Synthetic Impulse Response Functions

David Lundquist[*], Daniel Eck[†]

Department of Statistics, University of Illinois at Urbana-Champaign

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Abstract

We adopt techniques from an inferential procedure known as synthetic control to construct a new impulse response function (IRF) estimator. Distinct from estimates generated from the Wold decomposition or local projections (LP), synthetic IRFs leverage information from the context surrounding a shock with the goals of both reducing risk while also limiting bias. The method relies upon Wold and/or LP IRF estimates on multivariate time series from a "donor pool". These estimates in turn are aggregated using distanced-based weighting, a procedure in which the donor multivariate series are judged based on similarity to the target multivariate series. We also develop a procedure to discount the donor series based on signal-to-noise ratio. This adjustment supports the "unit-shock" convention used in impulse response function analysis. Simulations and empirical examples are provided.

1 Introduction

The technique of distanced-based weighting and synthetic, aggregation methods is naturally suited to IRF and vector autoregressions more generally due to the multivariate setting that comes for free. There is not necessarily a need to locate predictive or otherwise informative covariates.

Challenge: suppose a researcher knew that a countable set of donors $\mathcal{D}$ could be used to maximize fit (loosely defined) in the context of impulse response function estimation. However, for a particular donor $d_m, m \in \mathcal{D}$, the shock at time T^* is not observed and hence its magnitude must be inferred.

1. Estimate the shock ϵ_{i,T^*+1} for each donor.
   (a) Use estimated residuals
   (b) Use fixed effect to estimate shock. Now, is this counter to VAR analysis?
2. If the estimation technique parametric assumption, then we can scale estimates appropriately to satisfy the unit-shock assumption.
3. Confidence intervals? Strong assumptions needed?
4. Need to be very clear about whether this is an inferential or predictive tool. Or is it both?

2 Questions

1. Do we wait until the shock has already manifested?
   (a) If so, then it's different from other applications of post-shock forecasting.
   (b) If not, then are we combining post-shock forecasting with IRF estimation?

[*]davidl11@ilinois.edu
[†]dje13@illinois.edu

2.1 What are we estimating and what are we adjusting?

1. Do we need to estimate a model for the time series under study and then adjust that model?
   (a) Perhaps that for the TSUS, we can avoid assuming a particular model / method for the IRF and can instead just use dbw on the IRFs of the donors.
2. How do we get IRFs of donors? Either MA representation or local projections?
3. What is it exactly that we're trying to predict? The effect of a size-delta shock in series A on series B exactly K time points after the shock has manifested.
4. If we are willing to dispense with linearity (which seems like a reasonable choice, given that we're employing an approach that doesn't have a problem accommodating it)

1. Can we use the Wold Decomposition as ground truth?
2. read **?**
3. Can we use synthetic volatility forecasting for Value-at-Risk (VaR)?

3 The estimand

In a multivariate time series of dimension K, the response ith-step-ahead response of component j to a shock in component k at time t is

$$\frac{\partial y_{j,t+i}}{\partial w_{k,t}}$$

where w_t is the vector of innovations at time t for some model $y_t = f(\mathcal{F}_{t-1}) + w_t$.

4 Estimators

Estimators for an IRF may come in at least two forms: model-specific and model independent.

4.1 Unit shock - what does it mean to be scale-free or scale-independent?

5 The Special Case of Vector Autoregressions (VAR)