Synthetic Volatility Forecasting and Other Aggregation Techniques for Time Series Forecasting
Preliminary Exam

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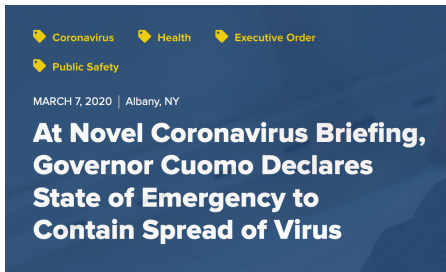April 10th, 2024

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A seemingly unprecedented event might make one ask

1. What does it resemble from the past?
2. What past events are most relevant?
3. Can we incorporate past events in a systematic, principled manner?

When would we ever have to do this?

- Event-driven investing strategies (unscheduled news shock)
- Pairs trading strategies
- Structural shock to macroeconomic conditions (scheduled news possibly pre-empted by news shock)
- Biomedical panel data subject to exogenous shock or interference

Example (Weekend of March 6th - 8th, 2020)

🏷 Coronavirus   🏷 Health   🏷 Executive Order

🏷 Public Safety

MARCH 7, 2020 | Albany, NY

At Novel Coronavirus Briefing, Governor Cuomo Declares State of Emergency to Contain Spread of Virus

Oil nose-dives as Saudi Arabia and Russia set off 'scorched earth' price war

PUBLISHED SUN, MAR 8 2020·9:01 AM EDT | UPDATED MON, MAR 9 2020·5:33 PM EDT

Oil crashes by most since 1991 as Saudi Arabia launches price war

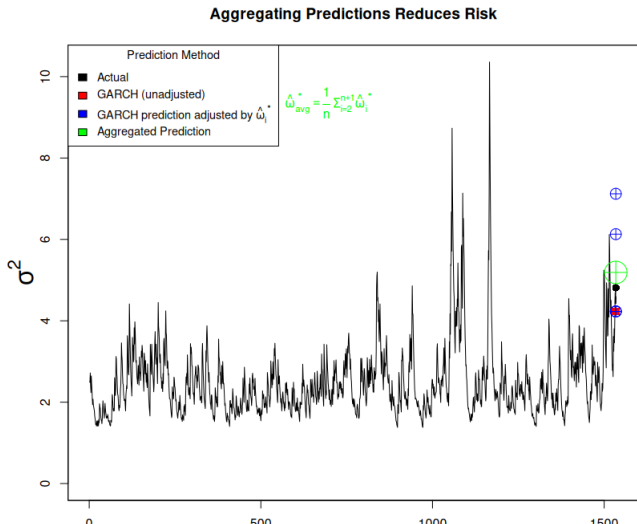By Matt Egan, CNN Business

⏱ 3 minute read · Updated 3:21 PM EDT, Mon March 9, 2020

Punchline of the paper

Credible forecasting is possible under news shocks, so long as we incorporate external information to account for the nonzero errors.

Aggregating Predictions Reduces Risk

Background and related methods

Volatility Modeling

- GARCH is slow to react to shocks (Andersen et al. 2003)
- Asymmetric GARCH models catch up faster but need post-shock data
- Realized GARCH (Hansen, Huang, and Shek 2012), in our setting, would require post-shock information and/or high-frequency data in order to outperform, and Realized GARCH is highly parameterized

Background and related methods

Forecast Augmentation

- Clements and Hendry 1998; Clements and Hendry 1996 laid the groundwork for modeling nonzero errors in time series forecasting
- Guerrón-Quintana and Zhong 2017 use a series' own errors to correct the forecast for that series
- Dendramis, Kapetanios, and Marcellino 2020 use a similarity-based procedure to correct linear parameters in time series forecasts
- Foroni, Marcellino, and Stevanovic 2022 adjust pandemic-era forecasts using intercept correction techniques and data from Great Financial Crisis
- Lin and Eck 2021 use distanced-based weighting (a similarity approach) to aggregate and weight fixed effects from a donor pool

Outline

Premise: News has broken but markets are closed

- After-hours trading provides a poor forum in which to digest news
- The news constitutes public, material information relevant to one or more traded assets
- The qualitative aspects of the news provide a basis upon which to match to past events

A Primer on GARCH

Let $\{a_t\}$ denote an observable, real-valued discrete-time stochastic process.
We say $\{a_t\}$ is a strong GARCH process with respect to $\{\epsilon_t\}$ iff

$$\sigma_t^2 = \omega + \sum_{k=1}^{m} \alpha_k a_{t-k}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2$$

$$a_t = \sigma_t \epsilon_t$$

$$\epsilon_t \overset{iid}{\sim} E[\epsilon_t] = 0, \, Var[\epsilon_t] = 1$$

$$\forall k, j, \alpha_k, \beta_j \geq 0$$

$$\forall t, \omega, \sigma_t > 0$$

Volatility Equation with an exogenous term: GARCH-X

$$\sigma_t^2 = \omega + \sum_{k=1}^{m} \alpha_k a_{t-k}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2 + \gamma^T \mathbf{x}_t \ .$$

Model Preliminaries

Let $I(\cdot)$ be an indicator function.

