Department of Physics
Umeå University

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SOUND VELOCITY MEASUREMENTS USING ULTRASOUND

Simon Agvik
siag0001@student.umu.se

David Ek
daek0011@student.umu.se

ABSTRACT

The purpose of this experiment was to measure the speed of sound in two different gases, air and argon. The velocities should then be used to determine if the particles in the gas rotate or vibrate at room temperature. The method used was a time of flight method in which ultrasound was sent through a tube with gas and the elapsed travel time of the sound wave was recorded. By using the time and the length of the tube the velocity of sound was found. Studying the degrees of freedom, calculated from the velocity, one can determine the behavior of the gas particles.

The velocity of sound in air was found to be 336 ± 6.3 m/s and in argon 314 ± 5.5 m/s. We suspect that there are systematical read off errors in these values. While reading the instruments it was difficult to determine when the recorded sound wave started, causing inaccurate measurements. Calculating the degrees of freedom for air we concluded that the molecules have translational and rotational motion at room temperature.

Noninvasive Measurement Techniques

Supervisors: Bertil Sundqvist, Isak Silander, Patrick Ehlers, Amir Khodabakhsh

1. INTRODUCTION

The purpose of this experiment is to determine the speed of sound waves in different gases. Using the experimentally determined velocities we should verify the theoretical relationship between the velocity and other physical properties of the gas. Also, for a molecular gas we should determine if the molecules in the gas rotate or vibrate at room temperature.

This experiment can be performed in different ways but we chose a time of flight method in which we register the elapsed time when a burst of sound travels a known distance. Having the time and distance one can calculate the speed of the sound. These measurements are made for different pressures with the two gases argon and air. Since argon is an atomic gas and air is mainly a mixture of molecular gases, this choice of gases will enable us to see a possible difference between atomic and molecular gases.

In order to get good results for the argon we have to make sure that no other gas is present in the tube during measurements, this require a properly sealed equipment.

2. THEORY

In this section a description of the necessary theory used in the experiment is given.

2.1 Ideal gas law

Theoretically a gas can be described as a set of randomly moving particles. If the particles are seen as non-interacting point particles the behavior of a gas can be approximated by the ideal gas law [1]:

$$PV = nRT \tag{1}$$

where P is the pressure, V is the volume, n is the number of moles, R is the universal gas constant and T is the temperature.

The ideal gas law is valid if the particles are non-interacting. Air and argon can be approximated as an ideal gas at room temperature and at low pressures. Since at high temperatures the particles kinetic energy is significantly larger than the intermolecular forces, and at low pressure the size of the molecules affect less because of the empty space between them.

2.2 Speed of sound

The motion of a wave propagating in one dimension is given by the wave equation [2]

$$\frac{\partial^2 \xi}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 \xi}{\partial t^2} \tag{2}$$

where c is the velocity of the wave. A sound wave, described by eq. (2), is a pressure wave in gas which phase velocity is given by

$$c_{sound} = \sqrt{\frac{B}{\rho}} \tag{3}$$

where B is the bulk modulus and ρ is the density [3]. The bulk modulus [4] for a sound wave is

$$B = -V\frac{dP}{dV}. \tag{4}$$

A sound wave traveling in a gas is an adiabatic process, i.e. a process that occurs without interaction of heat or matter with its surroundings. The equation of an adiabatic process of an ideal gas is

$$PV^{\gamma} = C \tag{5}$$

where, γ is the heat capacity ratio and C is a constant [5]. The heat capacity ratio for an ideal gas in terms of the degrees of freedom, f, for a particle is [6]

$$\gamma = 1 + \frac{2}{f}. \tag{6}$$

Inserting eq. (5) in eq. (4) and substituting this into eq. (3) together with eq. (1), the definition of density and the fact that mass can be expressed as the number of moles times the molar weight, M, yields

$$c_{sound} = \sqrt{\frac{\gamma RT}{M}}. \tag{7}$$

Thus for an ideal gas the speed of sound depends on the temperature, the molar mass and the heat capacity ratio which in turn depends on the degrees of freedom.

2.3 Degrees of freedom

The degrees of freedom for gas particles are associated with the translational, rotational and vibrational motion of the particles. From the translational motion in three dimensions the particles gain three degrees of freedom. An atomic gas can only have particles with three degrees of freedom since the particles cannot vibrate or rotate. However for diatomic gases the particles can rotate and/or vibrate. For the rotational motion the particles gain two extra degrees of freedom since they can rotate around two axes. Also, if the particles vibrate they will get one additional degree of freedom.

