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The condition for dynamic stability

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Abstract

The well-known condition for standing stability in static situations is that the vertical projection of the centre of mass (CoM) should be within the base of support (BoS). On the basis of a simple inverted pendulum model, an extension of this rule is proposed for dynamical situations: the position of (the vertical projection of) the CoM plus its velocity times a factor $\sqrt{l/g}$ should be within the BoS, l being leg length and g the acceleration of gravity. It is proposed to name this vector quantity 'extrapolated centre of mass position' (XcoM). The definition suggests as a measure of stability the 'margin of stability' b , the minimum distance from XcoM to the boundaries of the BoS. An alternative measure is the temporal stability margin τ, the time in which the boundary of the BoS would be reached without intervention.

Some experimental data of subjects standing on one or two feet, flatfoot and tiptoe, are presented to give an idea of the usual ranges of these margins of stability. Example data on walking are also presented.

Keywords: Balance; Inverted pendulum model; Base of support; Center of mass; Center of pressure

1. Introduction

In all human movements control of balance is an essential side issue. This being accepted, two questions emerge: what condition is to be fulfilled for balance to be maintained, and how good is the balance in a certain situation. The standard answer to the first question is, that the vertical projection of the body centre of mass (CoM) should be within the base of support (BoS) (Shumway-Cook and Woolacott, 1995; Winter, 1995a). The 'base of support', or 'supporting area', is defined as the possible range of the centre of pressure (CoP), the origin of the ground reaction vector.[1]

It is the merit of Pai and his group (Iqbal and Pai, 2000; Pai and Patton, 1997) to have brought to the attention, that this condition is insufficient in dynamical situations. The velocity of the CoM should also be accounted for. Even if the CoM is above the BoS, balance may be impossible if CoM velocity is directed outward. The reverse is also possible: even if the CoM is outside the BoS, but its velocity directed towards it, balance can be achieved. They have supported their point by simulations with a two-segment (Pai and Patton, 1997) and a four-segment body model (Iqbal and Pai, 2000). In the following we will show that many of their results can be predicted by a simple mechanical reasoning. The result to be derived also suggests a measure for the degree of stability.

2. Theory

The assumptions used in our model are those of the well-known inverted pendulum model of human standing balance (Geurtsen et al., 1975; Winter, 1995b): (1) the balance problem can completely be described by the

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[1] This statement is only true in approximation. In two dimensions it is possible to represent the ground reaction force as a single force, acting at a definite CoP. In three dimensions such a representation requires an additional couple. In this paper, we will assume that this couple is directed vertically, around the z-axis. In most movements the moment of this couple is small and does not influence stability. For a detailed discussion see Zatsiorsky (2002, Chapter 1).

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movement of the whole-body CoM, (2) the distance l from the axis of rotation to the CoM remains constant, and (3) the excursions of the CoM are small with respect to l. The gravity force vector $m\boldsymbol{g}$ is located at the CoM, pointing vertically downward, Fig. 1. The pressure on the feet can be represented by a single ground reaction force vector $\boldsymbol{F}_r$, located at the centre of pressure (CoM). In animate objects the position of the CoP can be voluntarily varied by means of muscle action, in the sagittal plane by the ankle plantar- and dorsiflexors ('ankle strategy'), in the frontal plane by the hip abductors (Winter, 1995a). CoP position is confined to a limited area, the 'base of support' (BoS) or 'supporting area', loosely equal to the area below and between the feet (in two-feet standing). This is because the ground reaction force is in fact the resultant of a pressure distribution under the foot or feet.

For the body modelled as an inverted pendulum, Fig. 1, Euler's equation holds:

$$\sum \boldsymbol{M} = I\alpha \tag{1}$$

with respect to the arbitrary origin of the coordinate system, at ground level. For a pendulum with mass m and effective length l, $I = ml^2$. When the vertical projection of the CoM is denoted as x and the position of the CoP as u, (1) can be written as (Winter, 1995b)

