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→ GCD and LCM

→ GCD (Greatest Common Divisor) also known as HCF (Highest Common Factor)

e.g: $a = 4$, $b = 12$

Do prime factorization

$$4 \rightarrow 2^2 \times 3^0$$

$$12 \rightarrow 2^2 \times 3$$

Now, for GCD, take lowest power elements from both (4, 12)

$$\therefore \text{GCD} = 2^2 \times 3^0 \Rightarrow 4$$

↓ ↓
Lowest in 2 Lowest in 3

Now, for LCM (Lowest Common Multiple) take highest power elements from both (4, 12)

$$\therefore \text{LCM} = 2^2 \times 3 \Rightarrow 12$$

e.g: $a = 12$, $b = 18$

$$12 \rightarrow 2^2 \times 3$$

$$18 \rightarrow 2 \times 3^2$$

$$\text{GCD} = 2 \times 3 \Rightarrow 6$$

$$\text{LCM} = 2^2 \times 3^2 \Rightarrow 36$$

→ R/P b/w GCD and LCM :

Multiply all Prime fac. divide them with lowest number of both of them (GCD).

$$\Rightarrow \frac{2^2 \times \cancel{2} \times \cancel{3} \times 3^2}{2 \times 3} \Rightarrow 36$$

$$\therefore \text{R/P is } \Rightarrow \boxed{\frac{a \times b}{\text{GCD}} = \text{LCM}}$$

So, now if we know GCD, this formula will calculate LCM in $O(1)$.

→ Another method to find GCD, Long division method.

e.g: $a = 12$, $b = 18$

$$\begin{array}{r} 12 \overline{) 18} \quad 1 \\ \underline{12} \\ 6 \end{array} \quad \begin{array}{r} 6 \overline{) 12} \quad 2 \\ \underline{12} \\ 0 \end{array}$$

This is GCD ← 6

→ We generally use this Long Division Method as recursive approach in coding to find GCD.

→ Recursive function to find GCD T.C = $O(\log(n))$

```
INT GCD (INT A, INT B)
{
    IF (B == 0) RETURN A;
    RETURN GCD(B, A % B);
}
```

→ Inbuilt function to find GCD: T.C = $O(\log(n))$

`cout << -- GCD(12, 18) << "12";`

→ Trick to find minimum fraction of two numbers:

e.g: $a = 12$ $b = 18$

$\frac{12}{18} = \frac{6}{9} = \frac{2}{3} \Rightarrow$ We can get this from:

$$\frac{a}{b} = \frac{a / \text{GCD}(a, b)}{b / \text{GCD}(a, b)}$$