2.4 Time of flight

A time of flight method can be set up in many ways, the simplest case is when the sound source and the receiver are arranged in a straight line, at a specific distance. The sound source sends out bursts of sound with a specific frequency. The time when the source sends out the sound and the time when the receiver receives the sound are recorded. The difference in time, i.e. the sound wave's travel time, t, is then used to calculate the velocity with equation

$$v = \frac{s}{t} \tag{8}$$

where s is the distance between the source and the receiver.

2.5 Variance formula

Assuming independent variables the propagation of error in an experimental result can be calculated by the variance formula

$$s_f = \sqrt{\left(\frac{\partial f}{\partial x_1}\right)^2 s_{x_1}^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 s_{x_2}^2 + \ldots} \tag{9}$$

where s_f is the standard deviation of the function f and s_{x_i} is the standard deviation of variable x_i when $i = 1, 2, \ldots$ [7].

3. EXPERIMENT

In the following sections the experimental setup and the method are described in more detail.

3.1 Experimental setup

The main part of the experimental setup was a sealed tube with one ultrasonic transducer, EFR-RSB40K2, mounted in each end. This tube was connected to a vacuum pump that could be used to lower the pressure. One of the transducers was connected to a digital signal generator and worked as an ultrasound transmitter, the other transducer was connected to an oscilloscope and acted as a receiver. The oscilloscope also measured the signal generated by the signal generator so that the two signals could be compared.

Using a measuring tape we measured the distance between the transmitter and the receiver to $0.940(5)$ m. The temperature in the room was measured using a thermometer with a resolution of 0.1 ℃.

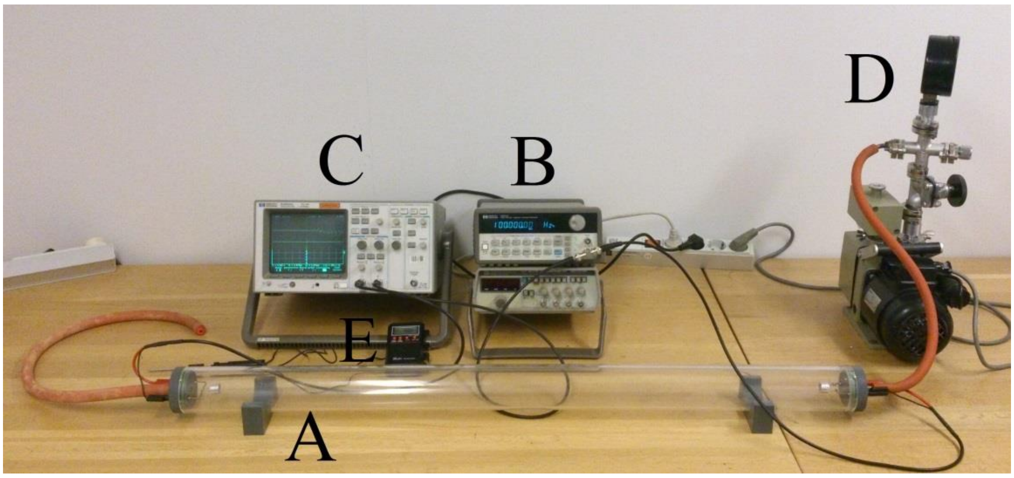

Figure 1. The experimental setup. A – Sealed tube filled with gas, B - digital signal generator, C - oscilloscope, D - vacuum pump, E - thermometer.

3.2 Method

The signal generator was adjusted to send out a square wave burst which was converted into sound by the transducer. The other transducer registered the burst and the signal was studied on the oscilloscope. Since the signal from the signal generator was measured simultaneously it was possible to measure the time difference between the transmitted and the received signal, see Figure 2. This time difference corresponds to the travel time of the sound and using eq. (8) the speed of the sound could be found.