$$(u - x)mg = I\alpha \approx -ml^2 \frac{\ddot{x}}{l}$$

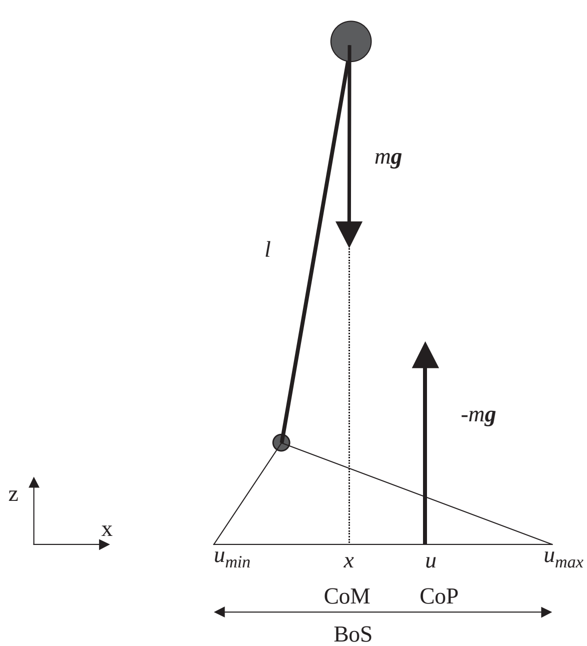

Fig. 1. Inverted pendulum model. The body is modelled as a single mass m balancing on top of a stick with length l. Indicated are the Centre of Pressure (CoP) u, the location of the effective ground reaction force, and the vertical projection of the Centre of Mass (CoM) x. The Base of Support (BoS) is the area to which the CoP is confined, and roughly equals the area of the footsole, see Fig. 3.

or

$$u - x = -\frac{l}{g}\ddot{x} = -\frac{\ddot{x}}{\omega_0^2}. \tag{2}$$

As long as the CoP is kept beyond the CoM, with respect to the rotation centre at the ankle, the body is accelerated back to the upright position. In Eq. (2) a new parameter $\omega_0 = \sqrt{g/l}$ has been introduced. It is equal to the (angular) eigenfrequency of a hanging, non-inverted, pendulum of length l and it has the dimension of time^{-1}.

In the following we will discuss what happens when the CoM has an initial velocity v_0. When, in the situation as depicted in Fig. 1, the CoM has a sufficient forward velocity, one may imagine that in some cases the backward acceleration is not sufficient to prevent that the CoM will eventually advance beyond the CoP. When this would happen, the acceleration will change sign, which means that the CoM will now be accelerated further forward, an evidently instable situation. To investigate this problem, we will solve the linear second-order differential equation (2) for the case that CoP position u remains constant, with position x_0 and velocity v_0 of the CoM as initial conditions. The solution is, cf. (Townsend, 1985)

$$x(t) = u + (x_0 - u)\cosh(\omega_0 t) + \frac{v_0}{\omega_0}\sinh(\omega_0 t) \tag{3}$$

in which 'sinh' and 'cosh' are the hyperbolic sine and cosine functions $\sinh(x) = \frac{1}{2}(e^x - e^{-x})$ and $\cosh(x) = \frac{1}{2}(e^x + e^{-x})$. The condition that the CoM will not pass the CoP means that $x(t) \leqslant u$ for any t. From (3) it follows that this is the case if:

$$(x_0 - u)\cosh(\omega_0 t) + \frac{v_0}{\omega_0}\sinh(\omega_0 t) \leqslant 0$$

$$\text{or} \qquad (u - x_0) \geqslant \frac{v_0}{\omega_0}\tanh(\omega_0 t). \tag{4}$$

As $-1 < \tanh(\omega_0 t) < 1$ for any t, this condition reduces to

$$x_0 + \frac{v_0}{\omega_0} \leqslant u. \tag{5}$$

As long as u can sufficiently fast be moved outward, this instability can be remedied. As soon as the quantity $x_0 + v_0/\omega_0$ exceeds the boundary of the BoS, u_{max}, however, stability can no longer be maintained by means pertaining to the inverted pendulum model. (A symmetrical argument can be followed for the case when CoM and CoP are both behind the axis of rotation.) The condition for static stability 'the projection CoM should be within the BoS', is thus not sufficient in dynamic situations. It should hold: 'The quantity $x_0 + v_0/\omega_0$ should be within the BoS'.

This expression (5) can be expanded into the two-dimensional form. Denoting the projection of the CoM on the ground plane as $\boldsymbol{r} = (x, y)$ and its velocity $\boldsymbol{v} = \mathrm{d}\boldsymbol{r}/\mathrm{d}t$, the condition for stability can now be stated as

$$\boldsymbol{r} + \boldsymbol{v}/\omega_0 \quad \text{should be within the BoS.} \tag{6}$$

Eq. (5) also suggests a measure for dynamical stability, viz. the 'margin of stability' b:

$$b = |u_{\max} - (x + v/\omega_0)|. \tag{7}$$

For the two-dimensional case b can be calculated as the shortest (perpendicular) distance between the position of $\boldsymbol{r} + \boldsymbol{v}/\omega_0$ and the boundaries of the BoS, see appendix for details. As a name for the vector quantity $\boldsymbol{r} + \boldsymbol{v}/\omega_0$ we propose 'position of the extrapolated centre of mass' (XcoM), because the CoM trajectory is extrapolated in the direction of its velocity.

