KNAPSACK PROBLEM

The Knapsack problem is an example of the combinational optimization problem. This problem is also commonly known as the "Rucksack Problem". The name of the problem is defined from the maximization problem as mentioned below:

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"Given a bag with maximum weight capacity of W and a set of items, each having a weight and a value associated with it. Decide the number of each item to take in a collection such that the total weight is less than the capacity and the total value is maximized."

→ Types of Knapsack Problem:

1) Fractional Knapsack problem
2) 0/1 Knapsack problem
3) Unbounded Knapsack problem.
4) Bounded Knapsack problem

1) Fractional Knapsack Problem:

Given the weights and values of N items, put these items in a knapsack of capacity W to get the maximum total value in the knapsack. In Fractional Knapsack, we can break items for maximizing the total value of the knapsack. It is solved using greedy approach.

2) 0/1 Knapsack Problem:

We are given N items where each item has some weight (W_i) and value (V_i) associated with it. We are also given a bag with capacity W. The target is to put the items into the bag such that the sum of values associated with them is the maximum possible.

Note that here we can either put an item completely into the bag or cannot put it at all.

Mathematically the problem can be expressed as:

$$\boxed{\text{Maximize } \sum_{i=1}^{N} v_i x_i \text{ subject to } \sum_{i=1}^{N} w_i x_i \leq W \text{ and } x_i \in \{0, 1\}}$$

3) Bounded Knapsack Problem :

Given N items, each item having a given weight w_i and a value v_i, the task is to maximize the value by selecting a maximum of K items adding up to a maximum weight W.

Mathematically the problem can be expressed as:

$$\boxed{\text{Maximize } \sum_{i=1}^{N} v_i x_i \text{ subject to } \sum_{i=1}^{N} w_i x_i \leq W \text{ and } x_i \in \{0, 1, \ldots, K\}}$$

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4) Unbounded Knapsack Problem :

Given a knapsack weight W and set of N items with certain values v_i and weight w_i, we need to calculate the maximum amount that could make up this quantity exactly. This is different from 0/1 Knapsack problem, here we are allowed to use an unlimited number of instances of an item.

Mathematically the problem can be expressed as :

$$\boxed{\text{Maximize: } \sum_{i=1}^{N} v_i x_i \text{ subject to } \sum_{i=1}^{N} w_i x_i \le W \text{ and } x_i \in \mathbb{Z} \text{ and } x_i \ge 0.}$$

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