

Synthetic Volatility Forecasting and Other Aggregation Techniques for Time Series Forecasting

Doctoral Defense

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- 3 Can we incorporate past events in a systematic, principled manner?

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- Scheduled macroeconomic news possibly pre-empted by a news leak

ECONOMY

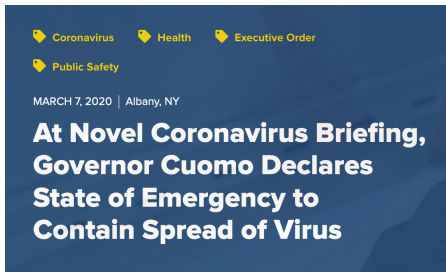
Fed Likely to Consider 0.75-Percentage-Point Rate Rise This Week

Officials had signaled plans to raise interest rates in half-point increments before recent deterioration in data

By Nick Timiraos [Follow](#)

Updated June 13, 2022 7:47 pm ET

Example (Weekend of March 6th - 8th, 2020)



Oil nose-dives as Saudi Arabia and Russia set off 'scorched earth' price war

PUBLISHED SUN, MAR 8 2020-9:01 AM EDT | UPDATED MON, MAR 9 2020-5:33 PM EDT

Oil crashes by most since 1991 as Saudi Arabia launches price war



By [Matt Egan](#), CNN Business

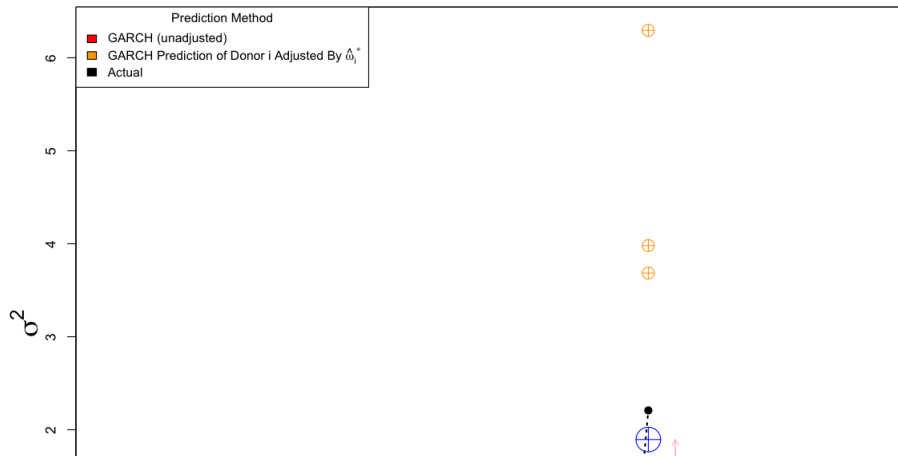
🕒 3 minute read · Updated 3:21 PM EDT, Mon March 9, 2020

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Background and related methods

Volatility Modeling

- GARCH is slow to react to shocks (Andersen et al. [2003](#))
- Asymmetric GARCH models catch up faster but need post-shock data
- Realized GARCH (Hansen, Huang, and Shek [2012](#)), in our setting, would require post-shock information and/or high-frequency data in order to outperform, and Realized GARCH is highly parameterized

Background and related methods

Forecast Adjustment

- Clements and Hendry 1998; Clements and Hendry 1996 laid the groundwork for modeling nonzero errors in time series forecasting
- Guerrón-Quintana and Zhong 2017 use a series' own errors to correct the forecast for that series
- Dendramis, Kapetanios, and Marcellino 2020 use a similarity-based procedure to correct $\hat{\beta}$ in time series forecasts
- Foroni, Marcellino, and Stevanovic 2022 adjust pandemic-era forecasts using intercept correction techniques and data from Great Financial Crisis
- Lin and Eck 2021 use distanced-based weighting (a similarity approach) to aggregate and weight fixed effects from a donor pool

Outline

- 1 Introduction
- 2 Setting
- 3 Post-shock Synthetic Volatility Forecasting Methodology
- 4 Properties of Volatility Shock and Shock Estimators
- 5 Real Data Example
- 6 Numerical Examples
- 7 Future directions for Synthetic Volatility Forecasting
- 8 Supplement

Premise: News has broken but markets are closed

- After-hours trading provides a poor forum in which to digest news
- News constitutes public, material information for one or more traded assets
- The **qualitative aspects** of the news provide a basis upon which to
 - match to past news shocks
 - match in a p -dimensional covariate space

A Primer on GARCH

Definition

Let $\{a_t\}$ denote an observable, real-valued discrete-time stochastic process.