The units of b and XcoM are of a distance (m or cm). An alternative interpretation is in terms of the minimum momentum needed to disturb the balance. It can be seen from (5) that the XcoM will reach the BoS boundary if we add an extra velocity $\Delta v = \omega_0 b$ in the proper direction. Thus when an impulse $m\Delta\boldsymbol{v}$ with a magnitude at least equal to

$$m\Delta v = m\omega_0 \cdot b \tag{8}$$

is applied to the CoM in the direction of the nearest boundary of the BoS, the CoM will pass the BoS, a potentially unstable situation. In other words: the margin of stability is proportional to the impulse needed to unbalance a subject.

2.1. Possible situations

Summing up, there are four quantities of interest (simplified to the two-dimensional case): CoM position x, CoP position u, XcoM position $x + v/\omega_0$, and the BoS interval $(u_{\min}, u_{\max})$. In the following we will assume that the BoS remains constant.When $v > 0$, there are three possibilities

Case a:

$$x < x + v/\omega_0 < u \quad < u_{\max},$$
$$\text{i.e.} \quad \text{CoM} < \text{XcoM} < \text{CoP} < \text{BoS}_{\max}.$$

In this case there is presently no problem and no action is needed. The CoM will never reach the present CoP position, but will return timely. After this, however, the velocity will change sign and the situation b (with $v < 0$) will have to be met. The inverted pendulum is in principle unstable, thus some action is needed from time to time.

Case b:

$$x < u < x + v/\omega_0 < u_{\max},$$
$$\text{i.e.} \quad \text{CoM} < \text{CoP} < \text{XcoM} < \text{BoS}_{\max}.$$

In this case the CoM will pass the CoP after some time, and it will then be accelerated forward. Here thus action is needed by bringing the CoP forward to a position in front of the XcoM. The time for this action is limited, however. The XcoM will reach the BoS in a 'time-to-contact' τ, about equal to

$$\tau \approx \frac{u_{\max} - (x + v/\omega_0)}{v} = \frac{b}{v}. \tag{9}$$

(The expression is not exact, in fact the differential equation (2) should be solved here for non-constant $u(t)$.)

Case c:

$$x + v/\omega_0 > u_{\max},$$
$$\text{i.e.} \quad \text{XcoM} > \text{BoS}_{\max}.$$

This is the unstable case: no movement of the CoP can prevent that the CoM will pass outward of the BoS. Possible actions to prevent a fall are to change the BoS by making a step or to move the trunk or the arms with respect to the CoM, actions that are not described within the inverted pendulum model (Otten, 1999). If τ is calculated in this case, one obtains negative values.

2.2. The equivalent pendulum length

In the above derivation no attention as yet has been given to the actual value of the effective pendulum length l. For the for-aft movement of humans balancing about the ankle, Geurtsen et al. (1975) have shown that it can be modelled in good approximation as the motion of a single pendulum and they give an expression for the length of this equivalent pendulum. Adopting the mass and length data as collected by Winter (1979), this results in $l = 1.20$ or 1.24 times trochanteric height, depending whether the subject keeps his trunk vertical, or moves it at the same angle as the legs, respectively. For movements in the frontal plane, Massen and Kodde (1979) give an expression resulting in $l = 1.34$ times trochanteric height.

As a first support of the theory presented here, in Fig. 2 the stability margins found by Pai and Patton (1997) (their Fig. 3) are given together with the predictions according to the above. Next to this, some results will be presented in which values for b and τ are given for postures of decreasing stability: standing on two feet, standing on one foot and standing on tiptoe on one foot. Finally, a few example data of b in walking will be presented.