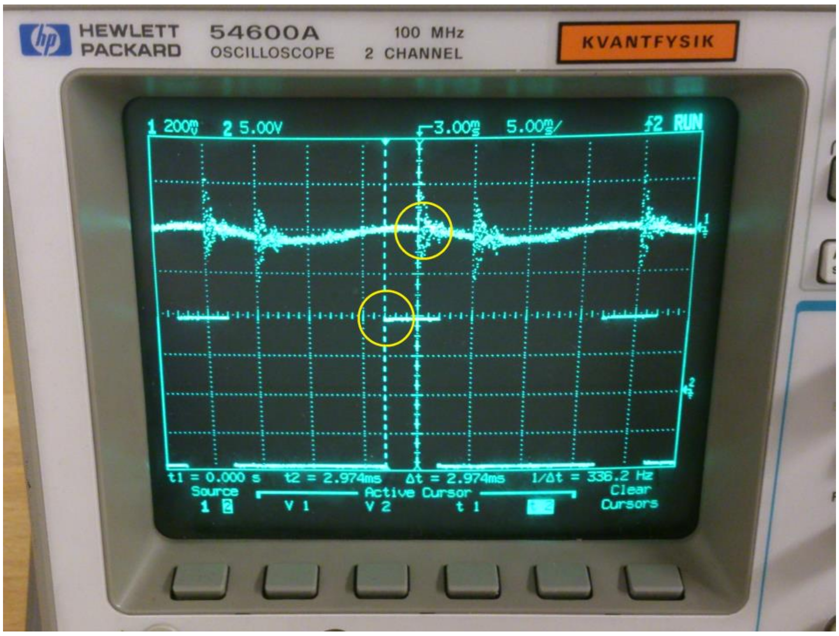

Figure 2. The measured signals on the oscilloscope. The lower curve is the signal from the signal generator and the upper curve is the received signal. The start of one pulse is marked by two yellow circles. Note that this picture is taken with a 100 Hz burst in order to see both the transmitted and received pulse in the same picture.

Filling the tube with air and using the vacuum pump we measured the elapsed time of the sound waves in the pressure range −90 kPa to 0 Pa, values given in gauge pressure.

To replace the air with argon we lowered the pressure down to −90 kPa and then filled the tube with argon. This process was repeated two times in order to make sure that we had high concentration of argon in the tube. When the tube was filled with argon the same measuring procedure was repeated, this time we had to make sure that the tube was properly sealed so that no air leaked in. Also, the temperature in the room was measured after the experiment.

4. RESULTS

The velocities were calculated for the two different gases and plotted versus the absolute pressure in Figure 3 for air and Figure 4 for argon. In the same figures the average velocity is marked with a line.

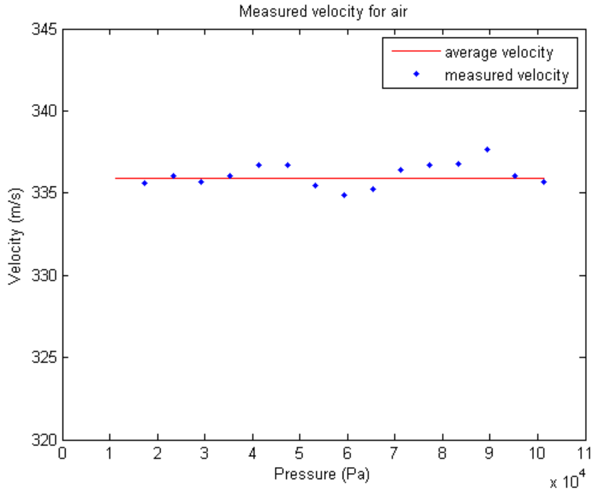

Figure 3. The measured velocity for air with the mean value displayed.

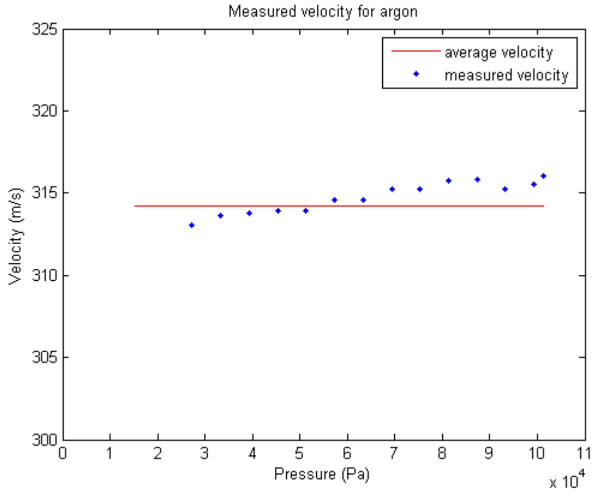

Figure 4. The measured velocity for argon with the mean value displayed.