We call $\{a_t\}$ a strong GARCH process (Francq and Zakoian 2019) with respect to $\{\epsilon_t\}$ iff

$$\sigma_t^2 = \omega + \sum_{k=1}^m \alpha_k a_{t-k}^2 + \sum_{j=1}^s \beta_j \sigma_{t-j}^2$$

$$a_t = \sigma_t \epsilon_t$$

$$\epsilon_t \stackrel{iid}{\sim} E[\epsilon_t] = 0, \text{Var}[\epsilon_t] = 1$$

$$\forall k, j, \alpha_k, \beta_j \geq 0$$

$$\forall t, \omega, \sigma_t > 0$$

Volatility Equation with an exogenous term: GARCH-X

$$\sigma_t^2 = \omega + \sum_{k=1}^m \alpha_k a_{t-k}^2 + \sum_{j=1}^s \beta_j \sigma_{t-j}^2 + \gamma^T \mathbf{x}_t .$$

We will be looking at only one exogenous term.

Model Preliminaries

Let $I(\cdot)$ be an indicator function.

Let T_i denote the time length of the time series i for $i = 1, \dots, n + 1$.

Let T_i^* denote the largest time index prior to news shock, with $T_i^* < T_i$ (i.e. we assume at least one post-shock observation).

Let $\delta, \mathbf{v}_{i,t} \in \mathbb{R}^p, \mathbf{x}_{i,t} \in \mathbb{R}^d$.

Model Setup

For $t = 1, \dots, T_i$ and $i = 1, \dots, n + 1$, the model \mathcal{M}_1 is defined as

$$\sigma_{i,t}^2 = \omega_i + \omega_{i,t}^* + \sum_{k=1}^{m_i} \alpha_{i,k} a_{i,t-k}^2 + \sum_{j=1}^{s_i} \beta_{i,j} \sigma_{i,t-j}^2 + \gamma_i^T \mathbf{x}_{i,t}$$

$$\mathcal{M}_1: a_{i,t} = \sigma_{i,t}((1 - D_{i,t}^{\text{return}})\epsilon_{i,t} + D_{i,t}^{\text{return}}\epsilon_i^*)$$

$$\omega_{i,t}^* = D_{i,t}^{\text{vol}}[\mu_{\omega^*} + \delta' \mathbf{v}_{i,t} + u_{i,t}],$$

with error structure

$$\epsilon_{i,t} \stackrel{iid}{\sim} \mathcal{F}_{\epsilon} \text{ with } E_{\mathcal{F}_{\epsilon}}(\epsilon) = 0, \text{Var}_{\mathcal{F}_{\epsilon}}(\epsilon) = 1$$

$$\epsilon_{i,t}^* \stackrel{iid}{\sim} \mathcal{F}_{\epsilon^*} \text{ with } E_{\mathcal{F}_{\epsilon^*}}(\epsilon) = \mu_{\epsilon^*}, \text{Var}_{\mathcal{F}_{\epsilon^*}}(\epsilon^*) = \sigma_{\epsilon^*}^2$$

$$u_{i,t} \stackrel{iid}{\sim} \mathcal{F}_u \text{ with } E_{\mathcal{F}_u}(u) = 0, \text{Var}_{\mathcal{F}_u}(u) = \sigma_u^2$$

$$\epsilon_{i,t} \perp\!\!\!\perp \epsilon_{i,t}^* \perp\!\!\!\perp u_{i,t}$$

where $D_{i,t}^{\text{return}} = I(t \in \{T_i^* + 1, \dots, T_i^* + L_{i,\text{return}}\})$ and $D_{i,t}^{\text{vol}} = I(t \in \{T_i^* + 1, \dots, T_i^* + L_{i,\text{vol}}\})$ and $L_{i,\text{return}}, L_{i,\text{vol}}$ denote lengths of log return and volatility shocks, respectively.

Note: we will be looking GARCH(1,1) only in this presentation.

