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Measurement of Heat Expansion by an Interferometer.

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Abstract

We measured the thermal expansion coefficient of two different metal rods in order to determine what metal it is. In order to do this we used a Michelson interferometer to measure the expansion of the metal rod and thermistors to measure the temperature of the rod at three points on its surface.

The metal rods was measured to have a thermal expansion coefficient of $26.605 \cdot 10^{-6} \pm 0.416 \cdot 10^{-6}$ 1/K and $19.371 \cdot 10^{-6} \pm 0.110 \cdot 10^{-6}$ 1/K. But we think the true values should have been $23 \cdot 10^{-6}$1/K and $8.6 \cdot 10^{-6}$1/K and correspond to aluminium and titanium respectively. So the results were not better than an order of magnitude estimate. In order to make more accurate measurements we believe we need a better way to measure the temperature of the rod and a better model for how the temperature is distributed in the rod.

Non-Invasive Measurement Techniques
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1 Introduction

The Michelson interferometer is a common configuration for optical interferometry. A beam of light is split into to two and made to travel different paths. The first path is used as reference and held fixed, whereas the second path is made to vary. When the beams from both these paths are joined together they will interfere with each other. The interference pattern can be measured over time and the difference in path length can be determined. If the path difference between the two paths changes by half the wavelength of light, we will move between two maxima in the interference pattern. So we can detect differences in path length that is less than the wavelength of light. We can also measure larger path differences by counting how many peaks you have seen.

In our lab we want to measure the coefficient of thermal expansion of unknown metal rods and from this identify what metal it is. We predict that the coefficient can be measured accurate enough to determine what metal we are testing on. We believe that the expansion of the metal rod can be measured very exactly. However it will not be as easy to measure the temperature of the rod since we can only measure a few points on the outside of the rod and we believe this will be the most significant source of error.

2 Theory

2.1 Heat expansion

The expression that describes thermal expansion for a volume in solids is

$$\alpha_v = \frac{1}{V}\frac{dV}{dT} \tag{1}$$

where α_v is the thermal expansion coefficient of the volume, V is the volume of the solid and T is the temperature of the solid. As the solid in this case is a long metallic rod eq. (1) can be approximated as

$$\alpha_L = \frac{1}{L}\frac{dL}{dT} \tag{2}$$

where α_L is the thermal expansion coefficient of the length L of the rod. Eq. (2) can then be approximated to

$$\alpha_L = \frac{1}{L}\frac{\Delta L}{\Delta T} \tag{3}$$

where ΔL and ΔT is the change in the length and temperature of the rod.

2.2 Michelson interferometer

The working principle behind the Michelson interferometer is that a beam of light is split so that the resulting beams can be led through two optical paths of different lengths and later be recombined to an interference pattern.

In fig. 1 the light emanating from the light source gets split up at a beam-splitter creating two beams. We will call these two beams $B1$ (the one travelling upwards in fig. 1) and $B2$ (the one travelling to the right). The beam $B1$ gets

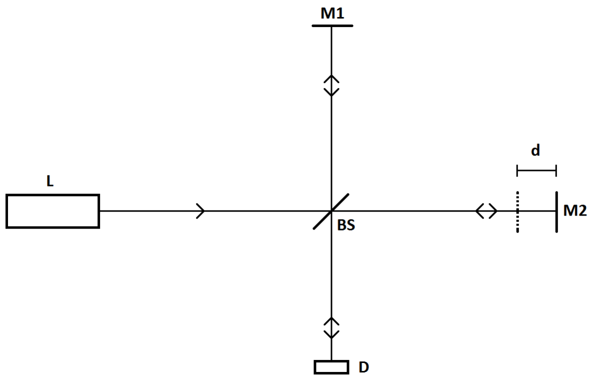

Figure 1: Schematic illustration of an Michelson interferometer. In the figure L is the light source, BS is the beam-splitter, $M1$ and $M2$ are the two mirrors, D is the detector and d is the displacement of $M2$

reflected in the beam-splitter and directed towards a mirror $M1$, travelling some distance l_1 to a mirror. It is then reflected back towards the beam-splitter, where it passes through and hit the detector after travelling a distance l_d.

