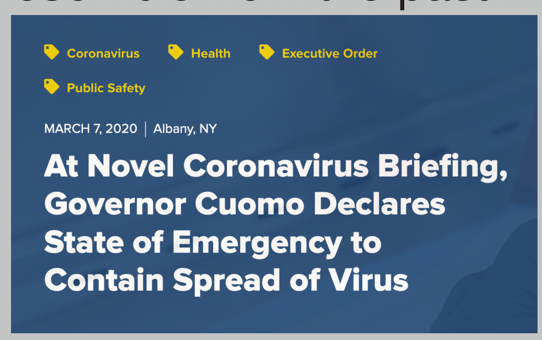
Volatility Forecasting Using Similarity-based Parameter Correction and Aggregated Shock Information

David P. Lundquist[1] and Daniel J. Eck[1]

[1]Department of Statistics, University of Illinois Urbana-Champaign

Introduction - Reacting to the Unprecedented

1. Reacting to a seemingly unprecedented event might involve the question: what, if anything, does it resemble from the past?


Coronavirus · Health · Executive Order · Public Safety

MARCH 7, 2020 | Albany, NY

At Novel Coronavirus Briefing, Governor Cuomo Declares State of Emergency to Contain Spread of Virus

2. Matching a crisis to past events is a problem with unsurprising statistical angles: identification, sample size, weighting, risk, robustness.
3. Here we employ a method to improve our GARCH-X volatility forecasts under unprecedented conditions.

The Family of GARCH-X Volatility Models

We define a family of $n+1$ univariate times series, each of length T_i, $i = 1, \ldots n+1$. For each i, there exists a news shock that occurs strictly between T_i^* and T_i^*+1, and for each i, there exists a GARCH-X model

$$\sigma_{i,t}^2 = \omega_i + \sum_{k=1}^{m_i} \alpha_{i,k} a_{i,t-k}^2 + \sum_{j=1}^{s_i} \beta_{i,j} \sigma_{i,t-j}^2 + \gamma_i^T \mathbf{v}_{i,t} + \omega_{i,t}^* \tag{1}$$

$$a_{i,t} = \sigma_{i,t} \epsilon_{i,t} \tag{2}$$

with the “unprecedented” shocks parameterized by

$$\omega_{i,t}^* = D_{i,t}^{vol} [\mu_{\omega^*} + \delta^T \mathbf{v}_{i,T_i^*+1} + u_{i,t}] \tag{3}$$

where $\mathbf{v}_{i,T_i^*+1}$ is a vector of variables, deterministic with respect to $\mathcal{F}_{T_i^*+1}$, that reflects and encodes prevailing risk conditions. All other errors are mean-zero and idiosyncratic.

Objective

Provide a one-step-ahead volatility forecast for the *time series under study*, i.e. the first time series in the family above. We will denote the series $i = 2, \ldots, n+1$ the *donor series*, from which we will extract information.

Forecast Methodology

We estimate the ”unprecedented” shocks in the donor pool (i.e., $i = 2, 3, \ldots n+1$) using fixed-effect estimation during the shock times, yielding shock estimators $\{\hat{\omega}_{i,*}\}_{i=2}^{n+1}$. The aggregated shock estimator is given by

$$\hat{\omega}^* = \sum_{i=2}^{n+1} \pi_i \hat{\omega}_{i,*}^* \tag{4}$$

where the weights $\{\pi_i\}_{i=2}^{n+1}$ are non-negative, sum to one, and chosen to minimize the L^2-norm of the difference of $\mathbf{v}_{i,T_i^*}$ and the convex hull of $\{\mathbf{v}_{i,T_i^*}\}_{i=2}^{n+1}$, similar to the Synthetic Control approach in causal inference [1, 2]. We then produce an adjusted one-step-ahead forecast of the time series under study:

$$\hat{\sigma}_{1,t+1}^2 = \hat{\omega}_1 + \sum_{k=1}^{m_1} \hat{\alpha}_{1,k} a_{1,t+1-k}^2 + \sum_{j=1}^{s_1} \hat{\beta}_{1,j} \sigma_{1,t+1-j}^2 + \hat{\gamma}_1^T \mathbf{v}_{1,t+1} + \hat{\omega}^* \tag{5}$$

Evaluating Forecasts with QL Loss

$$\mathrm{QL}_{method,groundtruth}^h = \frac{\hat{\sigma}_{h,groundtruth}^2}{\hat{\sigma}_{h,method}^2} - \log \frac{\hat{\sigma}_{h,groundtruth}^2}{\hat{\sigma}_{h,method}^2} - 1 \tag{6}$$

- Quasi-likelihood Loss is that it is multiplicative rather than additive.
- The technical properties of the QL Loss allow researchers to compare forecasts across heterogeneous time series, whereas additive loss functions like MSE unfairly penalize forecasts made under market turbulence [3].