Our Model is Nested inside a Factor Model

Consider \mathcal{M}_1 in the context of the factor model from Abadie, Diamond, and Hainmueller 2010, where an untreated unit is governed by:

$$Y_{i,t}^N = \delta_t + \theta_t' \mathbf{Z}_i + \lambda_t' \boldsymbol{\mu}_i + \varepsilon_{i,t}$$

which nests the GARCH model's volatility equation as well as the ARMA representation of a GARCH model, where

$\delta_t \sim \omega$, a location parameter shared across donors

$\theta_t \sim \boldsymbol{\alpha}_k$, a vector of ARCH parameters and other coefficients shared across donors

$\mathbf{Z}_i \sim \mathbf{a}_{i,t-k}$, a vector of observable quantities specific to each donor

$\lambda_t \sim \boldsymbol{\beta}_j$, a vector of GARCH parameters shared across donors

$\boldsymbol{\mu}_i \sim \boldsymbol{\sigma}_{i,t-j}^2$, a vector of latent quantities specific to each donor

Volatility Profile of a Time Series

Consider the $p \times n$ matrix that stores donor and covariate information at time t

$$\mathbf{V}_t = \begin{pmatrix} \hat{\alpha}_{1,t} & \hat{\alpha}_{t,2} & \cdots & \hat{\alpha}_{t,n} \\ \hat{\beta}_{1,t} & \hat{\beta}_{t,2} & \cdots & \hat{\beta}_{t,n} \\ \vdots & \vdots & \ddots & \vdots \\ RV_{1,t} & RV_{2,t} & \cdots & RV_{n,t} \\ RV_{1,t-1} & RV_{2,t-1} & \cdots & RV_{n,t-1} \\ \vdots & \vdots & \ddots & \vdots \\ IV_{1,t} & IV_{2,t} & \cdots & IV_{n,t} \\ IV_{1,t-1} & IV_{2,t-1} & \cdots & IV_{n,t-1} \\ \vdots & \vdots & \ddots & \vdots \\ |r_{1,t}| & |r_{2,t}| & \cdots & |r_{n,t}| \\ |r_{1,t-1}| & |r_{2,t-1}| & \cdots & |r_{n,t-1}| \end{pmatrix},$$

where RV denotes realized variance and IV the implied volatility

Significance of the Volatility Profile

Covariates chosen for inclusion in a given volatility profile may be any \mathcal{F}_t -measurable function, for example

- levels
- differences in levels
- log returns
- percentage returns
- measurable transformations of the above

Key criterion for inclusion: how plausible is the covariate as a **proxy for risk conditions** for the volatility series to be forecasted?

Forecasting

We present two forecasts:

$$\text{Forecast 1: } \hat{\sigma}_{unadjusted}^2 = \hat{\mathbb{E}}[\sigma_{1, T_1^*+1}^2 | \mathcal{F}_{T^*}] = \hat{\omega}_i + \sum_{k=1}^{m_i} \hat{\alpha}_{i,k} a_{i,t-k}^2 + \sum_{j=1}^{s_i} \hat{\beta}_{i,j} \sigma_{i,t-j}^2 + \hat{\gamma}_i^T \mathbf{x}_{i,t}$$

$$\text{Forecast 2: } \hat{\sigma}_{adjusted}^2 = \hat{\mathbb{E}}[\sigma_{1, T_1^*+1}^2 | \mathcal{F}_{T^*}] + \hat{\omega}^* = \hat{\omega}_i + \sum_{k=1}^{m_i} \hat{\alpha}_{i,k} a_{i,t-k}^2 + \sum_{j=1}^{s_i} \hat{\beta}_{i,j} \sigma_{i,t-j}^2 + \hat{\gamma}_i^T \mathbf{x}_{i,t} + \hat{\omega}^* .$$

Excess Volatility Estimators

- Observe the pair $(\{\hat{\omega}_i^*\}_{i=2}^{n+1}, \{\mathbf{v}_i\}_{i=2}^{n+1})$.

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- Following Abadie, Diamond, and Hainmueller [2010](#); Abadie and Gardeazabal [2003](#), let $\|\cdot\|_S$ denote any semi-norm on \mathbb{R}^P , and define

$$\{\pi\}_{i=2}^{n+1} = \arg \min_{\pi} \|\mathbf{v}_{1,T^*} - \mathbf{V}_{T^*} \pi\|_S .$$

Ground Truth Estimators

We use realized volatility (RV)

- “model-free” in the sense that it requires no modeling assumptions (Andersen and Benzoni 2010).
- RV can be decomposed into the sum of a **continuous component** and a **jump component**, with the latter being less predictable and less persistent (Andersen, Bollerslev, and Diebold 2007), cited in De Luca et al. 2006, two facts that further motivate our method.