Let T_i denote the time length of the time series i for $i = 1, \ldots, n + 1$.

Let T_i^* denote the largest time index prior to news shock, with $T_i^* < T_i$.

Let $\delta, \mathbf{x}_{i,t} \in \mathbb{R}^p$.

Model Setup

For $t = 1, \ldots, T_i$ and $i = 1, \ldots, n+1$, the model $\mathcal{M}_1$ is defined as

$$\sigma_{i,t}^2 = \omega_i + \omega_i^* + \sum_{k=1}^{m_i} \alpha_{i,k} a_{i,t-k}^2 + \sum_{j=1}^{s_i} \beta_{i,j} \sigma_{i,t-j}^2 + \gamma_i^T \mathbf{x}_{i,t}$$

$$\mathcal{M}_1: \quad a_{i,t} = \sigma_{i,t}((1 - D_{i,t}^{return})\epsilon_{i,t} + D_{i,t}^{return}\epsilon_{i,t}^*)$$

$$\omega_{i,t}^* = D_{i,t}^{vol}[\mu_{\omega^*} + \delta' \mathbf{x}_{i,t-1} + u_{i,t}],$$

with error structure

$$\epsilon_{i,t} \overset{iid}{\sim} \mathcal{F}_\epsilon \text{ with } \mathrm{E}_{\mathcal{F}_\epsilon}(\epsilon) = 0, \mathrm{Var}_{\mathcal{F}_\epsilon}(\epsilon) = 1$$

$$\epsilon_{i,t}^* \overset{iid}{\sim} \mathcal{F}_{\epsilon^*} \text{ with } \mathrm{E}_{\mathcal{F}_{\epsilon^*}}(\epsilon) = \mu_{\epsilon^*}, \mathrm{Var}_{\mathcal{F}_{\epsilon^*}}(\epsilon^*) = \sigma_{\epsilon^*}^2$$

$$u_{i,t} \overset{iid}{\sim} \mathcal{F}_u \text{ with } \mathrm{E}_{\mathcal{F}_u}(u) = 0, \mathrm{Var}_{\mathcal{F}_u}(u) = \sigma_u^2$$

$$\epsilon_{i,t} \perp\!\!\!\perp \epsilon_{i,t}^* \perp\!\!\!\perp u_{i,t}$$

where $D_{i,t}^{return} = I(t \in \{T_i^* + 1, ..., T_i^* + L_{i,return}\})$ and $D_{i,t}^{vol} = I(t \in \{T_i^* + 1, ..., T_i^* + L_{i,vol}\})$ and $L_{i,return}, L_{i,vol}$ denote lengths of log return and volatility shocks, respectively.

Model Details

Let $\mathcal{M}_0$ denote the subclass of $\mathcal{M}_1$ models such that $\delta \equiv 0$.

Note that $\mathcal{M}_0$ assumes that ω_i^* have no dependence on the covariates and are i.i.d. with $\mathbb{E}[\omega_i^*] = \mu_{\omega^*}$.

Our Model is Nested inside a Factor Model

Consider $\mathcal{M}_0$ in the context of the factor model from Abadie, Diamond, and Hainmueller 2010, where an untreated unit is governed by:

$$Y_{i,t}^N = \delta_t + \boldsymbol{\theta}_t \mathbf{Z}_i + \boldsymbol{\lambda}_t \boldsymbol{\mu}_i + \varepsilon_{i,t}$$

which nests the GARCH model's volatilty equation (putting aside that σ_t is latent in the GARCH model) as well as the ARMA representation of a GARCH model, where

$\delta_t \sim \omega$, a location parameter shared across donors

$\boldsymbol{\theta}_t \sim \boldsymbol{\alpha}_k$, a vector of ARCH parameters and other coefficients shared across donors

$\mathbf{Z}_i \sim \boldsymbol{a}_{i,t-k}$, a vector of observable quantities specific to each donor

$\boldsymbol{\lambda}_t \sim \boldsymbol{\beta}_j$, a vector of GARCH parameters shared across donors

