Synthetic Volatility Forecasting and Other Aggregation Techniques for Time Series Forecasting

Preliminary Exam

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A seemingly unprecedented event might provoke the questions

1. What does it resemble from the past?
2. What past events are most relevant?
3. Can we incorporate past events in a systematic, principled manner?

When would we ever have to do this?

- Event-driven investing strategies (unscheduled news shock)
- Pairs trading strategies
- Structural shock to macroeconomic conditions (scheduled news possibly pre-empted by news shock)
- Biomedical panel data subject to exogenous shock or interference

Example: weekend of March 7th and 8th, 2020

Punchline of the paper

Forecasting is possible under structural shocks, so long as we incorporate external information to account for the nonzero errors.

Background and related methods

Volatility Modeling

- GARCH is slow to react (Andersen et al. 2003)
- Asymmetric GARCH models catch up faster but need post-shock data
- Realized GARCH (Hansen, Huang, and Shek 2012), in our setting, would require post-shock information and/or high-frequency data in order to outperform, and the model is highly parameterized

Background and related methods

Forecast Augmentation

- Clements and Hendry 1998; Clements and Hendry 1996 laid the groundwork for modeling nonzero errors in time series forecasting
- Guerrón-Quintana and Zhong 2017 use a series' own errors to correct the forecast for that series
- Dendramis, Kapetanios, and Marcellino 2020 use a similarity-based procedure to correct linear parameters in time series forecasts
- Foroni, Marcellino, and Stevanovic 2022 adjust pandemic-era forecasts using intercept correction techniques and data from Great Financial Crisis
- Lin and Eck 2021 use distanced-based weighting (a similarity approach) to aggregate and weight fixed effects from a donor pool

Outline

1. Introduction
2. Setting
   - Model Setup
   - Volatility Profile of a Time Series
3. Post-shock Synthetic Volatility Forecasting Methodology
4. Properties of Volatility Shock and Shock Estimators
5. Real Data Example
6. Numerical Examples
7. Discussion
8. Future directions for Synthetic Volatility Forecasting
   - Signal Recovery
   - Synthetic Impulse Response Functions
9. Supplement

The news has broken but markets are closed

- $y \in \mathbb{R}^n$, a mean-zero, real-valued response to be predicted

A Primer on GARCH

$$\sigma_t^2 = \omega + \sum_{k=1}^{m} \alpha_k a_{t-k}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2$$

$$a_t = \sigma_t \epsilon_t$$

$$\epsilon_t \overset{iid}{\sim} E[\epsilon_t] = 0, Var[\epsilon_t] = 1$$

$$\forall k, j, \alpha_k, \beta_j \geq 0$$

$$\forall t, \omega, \sigma_t > 0$$

Our Model is Nested Within GARCH-X

Volatility Profile

$$
\mathbf{v}_{p,n} = \begin{pmatrix}
\alpha_{T^*,1} & \alpha_{T^*,2} & \cdots & \alpha_{T^*,n} \\
\beta_{T^*,1} & \beta_{T^*,2} & \cdots & \beta_{T^*,n} \\
\vdots & \vdots & \ddots & \vdots \\
RV_{T^*,1} & RV_{T^*,2} & \cdots & RV_{T^*,n} \\
RV_{T^*-1,1} & RV_{T^*-1,2} & \cdots & RV_{T^*-1,n} \\
\vdots & \vdots & \ddots & \vdots \\
IV_{T^*,1} & IV_{T^*,2} & \cdots & IV_{T^*,n} \\
IV_{T^*-1,1} & IV_{T^*-1,2} & \cdots & IV_{T^*-1,n} \\
\vdots & \vdots & \ddots & \vdots \\
AbsoluteReturn_{T^*,1} & AbsoluteReturn_{T^*,2} & \cdots & AbsoluteReturn_{T^*,n} \\
AbsoluteReturn_{T^*-1,1} & AbsoluteReturn_{T^*-1,2} & \cdots & AbsoluteReturn_{T^*-1,n} \\
\vdots & \vdots & \ddots & \vdots \\
Volume_{T^*,1} & Volume_{T^*,2} & \cdots & Volume_{T^*,n} \\
Volume_{T^*-1,1} & Volume_{T^*-1,2} & \cdots & Volume_{T^*-1,n}
\end{pmatrix}
$$

What's the method here?

$$2 = 3$$

Minimum Norm Estimator

Key Conceptual Innovation: Effective Rank

Main Result: Existence Proof, Dichotomy, and Bounds

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Remarks

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Some things to think about with papers like this

Simplest Simulation Setup

A more benign example

Example (Coverging at the slowest rate possible)

Fix $\alpha = 1, \beta > 1$. Let $\lambda_i = \frac{1}{i \log^{\beta}(i+1)}$.

How noise is hidden just right

After all of this waiting, we formalize the notion under discussion.

Definition (Asymptotically Benign)

We analyze the real-world example with Brexit included.

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