Realized Volatility Estimation

Examine K units of time; each unit is divided into m intervals of length $\frac{1}{m}$. Let $p_t = \log P_t$, and let $\tilde{r}(t, \frac{1}{m}) = p_t - p_{t-\frac{1}{m}}$ (Andersen and Benzoni 2008).

Estimate variance of i th log return series using Realized Volatility of the K consecutive trading days that conclude with day t , denoted $RV_{i,t}^{K,m}$, using

$$RV_{i,t}^{K,m} = \frac{1}{K} \sum_{v=1}^{Km} \tilde{r}^2(v/m, 1/m),$$

where the K trading days have been chopped into Km equally-sized blocks.

Assuming the K units $\tilde{r}(t, 1) = p_t - p_{t-1}$ are s.t. $\tilde{r}(t, 1) \stackrel{iid}{\sim} N(\mu, \delta^2)$, it is easily verified that

$$\mathbb{E}[RV^{K,m}] = \frac{\mu^2}{m} + \delta^2,$$

which is a biased but consistent estimator of the variance. We pick $m = 77$, corresponding to the 6.5-hour trading day chopped into 5-minute blocks, omitting first five-minutes of the day.

Loss Functions

Aim: point forecasts for $\sigma_{1,T^*+h}^2 | \mathcal{F}_{T^*}$, $h = 1, 2, \dots$, the h -step ahead conditional variance for the time series under study

Let L^h with the subscripted pair {prediction method, ground truth estimator}, denote the loss function for an h -step-ahead forecast using a given prediction function and ground truth estimator.

Loss Function Examples

For example, one loss possible function of interest in this study is the 1-step-ahead MSE using Synthetic **Volatility Forecasting** and **Realized Volatility**:

$$\text{MSE}_{\text{SVF, RV}}^1 = (\hat{\sigma}_{\text{SVF}}^2 - \hat{\sigma}_{\text{RV}}^2)^2$$

Also of interest in mean absolute percentage error for an h -step-ahead forecast, defined as

$$\text{MAPE}_{\text{method,groundtruth}}^h = \frac{|\hat{\sigma}_{h,\text{method}}^2 - \hat{\sigma}_{h,\text{groundtruth}}^2|}{\hat{\sigma}_{h,\text{groundtruth}}^2}$$

Our choice of Loss Function

Finally, we introduce the QL (quasi-likelihood) Loss (Brownlees, Engle, and Kelly [2011](#)):

$$QL_{method,groundtruth}^h = \frac{\hat{\sigma}_{h,method}^2}{\hat{\sigma}_{h,groundtruth}^2} - \log \frac{\hat{\sigma}_{h,method}^2}{\hat{\sigma}_{h,groundtruth}^2} - 1 .$$

What distinguishes QL Loss?

- Multiplicative rather than additive
- As Brownlees, Engle, and Kelly [2011](#) explain, “[a]mid volatility turmoil, large MSE losses will be a consequence of high volatility without necessarily corresponding to deterioration of forecasting ability. The QL avoids this ambiguity, making it easier to compare losses across volatility regimes.”

Two Consistency Results

Proposition

Assume

- ① For each i , $\{a_{i,t}\}_{t=0,\dots,T_i}$ obeys a GARCH- $X(m, s)$ with volatility shocks found in \mathcal{M}_1 , where T_i is the length of the i th series.
- ② For each i , $\{\omega_{i,t}^*\}_{t=0,\dots,T_i}$ is potentially non-zero at $\{T_i^* + 1, \dots, T_i^* + k\}$, $\omega_{i,T^*+1}^* \equiv \dots \equiv \omega_{i,T^*+k}^*$, and zero otherwise, where the arrival of T_i^* is governed by a time-invariant distribution on $\{a_{i,t}\}_{t=0,\dots,T_i-1}$.
- ③ The conditions in Assumption 0 of Han and Kristensen 2014 prevail.

Then for any i , $\hat{\omega}_{i,t}^* \xrightarrow{P} \omega_{i,t}^*$.

Proposition

Assume

- ① All conditions from the previous proposition.
- ② There exist weights $\{\pi_i\}_{i=2}^{n=1}$ such that $\mathbf{v}_{1,T^*} = \sum_{i=2}^{n+1} \pi_i \mathbf{v}_{i,T^*}$.