Figure 3 and Figure 4 show that the velocity in both gases does not depend on the pressure in the range we are measuring in. The average velocity was calculated as the mean value of the measured data.

We estimated the error in the measurement of the distance to 5 mm and the error in the time reading on the oscilloscope to 0.05 ms. The average values of the variables and their estimated errors are listed in Table 1 for air and Table 2 for argon. The error in the velocity was calculated using eq. (9), eq. (8), the mean values and standard deviations for the time and the distance. The average velocity together with the measured temperature, 22.2 ℃, were then, with eq. (7), used to calculate the heat capacity ratio. Since we neglect errors in the temperature, the standard deviation of the heat capacity ratio was found using the smallest and largest value in the confidence interval of the velocity, i.e. $\bar{v} \pm s_v$.

Table 1. Average values and standard deviation for the parameters used for air.

	Average value	Standard deviation
Time [s]	0.0028	0.0005
Velocity [m/s]	336	6.3
Heat capacity ratio	1.33	0.05

Table 2. Average values and standard deviation for the parameters used for argon.

	Average value	Standard deviation
Time [s]	0.0030	0.0005
Velocity [m/s]	314	5.5
Heat capacity ratio	1.61	0.06

From the heat capacity ratio we can determine the degrees of freedom for the gases using eq. (6). The degrees of freedom for air were calculated to 6.0 ± 1.1 and for argon 3.3 ± 0.3. The standard deviation was calculated in the same way as for the heat capacity ratio, by taking the largest and smallest value in the confidence interval, i.e. $\bar{\gamma} \pm s_\gamma$. The degrees of freedom for argon corresponds to a gas were the particles only have translational motion. For air we got 6 degrees of freedom which correspond to the particles having translational, rotational and vibrational motion. Taking the standard deviation into account the particles can also have either 5 or 7 degrees of freedom. 5 corresponding to translational and rotational motion while 7 give one additional rotational or vibrational degree.

5. Discussion and Conclusions

Our measured velocity for air was 336 ± 6.3 while the theoretical value is 344 m/s, calculated with eq. (7), the measured temperature, the tabulated value of the heat capacity ratio [6] and molar weight [8]. Our confidence interval does not include the theoretical value, probably because we have a systematic error in our time measurements. Since it was difficult to observe exactly when the sound wave was received there is a possibility that we underestimated when the pulse started and made measurements too late. For low pressures the received signal was weaker which made it even more difficult to determine the time.

The measured velocity for argon was 314 ± 5.5 m/s and the theoretical value is 320 m/s, calculated in the same way as for air with the argon molar mass 39.95 g/mol [9]. This measured value with confidence interval is also lower than the theoretical value which supports the systematic error assumption.

From our measurements we calculated the heat capacity ratio to 1.33 ± 0.05 which corresponds to, using eq. (6), 6.0 ± 1.1 degrees of freedom. According to this value the molecules in air should have translational, rotational and vibrational motion. However, assuming that the measured velocities are too low we can assume that the lower region of the confidence interval is more accurate. This would indicate that a value of 5 degrees of freedom is more correct. Under this assumption we conclude that air has 5 degrees of freedom and thus only have translational and rotational motion at room temperature. To verify this conclusion we compare to the tabulated value [6] for the heat capacity ratio of air, 1.40 which corresponds to 5 degrees of freedom.

Our experimentally determined value of the heat capacity ratio for argon was 1.61 ± 0.06. This corresponds to 3.3 ± 0.3 degrees of freedom which seems reasonable since argon is an atomic gas and should only have translational degrees of freedom. To verify that this is correct we looked at the tabulated value [6] for the heat capacity ratio, 1.67. This value gives 3 degrees of freedom for argon, i.e. only translational motion.

In addition to the error when reading the values on the oscilloscope we have other sources of error such as the temperature measurement and the argon concentration in the tube.

During the experiment the temperature in the room increased slightly since the vacuum pump got heated up. We made only one measurement of the temperature after the experiment even though it would have been better to measure both before and after the experiment. By doing multiple measurements it is possible to see how large the temperature difference is and for instance use a mean value of the temperatures in the calculations. Also, the error in the thermometer measurement is neglected in calculations. This is because the error in the velocity dominates the total error.

During the argon measurement we have a possible error if the tube was not properly sealed. This would enable air to enter the tube and change the concentration of argon which would affect the time measurements. Also, the standard deviation in the time measurement is only an estimation of how accurate we could read off the pulse on the oscilloscope, the real error could be much larger.

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