Computer Science & DA

Probability and Statistics

Permutation and Combination

Lecture No. 01

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Recap of previous lecture

Topic Prerequisites of Engineering Mathematics

Topics to be Covered

Topic Permutation and Combination

Topic:Permutation and Combination

① $y = \sin(e^{x^2})$

$\frac{dy}{dx} = \cos(e^{x^2}) \cdot (e^{x^2}) \cdot (2x)$ (using chain Rule)

$\int \sin(e^{x^2})\, dx = ?$ (Not possible) (∵ Chain Rule DIVE)

② $\boxed{\text{Area} = \iint (1)\, dx\, dy} = \int y\, dx = \int_a^b f(x)\, dx$

$\text{Volume} = \iiint (1)\, dx\, dy\, dz = \iint z\, dx\, dy = \boxed{\iint f(x,y)\, dx\, dy}$

$\text{Mass} = \iiint \rho \cdot dx\, dy\, dz \qquad \rho = \rho(x,y,z) \qquad D = \frac{M}{V}$

Counting Principle →

(1) **F.P of Addition** → if we have to perform only one of the job at a time then use this Principle & it's keywords are "Either or / only one / Anyone"

(2) **F.P of Multiplication** → if we have to perform all the jobs at a time then use this Principle and it's keywords are "All / Both / AND"

Eg: In a class there are 10 Boys and 8 Girls Then in how many ways we can select

① A Boy and a Girl = 10 ways × 8 ways = 80 ways

② Either a Boy or Girl = 10 ways + 8 ways = 18 ways

B_1G_1	B_2G_1	B_3G_1	----	$B_{10}G_1$
B_1G_2	B_2G_2	B_3G_2		$B_{10}G_2$
B_1G_3	⋮	⋮		⋮
⋮	⋮	⋮		⋮
B_1G_8	B_2G_8	B_3G_8		$B_{10}G_8$ = 80 ways.

Q. There are 5 students appearing in Maths scholarship test, 4 are appearing in Physics test and 3 are in Chemistry scholarship test then in how many way?

(i) these scholarships can be awarded = ? $= 5 \text{ ways} \times 4 \text{ ways} \times 3 \text{ ways} = 60 \text{ ways}$

(ii) one of these " " " " = ? $= 5 + 4 + 3 = 12 \text{ ways}$

Q. There are 5 letters and 4 letter Boxes then in how many ways we can post these letters?

Sol. Total ways of posting these letters = ? $= \underset{L_1}{4} \times \underset{L_2}{4} \times \underset{L_3}{4} \times \underset{L_4}{4} \times \underset{L_5}{4} = 4^5$

(RA)

5×4, $5+4$, $5!$, ${}^5P_4$, ${}^5C_4$, 4^5 ✓, 5^4 ✗

✗

Wrong Approach :-

Total ways of posting letters

$= \frac{5}{B_1} \times \frac{5}{B_2} \times \frac{5}{B_3} \times \frac{5}{B_4} = 5^4$

Here we are throwing L-B (R.P.)

Q- there are 3 R & 4 f then in how many ways we can worn these Rings ? if

(i) there is no Restriction = ?

Sol: Total ways (RA) $= \frac{4}{R_1} \times \frac{4}{R_2} \times \frac{4}{R_3} = 4^3 = 64$ ways

Wrong App: $= \frac{3}{f_1} \times \frac{3}{f_2} \times \frac{3}{f_3} \times \frac{3}{f_4} = 3^4 = 81$ ✗

Here we are Cracking fingers (Nope)

(ii) with at Most one R in a single finger = ?

$RNA = \frac{4f}{R_1} \times \frac{3f}{R_2} \times \frac{2f}{R_3} = 4 \times 3 \times 2 = 24$

Q: In how many ways 5 persons can be seated on 8 chairs?

Sol: Total seating Arrangements = $\underset{P_1}{8} \times \underset{P_2}{7} \times \underset{P_3}{6} \times \underset{P_4}{5} \times \underset{P_5}{4} = {}^8C_5 \times 5! = {}^8P_5$

(R.N.A)

(ii) if there are 8 persons & 8 chairs then T.S.A = ? = $\underset{P_1}{8} \times \underset{P_2}{7} \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 8!$