Then $\hat{\sigma}_{adjusted}^2 \xrightarrow{P} \sigma_{1,T^*+1}^2$.

Why apply our method to the 2016 US Election?

- You can win the US Presidency without a majority.
- No incumbent candidate
- Donald J. Trump espoused unorthodox, populist positions on healthcare, trade, foreign policy
- Donald J. Trump had no record to assess or criticize
- It was not predicted – hence it delivered news.

IYG

iShares U.S. Financial Services ETF

Figure: IYG includes JPM, BAC, WF, CITI, among other financial majors

- 1 **Model choice** GARCH(1,1) on the daily log return series of IYG in each donor
- 2 **Covariate Choice**
 - previous 30 log returns of IYG (large pre-treatment period, in the language of SC)
 - log return Crude Oil (CL.F)
 - VIX
 - log return of the VIX
 - log returns of the 3-month, 5-year, 10-year, and 30-year US Treasuries
 - return of the most recently available monthly spread between AAA and BAA corporate debt
 - log return in the trading volume of the ETF IYG itself
- 3 **Donor pool construction** US Elections from 2004, 2008, 2012
- 4 **Choice of estimator for volatility** Sum of 77 squared five-minute returns generated between 9:35am and 4pm on November 9th, 2016.

2016 Election

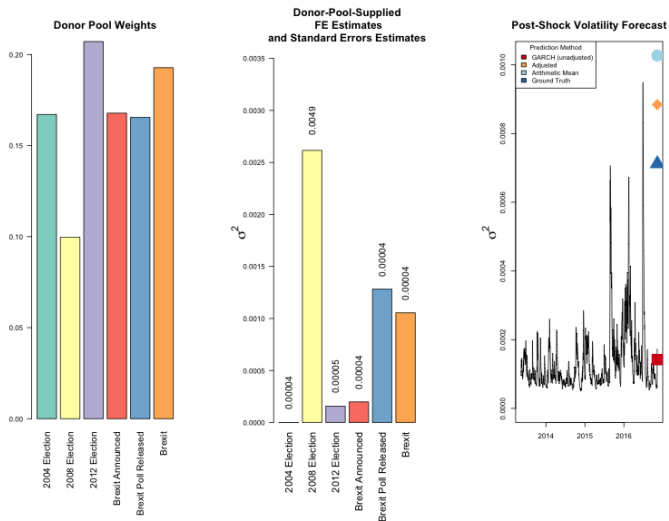


Figure: The volatility induced by the 2016 US election

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- 4 Distance-based weighting should **underperform** the arithmetic mean as μ_{ω^*} increases (importance of linear signal)

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- ④ Distance-based weighting should **underperform** the arithmetic mean as μ_{ω^*} increases (importance of linear signal)
- ⑤ When $D_{i,T^*+1}^{return} = 1 = D_{i,T^*+1}^{vol}$, i.e. when there is both a **return shock** and **volatility shock**, our adjustment methods should **underperform** due to failed identification in $a_{T^*+1} = \sigma_{T^*+1} \epsilon_{T^*+1}$

Simplest Simulation Setup

Most elementary simulation regime tests Hypothesis 1 and 2 by varying δ and $u_{i,t}$.

Recall an \mathcal{M}_1 model on the volatility, which is characterized by an exogenous shock to the volatility equation generated by an affine function of the covariates:

$$\begin{aligned}\sigma_{i,t}^2 &= \omega_i + \omega_{i,t}^* + \sum_{k=1}^{m_i} \alpha_{i,k} a_{i,t-k}^2 + \sum_{j=1}^{s_i} \beta_{i,j} \sigma_{i,t-j}^2 + \gamma_i^T \mathbf{x}_{i,t} \\ \mathcal{M}_1: \quad a_{i,t} &= \sigma_{i,t}((1 - D_{i,t}^{return})\epsilon_{i,t} + D_{i,t}^{return}\epsilon_i^*) \\ \omega_{i,t}^* &= D_{i,t}^{vol}[\mu_{\omega^*} + \delta' \mathbf{v}_{i,t} + u_{i,t}] \\ D_{i,t}^{return} &\equiv 0\end{aligned}$$

We shall group the extensions into five buckets:

- How much can we automate?
- Alternatives for fixed effect estimation
- Alternative estimators and estimands
- What can you do with a volatility forecast?
- Where else is distanced-based weighting useful?

How much can we automate?

