

→ BINARY EXPONENTIATION : RECURSIVE METHOD 47

- Why do we need binary exponentiation, we can simply use inbuilt `pow()` method?
- The inbuilt `pow()` method calculates in double, ~~and double~~, and double can give precision errors.

→ The other option is, we can use a for loop to compute ans in a variable but it will take $O(n)$ time.

But we can solve that problem in $O(\log(n))$ time using Binary exponentiation

→ Suppose we have to find 2^{16} , we can achieve this like this:

$$2^{16} \rightarrow 2^8 \times 2^8$$

$$2^8 \rightarrow 2^4 \times 2^4$$

$$2^4 \rightarrow 2^2 \times 2^2$$

$$2^2 \rightarrow 2^1 \times 2^1$$

$$a = 2, \quad b = 16$$

∴ In 4 steps we have got our ans and its T.C is $O(\log(b))$.

→ Now for odd power:

$$a = 3, \quad b = 13$$

$$3^{13} \rightarrow 3 \times 3^{12}$$

$$3^{12} \rightarrow 3^6 \times 3^6$$

$$3^6 \rightarrow 3^3 \times 3^3$$

$$3^3 \rightarrow 3 \times 3^2$$

$$3^2 \rightarrow 3^1 \times 3^1$$

$$3^1 \rightarrow 3 \times 3^0$$

∴ If b is even:

$$F(a, b) \longrightarrow F(a, b/2) \times F(a, b/2)$$

If b is odd :

$$F(a, b) \longrightarrow a \times F(a, b/2) \times F(a, b/2)$$

```
INT BINEXP RECUR (INT A, INT B)
```

```
{
```

```
    IF (B == 0)
```

```
    {
```

```
        RETURN 1;
```

```
    }
```

```
    LL RES = BINEXP RECUR (A, B/2);
```

```
    → If odd
```

```
    IF (B % 2 != 0)
```

```
    {
```

```
        RETURN A * RES * RES;
```

```
    }
```

```
    ELSE
```

```
    {
```

```
        RETURN RES * RES;
```

```
    }
```

```
}
```