3. Experimental methods

3.1. Subjects

Subjects for the standing study were 10 healthy subjects, 5 male, 5 female. Their age was 23.3 (1.3) years (mean, SD) , body mass 74.1 (12.4) kg, leg length

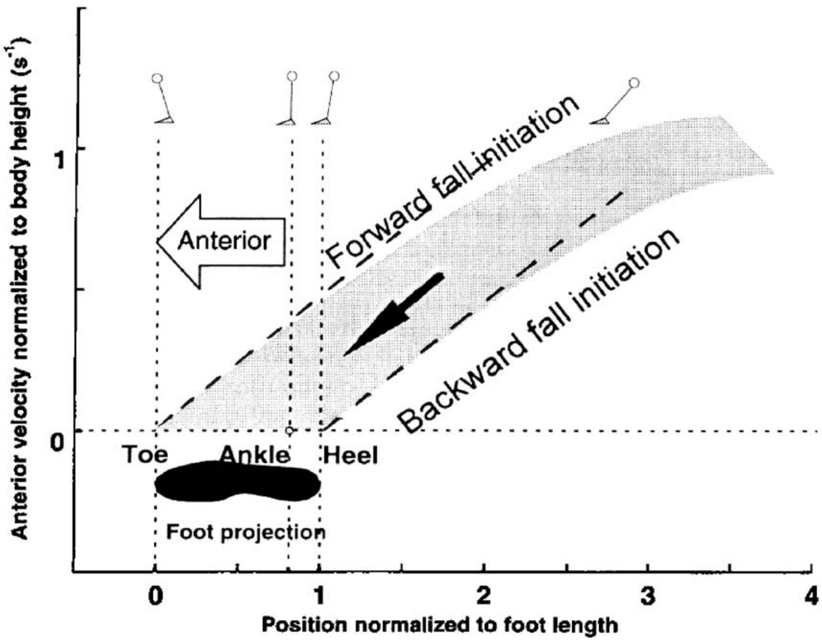

Fig. 2. Feasible horizontal centre of mass velocity-position (shaded diagonal band) for terminating anterior movement of a simple pendulum connected to a stationary base of support, calculated from the model of Pai and Patton (1997), their Fig. 3. Dashed lines: prediction from our model. Anthropometric data from their paper, Table 1.

0.936 (0.06) m. Informed consent was obtained according to rules of the local Ethical Committee.

Data for walking are given as an example, for one subject 23 years, mass 85 kg, leg length 1.06 m.

3.2. Force plate recordings

The CoP was recorded by means of a Bertec $40 \times 60\,\mathrm{cm}^2$ force plate. At first the BoS was determined by recording the extreme boundaries of the CoP. The subject stood on one foot and was asked to shift his weight as much as possible laterally, anteriorly, medially and posteriorly. He was allowed to lean on a support to maintain balance. In this way the boundary of the BoS is recorded as a loop of the CoP. Fig. 3 gives this loop, additionally recorded by a foot pressure recording system (RSscan Footscan, Romberg plot) so that the relation between BoS area and foot surface can be seen. For further processing BoS circumference (recorded by the force plate) was fitted by straight lines (Fig. 4). The position of the foot was marked on the force plate. The recordings during standing on one foot and on tiptoe were made with the preferred foot in the same position as in the determination of the BoS at the guide of the markers on the plate. For standing on two feet, the feet were positioned in the standardised posture, with the feet put against a wedge of 15° and the heels 10 cm from the top of the wedge. The BoS was taken as the area between the lateral boundaries of left and right feet, both positioned in the described way (Figs. 4a and c).

The x- and y-coordinates of the CoM were determined by a method which combines both the force plate CoP recordings and the horizontal components of the ground reaction force (Zatsiorsky and King, 1998).

CoP recordings during walking were made on a treadmill with built-in force transducers (Verkerke et al., in press). The CoM trajectory was determined by means of optokinetic registrations by means of an ELITE system (Ferrigno and Pedotti, 1985) and a 15-segment body model using the anthropometric data of Winter (1979).

4. Results

Fig. 4(a–d) shows the BoS and the trajectory of the XcoM in 30 s of quiet standing on two feet, one foot and on tiptoe, on two and one foot, respectively. Table 1 gives the average data of BoS diameter, the r.m.s. position variation of CoP and XcoM, and the stability margin b. It is seen that b becomes smaller going from Fig. 4(a–d) because of two effects: the motion of the XcoM increases and the area of the BoS decreases. Fig. 5 gives an example of the temporal variation of CoP, CoM and XcoM for a subject standing on tiptoe. In this case XcoM and CoM position may differ up to 1 cm.In more quiet standing the differences are less, about 1 mm in standing on two feet.