For the other beam, $B2$, we see that it passes through the beam splitter and travels some distance l_2, gets reflected back to the beam splitter and is directed to the detector.

When we consider the phase shift between these beams as they go through the system; both beams gets reflected the same number of times. This means that the optical path length difference—and therefore also the phase shift between the beams—will only depend on the actual travelling distances l_1 and l_2 (the travelling distances shared by both beams will not contribute to the optical path difference and are therefore not of interest). From this we can see that the optical path difference of the beams can be expressed as

$$\Delta l = 2l_1 - 2l_2 = 2(l_1 - l_2) \tag{4}$$

where Δl is the optical path difference. For this phase shift to result in constructive interference the optical path difference must be an integer multiple of the wavelength. This condition and eq. 4 gives us the following expression

$$m\lambda = \Delta l = 2(l_1 - l_2), \qquad m = 0, 1, 2, ... \tag{5}$$

where λ is the wavelength of the light and m is an integer numbering the interference fringes.

In our application of this interferometer the length l_2 is not going to be fixed but it will shift with some displacement, d, that is dependent on time. If this is inserted into eq. 5 we get

$$m\lambda = \Delta l = 2(l_1 - l_2 + d). \tag{6}$$

From this we see that as this displacement changes, the value of m will change. This will result in a movement of the interference fringes which can be detected and used to measure the displacement.

3 Experimental

The experiment is executed by directing a laser beam at a beam-splitter that splits the laser beam into two beams that takes different paths. The first path is held fixed and the second path may vary depending on how much a heated metal rod expands. The heated metal rod is held fixed at one end so that the metal rod only expand in one direction. This means that the varying path length decreases by two times the length the metal rod expands. The two beams then joins together and creates an interference pattern according to eq. (6). The beams goes through some optics—that is only used to separate the interference fringes so that the photo diode can detect it—and then hits the photo diode, that measures the intensity. A block diagram of the setup can be seen in fig. 2.

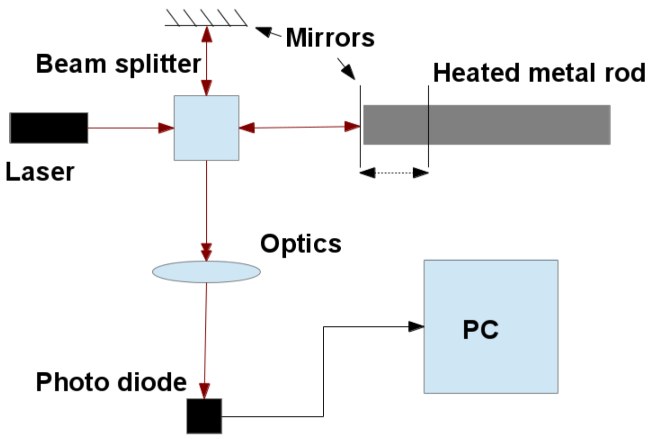

Figure 2: Block diagram of the experimental setup.

To determine the thermal expansion coefficient; we need to know the temperature of the rod, according to eq. (3). Three thermistors measure the temperature and are fitted on the surface of the metal rod; one in the middle and one in each end of the rod. From this the temperature distribution of the metal rod can be approximated.

4 Results

The experiment was conducted on two different metal rods that we denote as *rod 1* and *rod 2*.

4.1 Results for Rod 1

To determine the thermal expansion coefficients of rod 1, we analysed the normalized intensities measured by the photo diode. A sample of this data can be seen below in fig. 3.