$\boldsymbol{\mu}_i \sim \boldsymbol{\sigma}_{i,t-j}^2$, a vector of latent quantities specific to each donor

Volatility Profile of a Time Series

$$\mathbf{V}_t = \begin{pmatrix} \hat{\alpha}_{1,t} & \hat{\alpha}_{t,2} & \cdots & \hat{\alpha}_{t,n} \\ \hat{\beta}_{1,t} & \hat{\beta}_{t,2} & \cdots & \hat{\beta}_{t,n} \\ \vdots & \vdots & \ddots & \vdots \\ RV_{1,t} & RV_{2,t} & \cdots & RV_{n,t} \\ RV_{1,t-1} & RV_{2,t-1} & \cdots & RV_{n,t-1} \\ \vdots & \vdots & \ddots & \vdots \\ IV_{1,t} & IV_{2,t} & \cdots & IV_{n,t} \\ IV_{1,t-1} & IV_{2,t-1} & \cdots & IV_{n,t-1} \\ \vdots & \vdots & \ddots & \vdots \\ |r_{1,t}| & |r_{2,t}| & \cdots & |r_{n,t}| \\ |r_{1,t-1}| & |r_{2,t-1}| & \cdots & |r_{n,t-1}| \end{pmatrix},$$

where RV denotes realized variance, IV the implied volatility

Significance of the Volatility Profile

Covariates chosen for inclusion in a given volatility profile may be

- levels
- differences in levels
- log differences in levels
- percentage changes in levels
- absolute values of the above

Key criterion for inclusion: how plausible is the covariate as a proxy for risk conditions?

Forecasting

We present two forecasts:

Forecast 1: $\hat{\sigma}^2_{unadjusted} = \hat{\mathbb{E}}_{T^*}[\sigma^2_{1,T_1^*+1}|\mathcal{F}_{T^*}] = \hat{\omega}_i + \sum_{k=1}^{m_i} \hat{\alpha}_{i,k} a^2_{i,t-k} + \sum_{j=1}^{s_i} \hat{\beta}_{i,j} \sigma^2_{i,t-j} + \hat{\gamma}_i^T \mathbf{x}_{i,t}$

Forecast 2: $\hat{\sigma}^2_{adjusted} = \hat{\mathbb{E}}_{T^*}[\sigma^2_{1,T_1^*+1}|\mathcal{F}_{T^*}] + \hat{\omega}^* = \hat{\omega}_i + \sum_{k=1}^{m_i} \hat{\alpha}_{i,k} a^2_{i,t-k} + \sum_{j=1}^{s_i} \hat{\beta}_{i,j} \sigma^2_{i,t-j} + \hat{\gamma}_i^T \mathbf{x}_{i,t} + \hat{\omega}^*$.

Excess Volatility Estimators

Observe the pair $(\{\hat{\omega}_i^*\}_{i=2}^{n+1}, \{\mathbf{v}_i\}_{i=2}^{n+1})$.

Goal: recover weights $\{\pi_i\}_{i=2}^{n+1} \in \Delta^n$ and compute $\hat{\omega}^* \coloneqq \sum_{i=2}^{n+1} \pi_i \hat{\omega}_i^*$, our forecast adjustment term.

Following Abadie, Diamond, and Hainmueller 2010; Abadie and Gardeazabal 2003, let $\|\cdot\|_{\mathbf{s}}$ denote any semi-norm on $\mathbb{R}^p$, and define

$$\{\pi\}_{i=2}^{n+1} = \arg\min_{\pi} \|\mathbf{v}_{1,\mathcal{T}^*} - \mathbf{V}_{\mathcal{T}^*}\pi\|_{\mathbf{s}} .$$

Nota bene: the weights $\{\pi_i\}_{i=2}^{n+1}$ are deterministic with respect to $\mathcal{F}_{\mathcal{T}^*}$.

Ground Truth Estimators

We use realized volatility (RV)

- "model-free" in the sense that it requires no modeling assumptions (Andersen and Benzoni 2010).
- RV can be decomposed into the sum of a continous component and a jump component, with the latter being less predictable and less persistent (Andersen, Bollerslev, and Diebold 2007), cited in De Luca et al. 2006, two factors that further motivate our method.

Realized Volatility Estimation

Examine K units of of time; each unit is divided into m intervals of length $\frac{1}{m}$. Let $p_t = \log P_t$, and let $\tilde{r}(t, \frac{1}{m}) = p_t - p_{t-\frac{1}{m}}$ (Andersen and Teräsvirta 2009).