A recording of lateral CoP, CoM and XcoM position in walking, Fig. 6, shows that here the XcoM trajectory considerably deviates from that of the CoM. In fact the CoP of the stance foot is only some 2.5 cm lateral to the XcoM. Although the border of the foot BoS may have been 1–2 cm more lateral to the actual measured CoP,

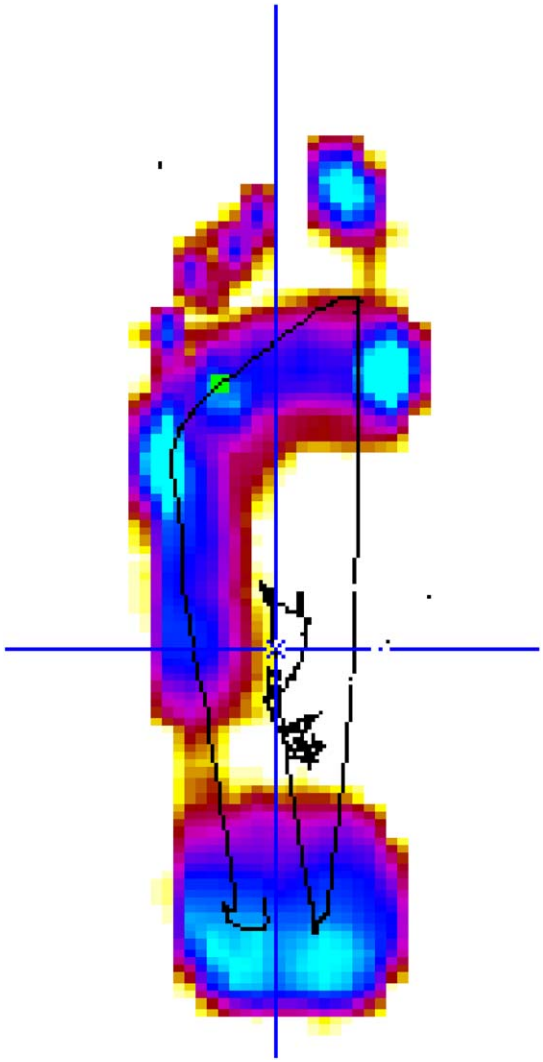

Fig. 3. Experimental determination of the BoS area. Maximum foot pressure is recorded by a RSscan™ Footscan system, while the subject leaned as far as possible to all sides. He was allowed to hold on to a support. The footscan system calculates the CoP trajectory from the pressure data, but the same trajectory is obtained from a force plate recording. For processing, the BoS boundary is approximated by a set of straight lines, cf. Fig. 4. Same subject as in Fig. 4.

this suggests that b is in the range of only 2–3 cm in walking.

5. Discussion

On the basis of mechanical arguments we have introduced here a new spatial variable, the XcoM, which enables to formulate a stability condition (6) which is valid for both static and dynamic situations. In our opinion, this new stability condition can be applied successfully in many experimental situations, especially those in which balance is disturbed by expected or unexpected movements. Examples are the model work of Pai and Patton (1997), discussed above, and their experiments in Patton et al. (1999).

In earlier work the variable critical for stability is as a rule taken to be either the CoM, the statical approximation (e.g. Nagy et al., 1994; Shumway-Cook and Woolacott, 1995), or the CoP, probably because it is relatively easy to measure by means of a force plate, see Prieto et al. (1996) for a review . Now CoM, CoP and XcoM are indeed closely related. If CoM position is denoted $\boldsymbol{x}$ XcoM and CoP positions are, respectively

$$\text{XcoM}: \qquad \boldsymbol{x} + \frac{\dot{\boldsymbol{x}}}{\omega_0}, \tag{10}$$

$$\text{CoP}: \qquad \boldsymbol{x} - \frac{\ddot{\boldsymbol{x}}}{\omega_0^2}. \tag{11}$$

Eq. (11) is found by rearranging (2) and is valid only under the conditions of the inverted pendulum model. In standing, velocities and accelerations turn out to be relatively small and the differences in r.m.s. values between CoM, XcoM, and CoP are minor (Table 1) but in walking (Fig. 5) or in disturbed standing (Pai and Patton, 1997; Patton et al., 1999) the differences are considerable and probably relevant.