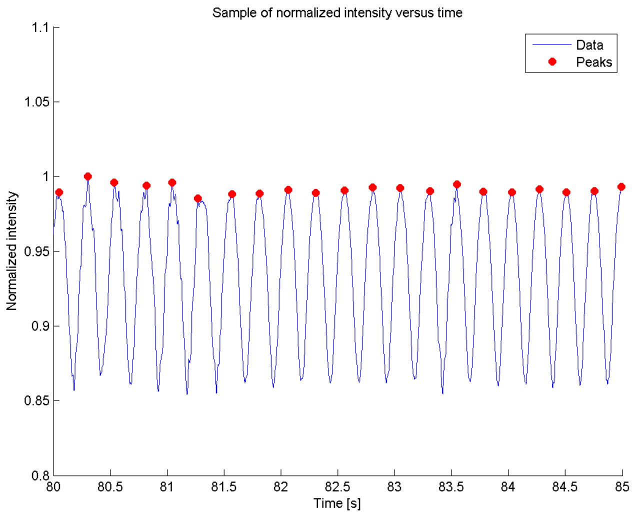

Figure 3: A sample of the normalized intensity data measured by the photo diode. Peaks of the data is marked by circles.

The peaks of the intensities were counted to calculate the linear expansion of the rod using eq. (6). Temperature measurements were also analysed, raw data from the three temperature sensors can be seen below in fig. 4 and a calculated mean temperature in fig. 5. From this mean temperature the thermal expansion coefficient of Rod 1, α_1, was calculated by using eq. (2). The value obtained was $\alpha_1 = 26.605 \cdot 10^{-6}$ 1/K. The error estimate was calculated using *Gauss propagation of uncertainty* and eq. (3) with $L = 0.280$ m, $\Delta T = 7.1377$ K and $\Delta L = 5.3172 \cdot 10^{-6}$ m with corresponding errors of 1 mm, 0.1 K and $3.165 \cdot 10^{-7}$ m respectively to be $\sigma_{\alpha_1} = 0.416 \cdot 10^{-6}$ 1/K.

A figure of the temperature dependency of the length expansion can be seen below in fig. 6, assuming that this dependence is linear; eq. (3) is equal to eq. (2) and evaluations can be made for large ΔT and ΔL.

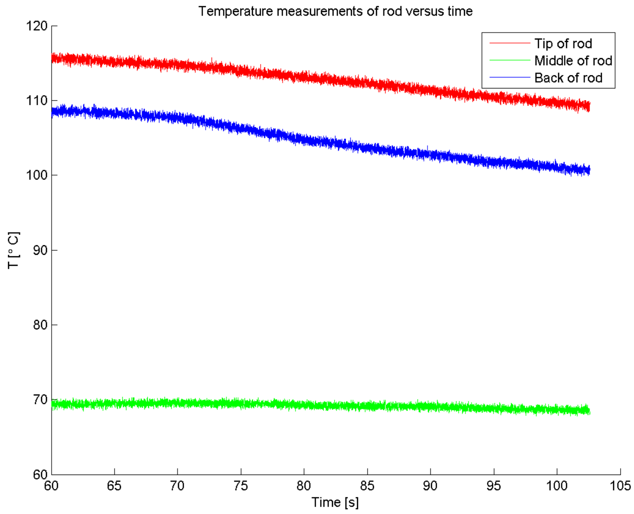

Figure 4: Temperature measurements from the three thermistors mounted on the rod versus time.

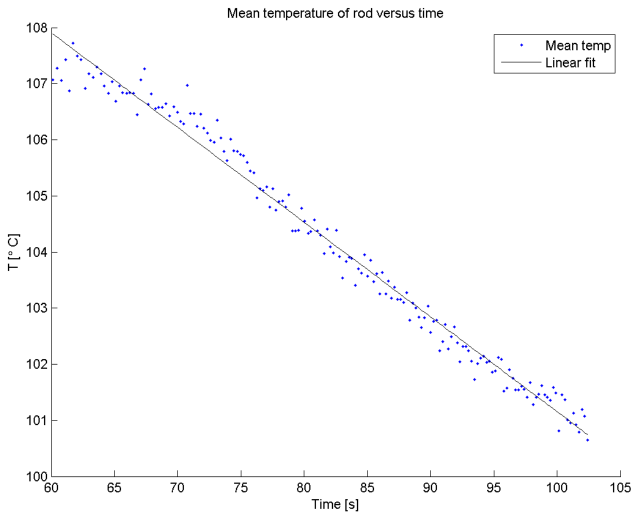

Figure 5: Mean temperature of the rod versus time.