Estimate variance of ith log return series using Realized Volatility of the K consecutive trading days that conclude with day t, denoted $RV_{i,t}^{K,m}$, using

$$RV_{i,t}^{K,m} = \frac{1}{K} \sum_{v=1}^{Km} \tilde{r}^2(v/m, 1/m),$$

where the K trading days have been chopped into Km equally-sized blocks.

Assuming the K units $\tilde{r}(t,1) = p_t - p_{t-1}$ are s.t. $\tilde{r}(t,1) \overset{iid}{\sim} N(\mu, \delta^2)$, it is easily verified that

$$\mathbb{E}[RV^{K,m}] = \frac{\mu^2}{m} + \delta^2,$$

which is a biased but consistent estimator of the variance. We pick $m = 77$, corresponding to the 6.5-hour trading day chopped into 5-minute blocks, omitting first five-minutes of the day.

Loss Functions

Aim: point forecasts for $\sigma^2_{1, T^*+h}|\mathcal{F}_{T^*}$, $h = 1, 2, ..., $, the h-step ahead conditional variance for the time series under study

Let L^h with the subscripted pair {prediction method, ground truth estimator}, denote the loss function for an h-step-ahead forecast using a given prediction function and ground truth estimator.

Loss Function Examples

For example, one loss function of interest in this study is the 1-step-ahead MSE using Synthetic Volatility Forecasting and Realized Volatility:

$$\text{MSE}^1_{\text{SVF, RV}} = (\hat{\sigma}^2_{SVF} - \hat{\sigma}^2_{RV})^2$$

Also of interest in mean absolute percentage error for an h-step-ahead forecast, defined as

$$\text{MAPE}^h_{method,groundtruth} = \frac{|\hat{\sigma}^2_{h,method} - \hat{\sigma}^2_{h,groundtruth}|}{\hat{\sigma}^2_{h,groundtruth}}$$

Our choice of Loss Function

Finally, we introduce the QL (quasi-likelihood) Loss (Brownlees, Engle, and Kelly 2011):

$$\text{QL}^h_{method,groundtruth} = \frac{\hat{\sigma}^2_{h,method}}{\hat{\sigma}^2_{h,groundtruth}} - \log \frac{\hat{\sigma}^2_{h,method}}{\hat{\sigma}^2_{h,groundtruth}} - 1 \ .$$

What distinguishes QL Loss?

- multiplicative rather than additive
- This has benefits, both practical and theoretical.
- As Brownlees, Engle, and Kelly 2011 explain, "[a]mid volatility turmoil, large MSE losses will be a consequence of high volatility without necessarily corresponding to deterioration of forecasting ability. The QL avoids this ambiguity, making it easier to compare losses across volatility regimes."

iShares U.S. Financial Services ETF

Figure: Includes: JPM, BAC, WF, among other financial majors

1. **Model choice** GARCH(1,1) on the daily log return series of IYG in each donor
2. **Covariate Choice** log return Crude Oil (CL.F), the VIX (VIX) and the log return of the VIX, the log returns of the 3-month, 5-year, 10-year, and 30-year US Treasuries, return of the most recently available monthly spread between AAA and BAA corporate debt, widely considered a proxy for lending risk (Goodell and Vähämaa 2013; Kane, Marcus, and Noh 1996). We also include the log return in the trading volume of the ETF IYG itself, which serves as a proxy for panic.
3. **Donor pool construction** the three most recent US presidential elections prior to the 2016 election. The three US presidential elections are the only presidential elections since the advent of the ETF IYG. We exclude the midterm congressional elections in the US, which generate far lower voter turnout and feature no national races.
4. **Choice of estimator for volatility** Sum of squared 5-minute log returns of IYG on November 9th, 2016, otherwise known as the Realized Volatility estimator of volatility (Andersen and Teräsvirta 2009), as our proxy. We exclude the first five minutes of the trading day, resulting in a sum of 77 squared five-minute returns generated between 9:35am and 4pm.
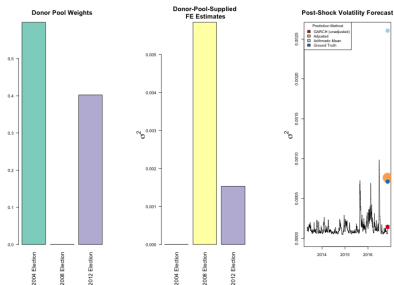
2016 Election

Figure: The volatility induced by the 2016 US election

Simplest Simulation Setup

Most elementary simulation uses $\mathcal{M}_1$ and varies only two parameters.