5.1. Spatial stability margin

Most current measures of stability are related to the excursion of the CoP (Prieto et al., 1996). A small CoP excursion is then considered as a good standing balance. For human standing this relation is indeed found experimentally, cf. Figs. 4(a–d) and Table 1. A strange consequence of using CoP excursion as a measure of stability is that a broomstick, which can stand on its end without motion of CoP or CoM (when the end is cut off squarely and it is put down carefully) thus would be an example of perfect balance, much better than a man standing on both feet. Only recently it has been proposed to relate CoP or CoM excursions to the BoS area (Patton et al., 2000; Popovic et al., 2000; Wegen et al., 2002). The margin of stability b, as proposed here, is more or less similar to the spatial stability margins proposed by these authors, but it accounts for the essential effect of CoM velocity, it can easily be visualised (cf. Figs. 4–6) and it has the advantage of a biomechanical meaning, as it bears direct relation to the minimal impulse needed to bring the subject out of balance, see Eq. (8). A major advantage is that it can be applied not only for standing but for all postures in which the body is more or less erect. It can be predicted, e.g. that a disturbance during walking has maximum effect when the impulse is directed outward and is timed early in single stance (Ferrigno and Pedotti, 1985; Hill et al., 2001), cf. Fig. 5. A possible application in sports might be the jumping down after a gymnastic exercise at the bridge or the high bar, where the gymnast has a

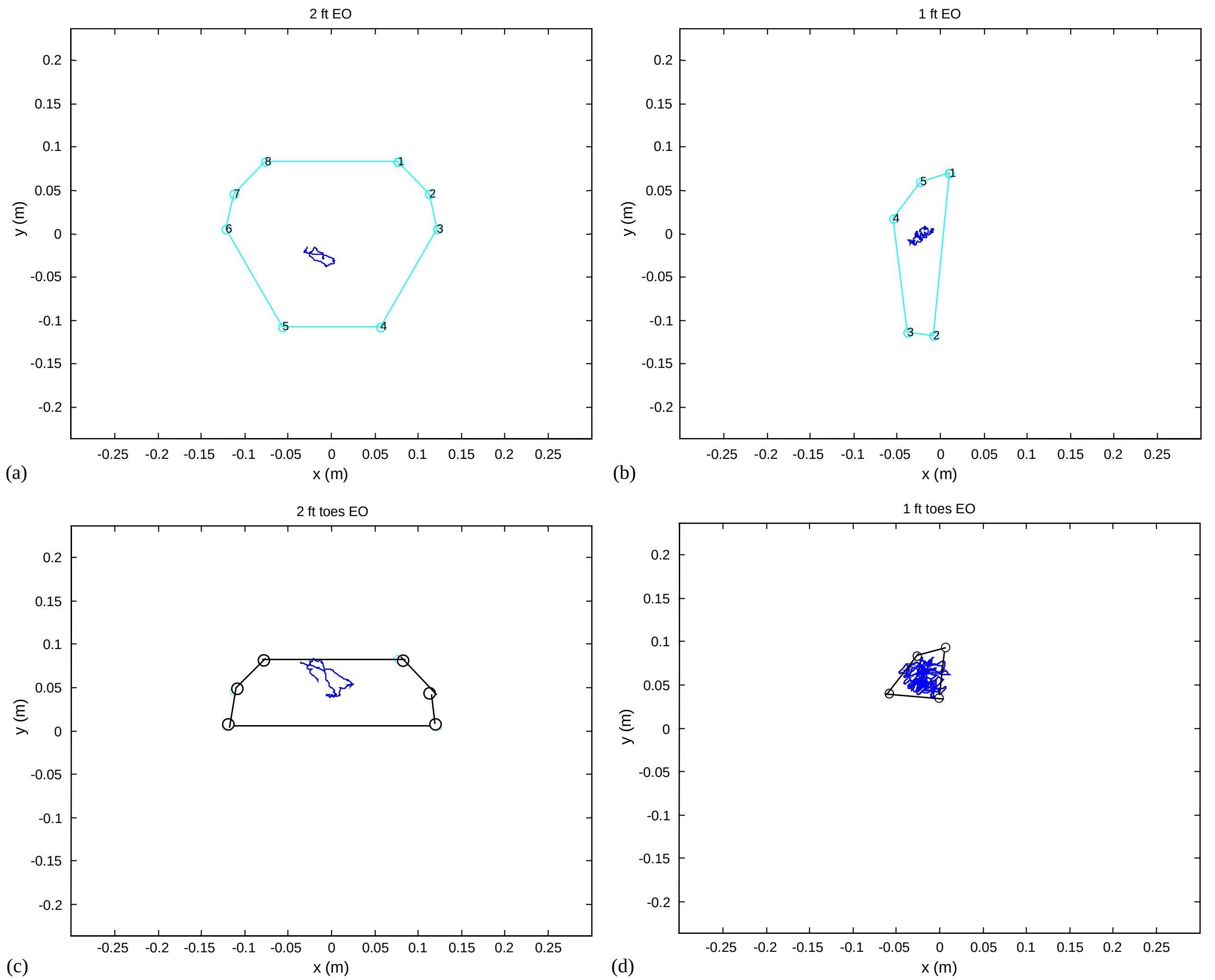

Fig. 4. Trajectory of the XcoM (thick line) and outline of the BoS (thin straight lines) for one subject (f, 20 years, 58 kg, leg length 0.89 m) (a) standing on two feet, (b) standing on one foot, (c) standing on tiptoe on two feet, and (d) standing on tiptoe with one foot. Eyes were open.

