

→ Sieve Algorithm : To find Prime Numbers  
Optimally. 54

→ Suppose we have  $q = 10^7$  queries and  
 $n = 10^7$  numbers to check prime in  
each queries.

So, if we use Square root method (which  
was quite fast), still we will have a  
Time Complexity of  $O(q) * O(\sqrt{n})$ .  
This will give TLE.

→ So, To solve this problem, we <sup>have</sup> compute all prime numbers using Sieve Algorithm.

CONST int N = 1e7 + 10;

→ Initializing a vector with N size, initially with all true values.

VECTOR<bool> isPrime(N, 1);

→ Sieve Algorithm

→ Suppose we have  $n = 30$

Write down numbers till 30.

<del>1</del>	2	3	<del>4</del>	5	<del>6</del>	7	<del>8</del>	<del>9</del>	<del>10</del>
11	<del>12</del>	13	<del>14</del>	<del>15</del>	<del>16</del>	17	<del>18</del>	19	<del>20</del>
<del>21</del>	<del>22</del>	23	<del>24</del>	<del>25</del>	<del>26</del>	<del>27</del>	<del>28</del>	29	<del>30</del>

STEPS:

0. Initially we consider all the numbers till  $n$  as prime numbers, and at the end:

~~CROSSED~~ = Non Prime

■ UNCROSSED = Prime

1. 1 is non-prime (we all know). So, simply cross it and move to next number.

2. The next number is 2, cross all the multiples of 2 till 30.

3. Now move to next uncrossed number which is 3, do same with 3, cross all its multiples.



(ignore already crossed.)

4. Same process again, we get 5 uncrossed.

5. Follow this process until we consider all the uncrossed numbers.

6. Finally, we are left with:

UNCROSSED = 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

These are all prime numbers till 30.

**NOTE:** Whenever we move to next uncrossed number, that number is always gonna be a prime number.

```

INT MAIN()
{

```

→ Sieve Algorithm, Time Complexity  $O(N * \log(\log(N)))$

→ Setting 0 and 1 as non prime (as we all know that).

```

ISPRIME[0] = ISPRIME[1] = FALSE;

```

→ Pre computing all the primes till N.

```

FOR (INT i = 2; i < N; i++)
{

```

→ If i is prime

```

IF (ISPRIME[i] == TRUE)
{

```

```

    FOR (INT j = 2 * i; j < N; j += i)
    {

```

→ changing all multiples of  $i$  to False.  
ISPRIME[j] = FALSE;

```
}  
}  
}
```

→ Query

```
INT Q;
```

```
CIN >> Q;
```

```
WHILE (Q--)
```

```
{
```

```
    INT M;
```

```
    CIN >> M;
```

```
    IF (ISPRIME[M])
```

```
    {
```

```
        cout << "PRIME" << "\n";
```

```
    }
```

```
    ELSE
```

```
    {
```

```
        cout << "NON PRIME" << "\n";
```

```
    }
```

```
}
```

→ Time Complexity =  $O(q) * O(1)$

```
RETURN 0;
```

```
}
```