

Synthetic Volatility Forecasting and Other Aggregation Techniques for Time Series Forecasting

Preliminary Exam

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A seemingly unprecedented event might make one ask

- ① What does it resemble from the past?
- ② What past events are most relevant?
- ③ Can we incorporate past events in a systematic, principled manner?

When would we ever have to do this?

- Event-driven investing strategies (unscheduled news shock)
- Pairs trading strategies
- Structural shock to macroeconomic conditions (scheduled news possibly pre-empted by news shock)
- Biomedical panel data subject to exogenous shock or interference

Example

Oil nose-dives as Saudi Arabia and Russia set off 'scorched earth' price war

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Example

Oil crashes by most since 1991 as Saudi Arabia launches price war



By [Matt Egan](#), CNN Business

🕒 3 minute read · Updated 3:21 PM EDT, Mon March 9, 2020

Punchline of the paper

Forecasting is possible under structural shocks, so long as we incorporate external information to account for the nonzero errors.

Background and related methods

Volatility Modeling

- GARCH is slow to react (Andersen et al. [2003](#))
- Asymmetric GARCH models catch up faster but need post-shock data
- Realized GARCH (Hansen, Huang, and Shek [2012](#)), in our setting, would require post-shock information and/or high-frequency data in order to outperform, and Realized GARCH is highly parameterized

Background and related methods

Forecast Augmentation

- Clements and Hendry [1998](#); Clements and Hendry [1996](#) laid the groundwork for modeling nonzero errors in time series forecasting
- Guerrón-Quintana and Zhong [2017](#) use a series' own errors to correct the forecast for that series
- Dendramis, Kapetanios, and Marcellino [2020](#) use a similarity-based procedure to correct linear parameters in time series forecasts
- Foroni, Marcellino, and Stevanovic [2022](#) adjust pandemic-era forecasts using intercept correction techniques and data from Great Financial Crisis
- Lin and Eck [2021](#) use distanced-based weighting (a similarity approach) to aggregate and weight fixed effects from a donor pool

Outline

- 1 Introduction
- 2 Setting
- 3 Post-shock Synthetic Volatility Forecasting Methodology
- 4 Properties of Volatility Shock and Shock Estimators
- 5 Real Data Example
- 6 Numerical Examples
- 7 Discussion
- 8 Future directions for Synthetic Volatility Forecasting
- 9 Supplement

The news has broken but markets are closed

- After-hours trading provides a poor forum in which to digest news
- The news constitutes public, material information relevant to one or more traded assets
- The qualitative aspects of the news provide basis upon which to match to past events

A Primer on GARCH

Let $\{a_t\}$ denote an observable, real-valued discrete-time stochastic process. We say $\{a_t\}$ is a strong GARCH process with respect to $\{\epsilon_t\}$ iff

$$\sigma_t^2 = \omega + \sum_{k=1}^m \alpha_k a_{t-k}^2 + \sum_{j=1}^s \beta_j \sigma_{t-j}^2$$

$$a_t = \sigma_t \epsilon_t$$

$$\epsilon_t \stackrel{iid}{\sim} E[\epsilon_t] = 0, \text{Var}[\epsilon_t] = 1$$

$$\forall k, j, \alpha_k, \beta_j \geq 0$$

$$\forall t, \omega, \sigma_t > 0$$

Model Setup

populate once model choice is firm

Our Model is Nested Within GARCH-X

Populate once notational details are decided.

Volatility Profile of a Time Series

$$V_{\tau,n} = \begin{pmatrix} \alpha_{\tau,*_1} & \alpha_{\tau,*_2} & \cdots & \alpha_{\tau,*_n} \\ \beta_{\tau,*_1} & \beta_{\tau,*_2} & \cdots & \beta_{\tau,*_n} \\ \vdots & \vdots & \ddots & \vdots \\ RV_{\tau,*_1} & RV_{\tau,*_2} & \cdots & RV_{\tau,*_n} \\ RV_{\tau,*_1} - 1_{1,1} & RV_{\tau,*_2} - 1_{1,2} & \cdots & RV_{\tau,*_n} - 1_{1,n} \\ \vdots & \vdots & \ddots & \vdots \\ IV_{\tau,*_1} & IV_{\tau,*_2} & \cdots & IV_{\tau,*_n} \\ IV_{\tau,*_1} - 1_{1,1} & IV_{\tau,*_2} - 1_{1,2} & \cdots & IV_{\tau,*_n} - 1_{1,n} \\ \vdots & \vdots & \ddots & \vdots \\ \text{AbsoluteReturn}_{\tau,*_1} & \text{AbsoluteReturn}_{\tau,*_2} & \cdots & \text{AbsoluteReturn}_{\tau,*_n} \\ \text{AbsoluteReturn}_{\tau,*_1} - 1_{1,1} & \text{AbsoluteReturn}_{\tau,*_2} - 1_{1,2} & \cdots & \text{AbsoluteReturn}_{\tau,*_n} - 1_{1,n} \\ \vdots & \vdots & \ddots & \vdots \\ \text{Volume}_{\tau,*_1} & \text{Volume}_{\tau,*_2} & \cdots & \text{Volume}_{\tau,*_n} \\ \text{Volume}_{\tau,*_1} & \text{Volume}_{\tau,*_2} & \cdots & \text{Volume}_{\tau,*_n} \end{pmatrix}$$

What's the method here?

$$2 = 2$$

Forecasting

Excess Volatility Estimators

Ground Truth Estimators

Loss Functions

Some things to think about with papers like this

Simplest Simulation Setup

A more benign example

Example (Coverging at the slowest rate possible)

Fix $\alpha = 1, \beta > 1$. Let $\lambda_i = \frac{1}{i \log^\beta(i+1)}$.

How noise is hidden just right

After all of this waiting, we formalize the notion under discussion.

Definition (Asymptotically Benign)

Alternative Data-Generating Processes

- Could we do all of the above with high-frequency data?
- Realized GARCH with High-Frequency Data
- Stochastic Volatility

Alternative Estimators and Estimands in Volatility Modeling



- Realized GARCH with High-Frequency Data
- Signal Recovery Perspective
- Stochastic Volatility: Correlation between errors

New Frontiers in Aggregation Methods

- Synthetic Impulse Response Functions

We analyze the real-world example with Brexit included.

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