence interval, and the computational time in seconds**. Construct a table showing the quantities in bold in the above for sample sizes {1000, 10,000, 100,000, 1,000,000, 10,000,000, 100,000,000}.
3. (4 points) Write a C++ program to compute the call price using Monte Carlo simulation with antithetic variates. Construct a table comparing the standard approach in Part 2 and the antithetic approach in Part 3. Your table must contain the following:
    a. An increasing sequence of sample sizes {4000, 40,000, 400,000, 4,000,000, 40,000,000}.
    b. For each sample size, the call price computed using the antithetic approach, the estimated standard error, the 95% CI, the total computational time in seconds, the efficiency measure $efficiency = (standard\ error)^2 \times total\ computational\ time$. Report the same for the standard approach side by side.
4. (1 point) Which method is more efficient? Describe what you have done to improve the speed of your implementation.