

→  $n^{\text{th}}$  Root using Binary Search

→ Logic

Suppose we have to find the sq. root of 4.

Then in Binary Search our search space will be from 1 to 4

1 2 3 4

And if we want decimal places as well then our search space will increase e.g.:

For  $x = 2$  if we

want 1.4 as ans then our search space will be:

1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2

(Now, we will apply Binary search and the condition will be if

$MID \times MID < x$  ~~is~~ <sup>the greater</sup>

Then,  $LO = MID$

ELSE,  $H1 = MID$ )

$\{x = 4 \text{ here}\}$

It's time complexity will be:

$P \times \log(N \times 10^d)$

(For  $P^{\text{th}}$  root of number  $N$  with  $d$  decimal places)

## CODE

DOUBLE EPS = 1e-7; // precision of our ans.

→ function to multiply our no till  $n^{\text{th}}$  root.

```
DOUBLE MULTIPLY (DOUBLE MID, INT N)
{
```

```
    DOUBLE ANS = 1;
```

```
    FOR (INT i = 0; i < N; i++)
    {
```

```
        ANS *= MID;
```

```
    }
```

```
    RETURN ANS;
```

```
}
```

```
VOID NTH-BINARY (DOUBLE x, INT N)
```

```
{
```

```
    DOUBLE LO = 1, HI = x, MID;
```

→ e.g. here, ~~EPS~~ EPS = 0.0000001, so,  
it will determine the accuracy

```
    WHILE (HI - LO > EPS)
```

```
    {
```

```
        MID = (HI + LO) / 2;
```

```
        IF (MULTIPLY (MID, N) < x)
```

```
        {
```

```
            LO = MID;
```

```
        }
```

```
    ELSE
```

```
    {
```

```
        HI = MID;
```

```
    }
```



}

→ Now we can print LO or HI any, they both will give same ans till given precision of  $10^{-7}$  here.

```
cout << LO << " " << HI << "\n";
```

→ Finding  $n^{\text{th}}$  root using inbuilt pow function.

```
cout << pow(x, 1.0/n) << "\n";
```

}

```
int main()
```

{

→ no. for which we have to find the root of  
DOUBLE x;

→ value of n in the  $n^{\text{th}}$  root

```
int N;
```

```
cin >> x >> N;
```

```
Nth-BINARY(x, N);
```

```
return 0;
```

}

**NOTE:**

Predicate function → A function which returns TRUE/FALSE.

Binary search is applied on Monotonic predicate function. e.g.:

```
FFF TTT
TTT FFF
```