Table 1
Stability in standing. b_{min}: minimum spatial margin of stability, i.e. minimum distance of Xcom to BoS. τ_{min}: minimum time-to-boundary of BoS. Diameter BoS has been defined as distance from average position CoM to BoS margin. Sway of CoP and XcoM has been given as r.m.s. value $= \sqrt{\text{mean}(x^2 + y^2)}$. Average values (SD) for 10 healthy young subjects.

Stance	Condition	Diam BoS (mm)	b_{min} (mm)	τ_{min} (s)	CoP sway (mm r.m.s.)	XcoM sway (mm r.m.s.)	CoM sway (mm r.m.s.)	V CoM (mm.s^{-1} r.m.s.)
Two feet	Eyes open	76 (11)	64.2 (8)	14.1 (3)	5.0 (2)	5.0 (2)	4.9 (2)	2.6 (1)
	Eyes closed		64.7 (10)	12.9 (5)	5.2 (2)	4.4 (1)	4.4 (1)	2.8 (1)
One foot	Eyes open	22 (5)	15.5 (6)	2.3 (1)	8.1 (3)	5.3 (3)	5.0 (3)	5.1 (1)
	Eyes closed		7.3 (6)	0.4 (0.5)	15.9 (2)	10.2 (2)	9.0 (2)	14.3 (4)
2 ft. toes	Eyes open	16 (4)	5.6 (12)	1.3 (6)	7.7 (2)	8.6 (4)	8.5 (5)	4.4 (1)
	Eyes closed		12.2 (11)	0.7 (3)	16.5 (2)	11.8 (5)	11.4 (5)	10.1 (3)
1 ft. toes	Eyes open	15 (4)	−9.3 (14)	−1.8 (2)	10.6 (3)	8.3 (3)	6.1 (2)	17.7 (7)

considerable horizontal velocity. Movements in which the distance from foot to CoM shows major changes will probably not follow the rules put forward here, e.g. (Iqbal and Pai, 2000).

A related measure has been used by Babic et al. (2001) and Karčnik and Kralj (1997). They defined a 'critical speed' and a 'relative index of dynamic stability' based on this speed. We see a minor advantage in position-

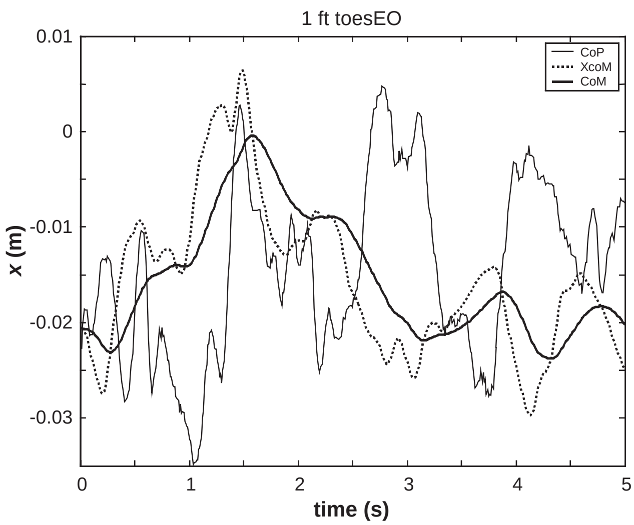

Fig. 5. Lateral position of CoP (thin line) CoM (thick line), and XcoM (dotted) vs. time in standing on tiptoe. Part of the registration of Fig. 3d.

