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→ Best Case, Worst Case & Expected Case
of an Algorithm:

→ Sometimes we get lucky in life. Exams
cancelled when you were not prepared,
surprise test when you were prepared
etc. \Rightarrow BEST CASE

Date:

→ Sometimes we get unlucky. Questions you never prepared asked in exams, train during sports period etc. \Rightarrow Worst case

→ But overall the life remains balanced with the mixture of lucky & unlucky events \Rightarrow EXPECTED CASE

→ Analysis of a search algorithm:

→ Consider an array which is sorted in \uparrow ing order.

1	7	18	28	50	180
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→ We have to search a given number in this array & report whether its present in the array or not.

→ Algo 1 \rightarrow Start from first element until an element \uparrow ter than or equal to the number to be searched is found.

→ Algo 2 \rightarrow Check whether the first or the last element is equal to the number. If not find the number b/w these two elements (center of the array). If the center element is \uparrow ter than the number to be searched, repeat the process for first half else repeat for second half until

the number is found.

→ Analyzing Algo 1:

→ If we really get lucky, the first element of the array might turn out to be the element we are searching for. Hence, we made just one comparison.

Best Case Complexity = $O(1)$ | $(I_m = K)$

→ If we are really unlucky, the element we are searching for might be the last one.

Worst Case Complexity = $O(n)$ | $(I_m = nK)$

→ For calculating Average case time, we sum the list of all the possible cases runtimes & divide it with the total number of cases:

$$\text{Avg Case} = O \left(\frac{\sum \text{All possible runtimes}}{\text{Total no. of possibility}} \right)$$

$$\text{Avg Case} = O \left[\frac{K + 2K + 3K + \dots + nK}{n+1} \right]$$

$$\Rightarrow O \left[\frac{K(1 + 2 + 3 + \dots + n)}{n+1} \right]$$

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$$\Rightarrow O K \left[\frac{n^2 + n + 2n}{2(n+1)} \right] \downarrow \text{Ignoring } 2n, \text{ Non-dominant.}$$

$$O K \frac{n(n+1)}{2(n+1)} \Rightarrow O \left(\frac{K}{2} n \right) \text{ '3' is constant}$$

$$\therefore \text{Avg Case} \Rightarrow O(n)$$

→ Analyzing Algo 2:

→ If we get really lucky, the first element will be the only one which gets compared!

$$\text{Best Case Complexity} = O(1)$$

→ If we get unlucky, we will have to keep dividing the array into halves until we get a single element (the array gets finished).

$$\text{Worst Case Complexity} = O(\log n)$$

→ What $\log(n)$? What is that!

$\log(n) \rightarrow$ Number of time we need to half the array of size n before it gets exhausted.

$$\log 8 = 3 \Rightarrow \frac{8}{2} \rightarrow \frac{4}{2} \rightarrow \frac{2}{2} \rightarrow \text{can't repeat anymore}$$

1 + 1 + 1

$$\log 4 = 2 \Rightarrow \frac{4}{2} \rightarrow \frac{2}{2} \rightarrow \text{can't break anymore.}$$

1 + 1

→ $\log(n)$ simply means how many times, I need to divide n units such that we cannot divide them (into halves) anymore.

→ Space Complexity: Time is not the only thing we worry about while analyzing algorithms. Space is equally important.

→ Creating an array of size $n \rightarrow O(n)$ space

↓
SIZE OF INPUT

→ If a function calls itself recursively n times its space complexity is $O(n)$.

→ Why can't we calculate complexity in seconds?

→ Not everyone's computer is equally powerful.

→ Asymptotic analysis is the measure of how time (runtime) grows with input.