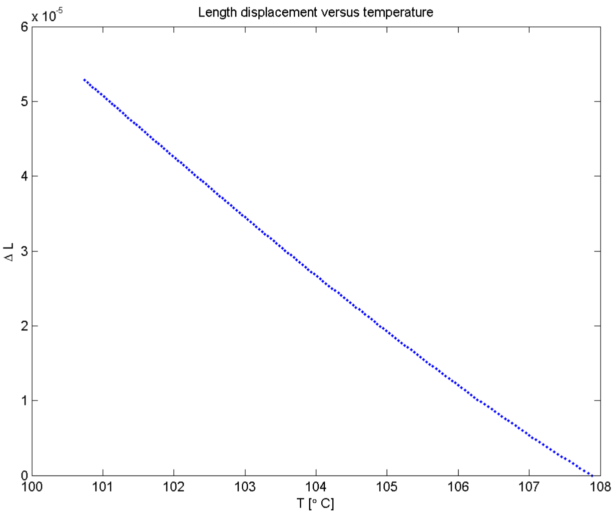

Figure 6: Length expansion of the rod as a function of temperature.

4.2 Results for Rod 2

The calculations described in the above sections where repeated for the second rod. Below we can find the corresponding figures, fig. 7,8,9 and 10 for these calculations.

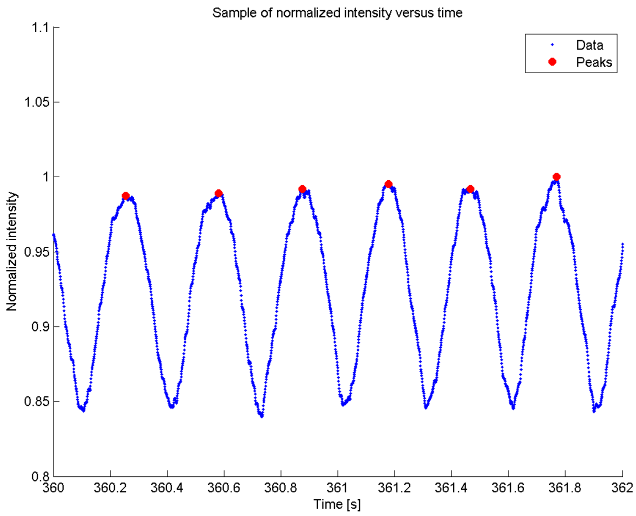

Figure 7: A sample of the normalized intensity data measured by the photo diode. Peaks of the data is marked by circles.

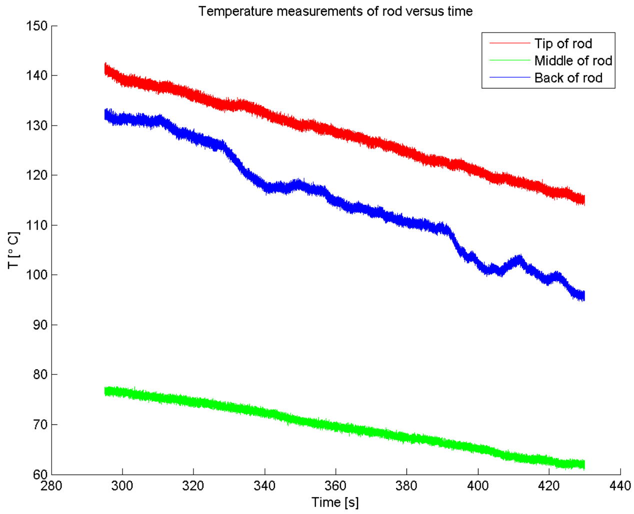

Figure 8: Temperature measurements from the three thermistors mounted on the rod versus time.