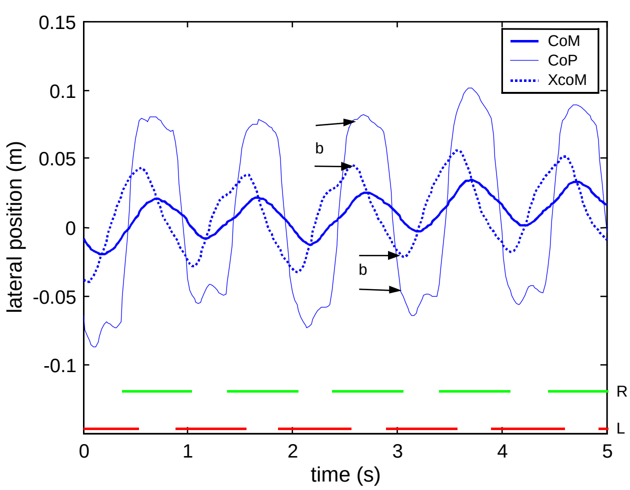

Fig. 6. Lateral position of CoP (thin line) CoM (thick line), and XcoM (dotted) vs. time in walking at 1.5 m/s on a treadmill. For subject data see Methods. The margin of stability b at the instant of contralateral toe-off has been indicated with two arrows. Below: foot contacts.

related variables XcoM and b in that they can easily been visualised and related to CoM and CoP position.

5.2. Temporal stability margin

Next to a spatial stability margin, a temporal margin, the time-to-boundary τ, i.e. the time needed to reach the boundary of the BoS, may be useful for quantifying the quality of balance. Such measures have been proposed previously for the CoP (Patton et al., 2000; Slobounov et al., 1998; Wegen et al., 2002). The equivalent variable for the XcoM can also be given (9). When comparing our τ-data for the XcoM with the CoP data from literature, a considerable discrepancy is seen. In quiet standing Wegen et al. (2002) reported τ values around 0.5 s, while our Xcom data give values over 10 s (Table 1). The reason for the difference will be that the velocity of CoP movement is considerably higher than that of the CoM., cf. (9) and (11), which gives rise to wrong estimates.

5.3. Limitations of the model

From the theory it might be inferred that b should in all cases be positive, that the XcoM should always be within the BoS. It is not this strict. In the theory described here, we already made the reservation that the inverted pendulum model was assumed. Next to the movement of the whole-body CoM, which is the only variable in inverted pendulum models, the segments can also move with respect to this CoM, and the acceleration of these movements can give an appreciable contribution to the moment equation (Zatsiorsky and King, 1998). By using these mechanisms, e.g. by bending the hips or by moving the arms, a disbalance with the XcoM outside the BoS, thus with negative b, can still be restored (Otten, 1999). The data of Table 1 give an indication of the range of b values to be expected in some standing positions of various stability. Standing on tiptoe, e.g. shows on average a negative b and indeed major trunk and arm movements were observed in that posture.

In quiet two-legged standing, CoM velocities turned out to be quite low and as a consequence XcoM amplitude is not much larger than of CoM, some 10% see also Table 1. In tiptoe standing, Fig. 5, the difference can be up to 1 cm.In walking this difference between XcoM and CoM can be up to 4 cm, Fig. 6. In that case the XcoM has not only a markedly higher amplitude than CoM, but is also about 90° shifted in phase. While CoM keeps a distance of at least 5 cm to the CoP, suggesting that balance is very stable, it is seen that XcoM and CoP approach each other quite closely, around 2.5 cm at the instant of foot contact. This suggests that balance in walking is more critical than would be realized when only CoM trajectory is considered and that, provided that the assumptions of the inverted pendulum model are valid in the case, the margin of stability b as proposed here (7) may indeed be a useful measure of stability in dynamical situations.

Appendix

Distance of a point to a line

The distance of a point $\boldsymbol{r} = (x, y)$ to a line between points $\boldsymbol{r}_1 = (x_1, y_1)$ and $\boldsymbol{r}_2 = (x_2, y_2)$ is easiest calculated by means of the cross product

$$b = \frac{(\boldsymbol{r}_2 - \boldsymbol{r}_1) \times (\boldsymbol{r} - \boldsymbol{r}_1)}{|\boldsymbol{r}_2 - \boldsymbol{r}_1|}. \tag{A.1}$$

For the two-dimensional case this can be written as

$$b = \frac{(x_2 - x_1)(y - y_1) - (y_2 - y_1)(x - x_1)}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}. \tag{A.2}$$

When the BoS is staked out by points $\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_N$ in clockwise order and $\boldsymbol{r}(t)$ is the position of the XcoM (A.2) gives distances positive for inside and negative for outside points. The $b_{\min}$ as given in Table 1 is the over all minimum of all $b_i(t)$ for these $i = 1, \ldots, N$ line segments.

The velocity normal to the line between $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ is calculated in a similar way as

$$v_{1,2} = \frac{(\boldsymbol{r}_2 - \boldsymbol{r}_1) \times \boldsymbol{v}}{|\boldsymbol{r}_2 - \boldsymbol{r}_1|}. \tag{A.3}$$

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