Use NLP to identify donors.

You

What are some events that are similar to Chinese government's ban on iPhones for government officials in September 2023?

ChatGPT

The Chinese government's ban on iPhones for government officials in September 2023 is a notable example of government-led restrictions or bans on certain technology products or services due to security concerns or political reasons. Here are some events or actions that are somewhat similar:

1. **Huawei Ban in the United States:** The United States government banned Huawei, a Chinese telecommunications company, from selling its equipment and infrastructure in the U.S. due to concerns about national security and espionage. This ban was initiated in 2019 and has had significant repercussions for Huawei's business globally.
2. **Russian Ban on Telegram:** In 2018, the Russian government banned Telegram, a popular messaging app, due to the company's refusal to provide encryption keys to Russian security agencies. The ban resulted in the disruption of Telegram services within Russia and sparked debates about privacy and government surveillance.
3. **India's Ban on Chinese Apps:** In 2020, the Indian government banned several Chinese mobile apps, including TikTok and WeChat, citing concerns about data privacy and national security after a border clash between Indian and Chinese troops. The ban affected millions of users and led to diplomatic tensions between India and China.
4. **Turkey's Wikipedia Ban:** In 2017, the Turkish government banned access to Wikipedia, alleging that the website was involved in a smear campaign against Turkey. The ban stemmed from Wikipedia's refusal to remove content that the Turkish government deemed offensive. Despite international criticism, the ban remained in place for over two years before being lifted in 2020.
5. **Iran's Ban on Messaging Apps:** The Iranian government has intermittently banned messaging apps such as Telegram and WhatsApp over concerns about the spread of misinformation, political dissent, and national security threats. These bans have been part of broader efforts to control online communication and social media platforms within Iran.

These examples demonstrate how governments may impose bans or restrictions on technology products, services, or platforms for various reasons, including national security, political control, and regulatory compliance.



How much can we automate?

What if the variables in the volatility profile are difficult to specify?

Proposed solution:

Use shrinkage estimation to detect fleeting signals in the cross section of a_t^2 (Chinco, Clark-Joseph, and Ye [2019](#)).

Alternative Ways of Estimating Fixed Effects

High-frequency data?

- Realized GARCH with High-Frequency Data
- Stochastic Volatility

Alternative Estimators and Estimands in Volatility Modeling

- Factors in volatility profile
- Overnight returns instead of open-to-close
- Signal Recovery Perspective (Ferwana and Varshney [2022](#))
- Stochastic Volatility: Correlation between errors
- Multivariate GARCH

What can you do with a volatility forecast?

- Value-at-Risk using SVF-based $\hat{\sigma}_t^2$

New Frontiers in Distance-based Weighting

- Integrate lessons from literature on under/over reactions to information shocks (Jiang and Zhu [2017](#))
- Synthetic Impulse Response Functions

Synthetic Impulse Response Functions: A Proposal

Suppose

- We have a collection of p -variate time series of lengths $T_i, i = 1, 2, \dots, n + 1$.
- We are interested in the response of variable r to shocks in variable $j, 1 \leq r \leq j \leq p$.

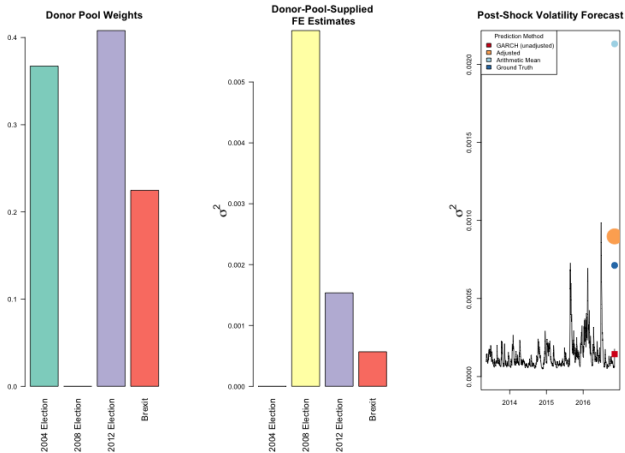
There many ways to estimate $IRF_1(r, j)$.

Can we somehow aggregate the estimates $\widehat{IRF}_i(r, j), i = 2, 3, \dots, n + 1$?

Additional research questions:

- What DGP would best motivate/justify such a method?
- Which method of IRF estimation would perform best?

We analyze the real data example with Brexit included.



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