(R.N.A) $\quad P_1\ P_2\ P_3 \dots P_8$

(iii) if there are 8 persons and 5 chairs then T.S.A = ? = Senselen 8

Q. How many 5-digit Numbers can be formed using the digits 1, 3, 5, 7, 9

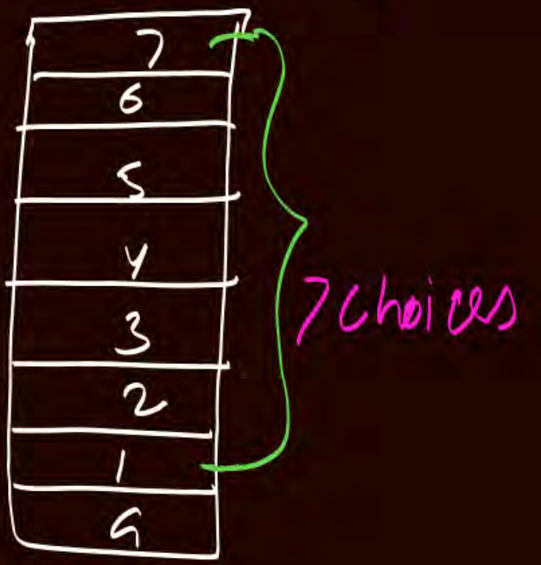
(i) If R.A = ? $= \underline{5} \times \underline{5} \times \underline{5} \times \underline{5} \times \underline{5} = 5^5$
$\quad P_1 \quad P_2 \quad P_3 \quad P_4 \quad P_5$

(ii) If RNA = ? $= \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 5!$

Q. How many 3 digit Numbers can be formed using 1, 3, 5, 7, 9 if

RNA = ? $= \underline{5} \times \underline{4} \times \underline{3} = {}^5C_3 \times 3! = {}^5P_3$
$\quad P_1 \quad P_2 \quad P_3$

Q: 5 persons entered in a lift at Ground floor in an 8 floor house then in how many ways they can leave the lift ?



(1) At any floor = ? (RA) $= \frac{7}{P_1} \times \frac{7}{P_2} \times \frac{7}{P_3} \times \frac{7}{P_4} \times \frac{7}{P_5} = 7^5$ ways

(2) At different floors = ? (RNA) $= \frac{7}{P_1} \times \frac{6}{P_2} \times \frac{5}{P_3} \times \frac{4}{P_4} \times \frac{3}{P_5} = {}^7C_5 \times 5!$

$= {}^7P_5$

Q: In how many ways B'days of 6 different persons will fall in Calendar months?

Sol: Total ways of falling B'day = $\frac{12}{P_1} \times \frac{12}{P_2} \times \frac{12}{P_3} \times \frac{12}{P_4} \times \frac{12}{P_5} \times \frac{12}{P_6} = 12^6$ ways

(RA) (12 months)

Note: ① if $n > r$ & RNA then Multi Rule ≃ Permutation Rule

② if $n = r$ & RNA then Multi Rule ≃ Perm. Rule ≃ Factorial Rule

③ if RA then only use Multi Rule

i.e the Concept of ${}^nP_r$, ${}^nC_r$, $r!$ is applicable only when R.N.A

Combination (If in a Question, only selection is required then use ^nC_r) →

$$^nC_r = \frac{n!}{r!(n-r)!}$$

Note ① $^nC_r = {}^nC_{n-r}$ eg $^{11}C_3 = \frac{11\times10\times9}{3\times2\times1}$

eg $^{22}C_4 = \frac{22\times21\times20\times19}{4\times3\times2\times1}$, eg $^{15}C_{12} = ? = {}^{15}C_3 = \frac{15\times14\times13}{3\times2\times1}$

Permutation (Selection & Arrangement) →

$$^nP_r = \frac{n!}{(n-r)!} = {}^nC_r \times r!$$

$^{11}P_3 = 11\times10\times9$, $^{22}P_4 = 22\times21\times20\times19$

eg of Permⁿ formation of words, formation of signals, Seating arrangement etc
" " Number, " of Photos,

Qe there are (5) flags of different colours, then how many different signals can be formed using R, B, G, Y, O

(1) (3) flag at a time = ? (R.N.A)

$$= \frac{5}{P_1} \times \frac{4}{P_2} \times \frac{3}{P_3} = {}^5C_3 \times 3! = {}^5P_3 = 60 \text{ ways}$$

${}^5C_3$: Selection of 3 flag

$3!$: Arrangement of 3 flag at 3 places

(2) Using any Number of flags at a time = ?

We can make signal either by choosing = 1 flag or 2 flag or 3 flag or 4 flag or 5 flag

n = 1f or 2f or 3f or 4f or 5f

M-I $= (5) + (5\times4) + (5\times4\times3) + (5\times4\times3\times2) + (5\times4\times3\times2\times1)$

M-II $= ({}^5C_1\times1!) + ({}^5C_2\times2!) + ({}^5C_3\times3!) + ({}^5C_4\times4!) + ({}^5C_5\times5!)$

M-III $= {}^5P_1 + {}^5P_2 + {}^5P_3 + {}^5P_4 + {}^5P_5$

Q. Salads are made from one or more eatables then how many different salads can be made using onion, Tomato, Carrot, Radish, Cucummer ?

we can make Salad either by choosing selections.

$= 1V \text{ or } 2V \text{ or } 3V \text{ or } 4V \text{ or } 5V$

$= {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5 = 31$ different Salads.

M-II

THANK - YOU