Recall an $\mathcal{M}_1$ model on the volatility, which is characterized by an exogenous shock to the volatility equation generated by an affine function of the covariates:

$$\mathcal{M}_1: \begin{aligned} &\sigma_{i,t}^2 = \omega_i + \omega_i^* + \sum_{k=1}^{m_i} \alpha_{i,k} a_{i,t-k}^2 + \sum_{j=1}^{s_i} \beta_{i,j} \sigma_{i,t-j}^2 + \gamma_i^T \mathbf{x}_{i,t} \\ &a_{i,t} = \sigma_{i,t}((1 - D_{i,t}^{return})\epsilon_{i,t} + D_{i,t}^{return}\epsilon_i^*) \\ &\omega_{i,t}^* = D_{i,t}^{vol}[\mu_{\omega^*} + \delta' \mathbf{x}_{i,t-1} + u_{i,t}] \\ &D_{i,t}^{return} \equiv 0 \end{aligned}$$

What do we vary?

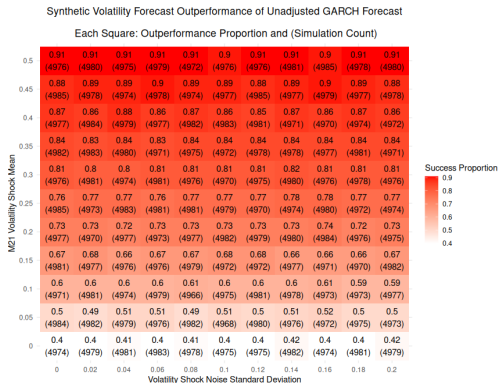The volatility shock signal δ and the variance of volatility shock noise u_t

Figure: Fixed parameter values: $\alpha = .1, \beta = .82, \mu_x = 1, \sigma_x = .1$

If we switch the values of α and β, we see similar behavior.
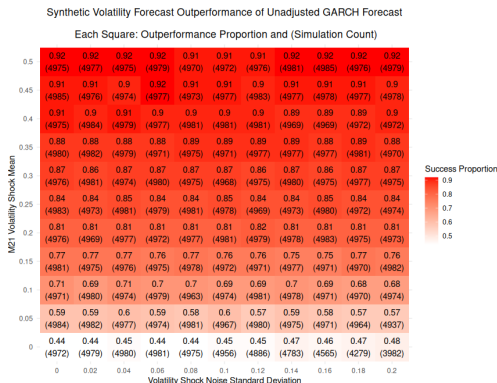

Figure: Fixed parameter values: $\alpha = .82, \beta = .1, \mu_x = 1, \sigma_x = .1$

We shall group the extensions into five buckets:

- How much can we automate?
- Alternatives for fixed effect estimation
- Alternative estimators and estimands
- What can you do with a volatility forecast?
- Where else is distanced-based weighting useful?

Alternative Ways of Estimating Fixed Effects

High-frequency data?

- Realized GARCH with High-Frequency Data
- Stochastic Volatility

Alternative Estimators and Estimands in Volatility Modeling

- Factors in volatility profile
- Overnight returns instead of open-to-close
- Signal Recovery Perspective (Ferwana and Varshney 2022)
- Stochastic Volatility: Correlation between errors
- Multivariate GARCH

What can you do with a volatility forecast?

- Value-at-Risk using SVF-based $\hat{\sigma}_t^2$

New Frontiers in Distance-based Weighting

- Integrate lessons from literature on under/over reactions to information shocks (Jiang and Zhu 2017)
- Synthetic Impulse Response Functions

Synthetic Impulse Response Functions: A Proposal

- Suppose we have a multivariate time series of dimension $p \times T$ subject to shocks from a common shock distribution
- Using an IRF estimate aggregated from the first n shocks of interest, we predict the response of variable i from variable j, $1 \leq i \leq j \leq p$.

We analyze the real data example with Brexit included.
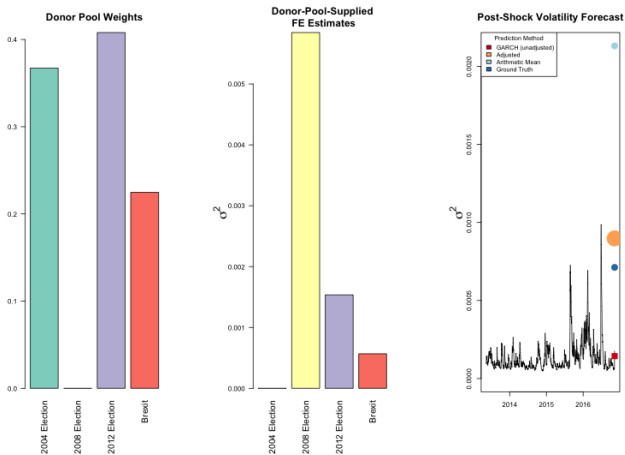
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