

# Synthetic Volatility Forecasting and Other Aggregation Techniques for Time Series Forecasting

## Preliminary Exam

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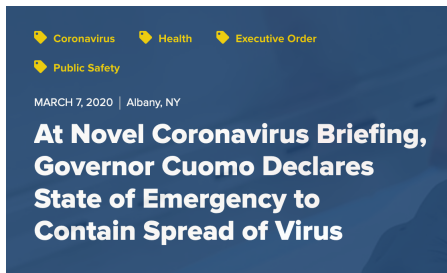
# A seemingly unprecedented event might make one ask

- ① What does it resemble from the past?
- ② What past events are most relevant?
- ③ Can we incorporate past events in a systematic, principled manner?

# When would we ever have to do this?

- Event-driven investing strategies (unscheduled news shock)
- Pairs trading strategies
- Structural shock to macroeconomic conditions (scheduled news possibly pre-empted by news shock)
- Biomedical panel data subject to exogenous shock or interference

## Example (Weekend of March 6th - 8th, 2020)



# Oil nose-dives as Saudi Arabia and Russia set off 'scorched earth' price war

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## Oil crashes by most since 1991 as Saudi Arabia launches price war



By [Matt Egan](#), CNN Business

🕒 3 minute read · Updated 3:21 PM EDT, Mon March 9, 2020

# Punchline of the paper

Forecasting is possible under structural shocks, so long as we incorporate external information to account for the nonzero errors.

# Background and related methods

## Volatility Modeling

- GARCH is slow to react (Andersen et al. [2003](#))
- Asymmetric GARCH models catch up faster but need post-shock data
- Realized GARCH (Hansen, Huang, and Shek [2012](#)), in our setting, would require post-shock information and/or high-frequency data in order to outperform, and Realized GARCH is highly parameterized

# Background and related methods

## Forecast Augmentation

- Clements and Hendry [1998](#); Clements and Hendry [1996](#) laid the groundwork for modeling nonzero errors in time series forecasting
- Guerrón-Quintana and Zhong [2017](#) use a series' own errors to correct the forecast for that series
- Dendramis, Kapetanios, and Marcellino [2020](#) use a similarity-based procedure to correct linear parameters in time series forecasts
- Foroni, Marcellino, and Stevanovic [2022](#) adjust pandemic-era forecasts using intercept correction techniques and data from Great Financial Crisis
- Lin and Eck [2021](#) use distanced-based weighting (a similarity approach) to aggregate and weight fixed effects from a donor pool

# Outline

- 1 Introduction
- 2 Setting
- 3 Post-shock Synthetic Volatility Forecasting Methodology
- 4 Properties of Volatility Shock and Shock Estimators
- 5 Real Data Example
- 6 Numerical Examples
- 7 Discussion
- 8 Future directions for Synthetic Volatility Forecasting
- 9 Supplement



# The news has broken but markets are closed

- After-hours trading provides a poor forum in which to digest news
- The news constitutes public, material information relevant to one or more traded assets
- The qualitative aspects of the news provide basis upon which to match to past events

# A Primer on GARCH

Let  $\{a_t\}$  denote an observable, real-valued discrete-time stochastic process. We say  $\{a_t\}$  is a strong GARCH process with respect to  $\{\epsilon_t\}$  iff

$$\sigma_t^2 = \omega + \sum_{k=1}^m \alpha_k a_{t-k}^2 + \sum_{j=1}^s \beta_j \sigma_{t-j}^2$$

$$a_t = \sigma_t \epsilon_t$$

$$\epsilon_t \stackrel{iid}{\sim} E[\epsilon_t] = 0, \text{Var}[\epsilon_t] = 1$$

$$\forall k, j, \alpha_k, \beta_j \geq 0$$

$$\forall t, \omega, \sigma_t > 0$$

# Model Setup

populate once model choice is firm

# Our Model is Nested Within GARCH-X

Populate once notational details are decided.

# Volatility Profile of a Time Series

$$\mathbf{V}_{\tau,n} = \begin{pmatrix}
 \alpha_{\tau,*_1} & \alpha_{\tau,*_2} & \cdots & \alpha_{\tau,*_n} \\
 \beta_{\tau,*_1} & \beta_{\tau,*_2} & \cdots & \beta_{\tau,*_n} \\
 \vdots & \vdots & \ddots & \vdots \\
 RV_{\tau,*_1} & RV_{\tau,*_2} & \cdots & RV_{\tau,*_n} \\
 RV_{\tau,*_1} - 1_{1,1} & RV_{\tau,*_2} - 1_{1,2} & \cdots & RV_{\tau,*_n} - 1_{1,n} \\
 \vdots & \vdots & \ddots & \vdots \\
 IV_{\tau,*_1} & IV_{\tau,*_2} & \cdots & IV_{\tau,*_n} \\
 IV_{\tau,*_1} - 1_{1,1} & IV_{\tau,*_2} - 1_{1,2} & \cdots & IV_{\tau,*_n} - 1_{1,n} \\
 \vdots & \vdots & \ddots & \vdots \\
 AbsoluteReturn_{\tau,*_1} & AbsoluteReturn_{\tau,*_2} & \cdots & AbsoluteReturn_{\tau,*_n} \\
 AbsoluteReturn_{\tau,*_1} - 1_{1,1} & AbsoluteReturn_{\tau,*_2} - 1_{1,2} & \cdots & AbsoluteReturn_{\tau,*_n} - 1_{1,n} \\
 \vdots & \vdots & \ddots & \vdots \\
 Volume_{\tau,*_1} & Volume_{\tau,*_2} & \cdots & Volume_{\tau,*_n} \\
 Volume_{\tau,*_1} & Volume_{\tau,*_2} & \cdots & Volume_{\tau,*_n}
 \end{pmatrix}$$

# What's the method here?

$$2 = 2$$

# Forecasting

# Excess Volatility Estimators



# Ground Truth Estimators

# Loss Functions

# Simplest Simulation Setup

We also have simulations for...

# Additional Simulations

Example (Coverging at the slowest rate possible)

Fix  $\alpha = 1, \beta > 1$ . Let  $\lambda_i = \frac{1}{i \log^\beta(i+1)}$ .

# Alternative Data-Generating Processes

- Could we do all of the above with high-frequency data?
- Realized GARCH with High-Frequency Data
- Stochastic Volatility

# Alternative Estimators and Estimands in Volatility Modeling

- Realized GARCH with High-Frequency Data
- Overnight returns instead of open-to-close
- Signal Recovery Perspective
- Stochastic Volatility: Correlation between errors

# New Frontiers in Aggregation Methods

- Integrate lessons from literature on under/over reactions to information shocks (Jiang and Zhu [2017](#))
- Synthetic Impulse Response Functions

# Synthetic Impulse Response Functions: A Proposal

- Suppose we have a multivariate time series of dimension  $p \times \text{times } T$  subject to shocks from a common shock distribution
- Using an IRF estimate aggregated from the first  $n$  shocks of interest, we predict the response of variable  $i$  from variable  $j$ ,  $1 \leq i \leq j \leq p$ .



We analyze the real-world example with Brexit included.

# Bibliography

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