Real Data Example: Aftermath of Donald Trump’s 2016 Victory

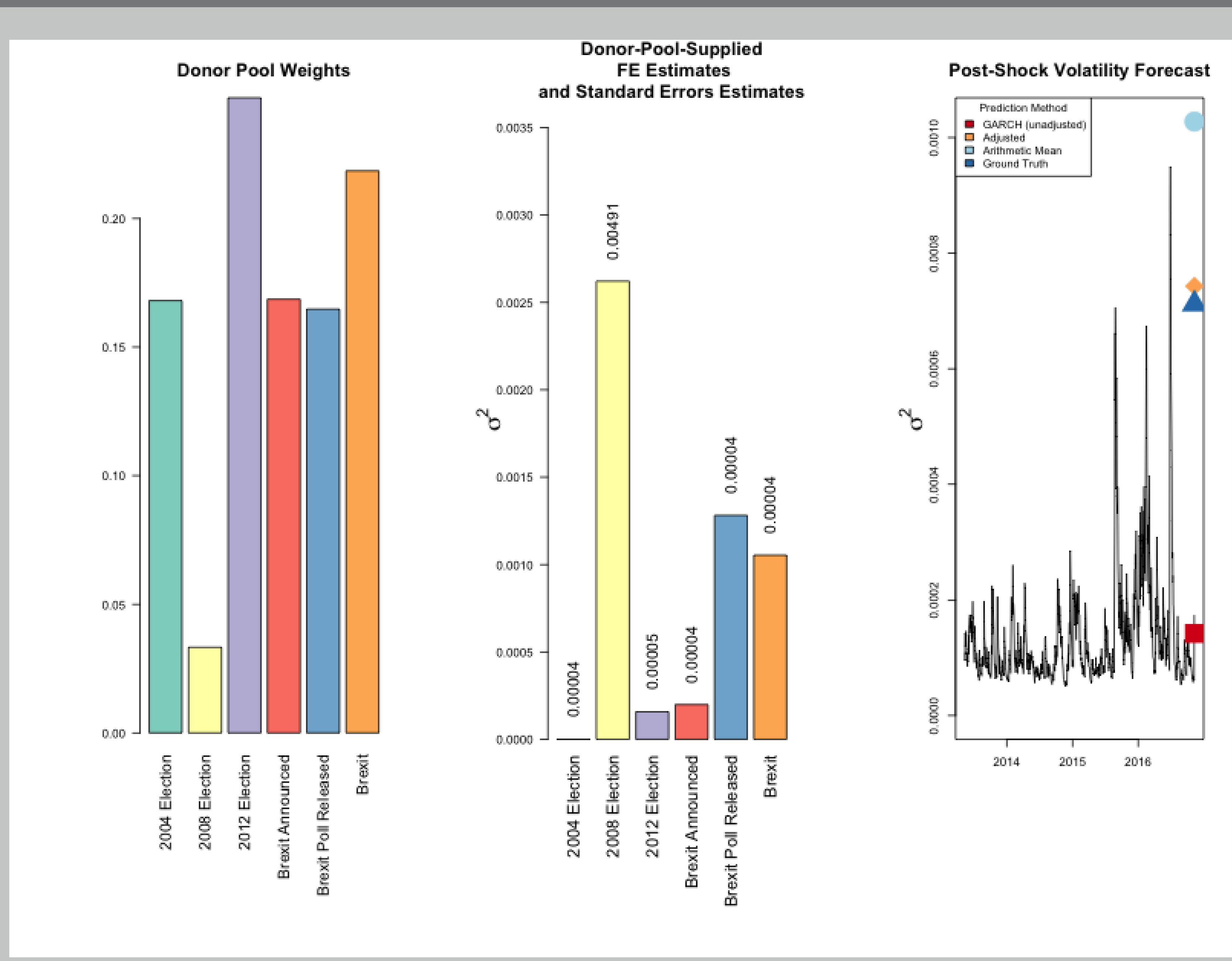


Figure 1: Donor pool weights (left panel), individual shock effects (center panel), and forecasts (right panel). In the right panel, we predict the volatility of the financial services exchange-traded fund IYG on November 9th, 2016. The adjusted prediction nearly recovers the realized volatility, which is consistently estimated using high-frequency data.

Conclusions

1. When news shocks undermine the credibility of the default forecasting model, a new method is called for.
2. Here we have attempted to model news shocks as parameterized by prevailing risk conditions.
3. We have also proposed an aggregation mechanism that allows the incorporation of past shock information into current forecasts.
4. Future directions may involve applications to HAR forecasts, VAR-based forecasts of multivariate time series, and impulse response functions.

References

[1] Alberto Abadie and Javier Gardeazabal.
The economic costs of conflict: A case study of the basque country.
American Economic Review, 93(1):113–132, 2003.

[2] Alberto Abadie, Alexis Diamond, and Jens Hainmueller.
Synthetic control methods for comparative case studies: Estimating the effect of california’s tobacco control program.
Journal of the American Statistical Association, 105(490):493–505, 2010.

[3] Christian T Brownlees, Robert F Engle, and Bryan T Kelly.
A practical guide to volatility forecasting through calm and storm.
Available at SSRN 1502915, 2011.

[4] David Lundquist and Daniel Eck.
Volatility forecasting using similarity-based parameter correction and aggregated shock information.
arXiv preprint arXiv:2406.08738, 2024.

Contact Information

- Web: https://lunddave.github.io
- Email: davidpatricklundquist@gmail.com
- Find our manuscript on ArXiv [4]