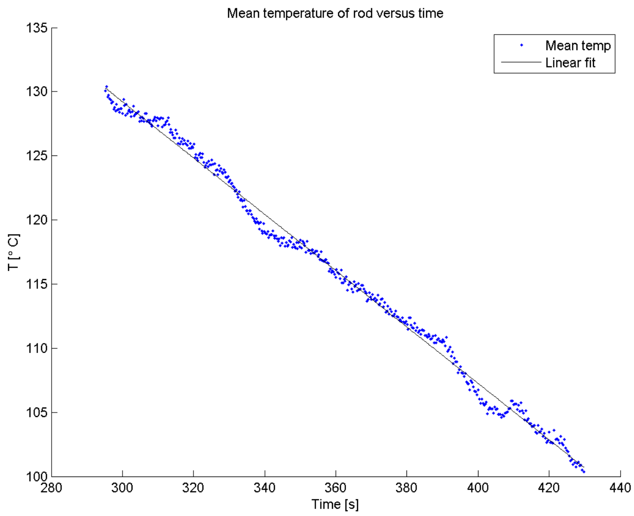

Figure 9: Mean temperature of the rod versus time.

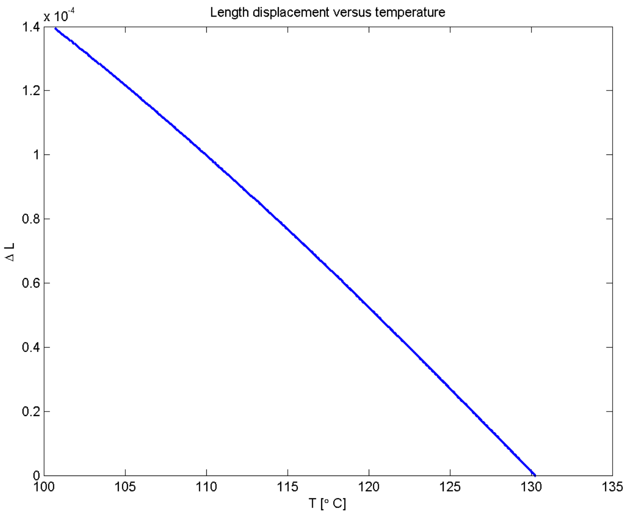

Figure 10: Length expansion of the rod as a function of temperature.

For this rod the thermal expansion coefficient was found to be $\alpha_2 = 19.371 \cdot 10^{-6}; 1/K$. This error estimate was also calculated using *Gauss propagation of uncertainty* and eq. (3). But with $L = 0.245$ m, $\Delta T = 30.6110$ K and $\Delta L = 1.4527 \cdot 10^{-4}$ m with corresponding errors of 1 mm, 0.1 K and $3.165 \cdot 10^{-7}$ m respectively to be $\sigma_{\alpha_2} = 0.110 \cdot 10^{-6}$ 1/K.

5 Discussion

To get a better picture of the expected results of this experiment we identified the metals the rods were constituted of. This was done by measuring the densities of both rods and comparing it to tabulated values. From this we found rod 1 to be made out of aluminium and rod 2 to be made out of titanium.

After the rods where identified we could also look up their respective linear thermal expansion coefficients. For the aluminium rod the tabulated value were $23 \cdot 10^{-6}$1/K [1]. This is outside the measured value of $26.605 \cdot 10^{-6} \pm 0.416 \cdot 10^{-6}$ 1/K which indicate some systematic error. For the titanium rod which has a tabulated coefficient of linear thermal expansion of $8.6 \cdot 10^{-6}$1/K [1]; we see that our calculated value of about $19.371 \cdot 10^{-6} \pm 0.110 \cdot 10^{-6}$ 1/K is not reasonable either.

One of the main reasons why our measurements suffered significant errors could possibly be due to an uneven temperature gradient in both the radial and axial direction. As a result the temperature measurements we got from the thermistors will not give a good representation of the rods true temperature.

6 Conclusions

To get better results we need to determine the temperature distribution much better. One problem was that the thermistors only measured the temperature at three points on the surface of the metal. Then we assumed that the temperature gradient was linear between the measured points and that the temperature was the same in the core of the rod as on the surface. We don't have any error estimate of how large this error might be and if it might explain why the results did not correspond with tabulated values.

References

[1] Thermal expansion coefficient, `http://en.m.wikipedia.org/wiki/Thermal_expansion_coefficients_of_the_elements_(data_page)`, (January 1, 2014).