Forecast Adjustment Under Shocks: Similarity-based Solutions to Unprecedented Events
Doctoral Defense

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A High-level Summary Uniting All My Projects

Forecasting under non-ideal conditions like

1. a rupture in the DGP
2. lack of past information for units we wish to forecast, i.e. a so-called "cold start" (**fatemi2023mitigating**)

Central technique explored: *correction* of model parameters using an aggregation strategy, where aggregation means that we make use of external data.

We began with particular models; then built upwards, establishing an abstract, general framework for correcting forecasts that is widely applicable for forecasting and beyond.

Why is this valuable?

We are weighing-in on a puzzle:

conflict between the intuitive notion that more relevant information should help in forecasting, and the hard reality that attempts to make it do so have not been uniformly successful (**clements2005guest**)

Why is this valuable?

We provide a precise proposal to a well-specified challenge:

incomplete information by itself is unlikely to play a key role in forecast failure (except if that information would forecast breaks). Consequently, using large amounts of data may not correct one of the main problems confronting forecasters, namely location shifts, unless that additional information is directly pertinent to forecasting breaks (**castle2013forecasting**)

Why is this valuable?

- Canonical problems: predicting based on a limited feature space and/or small n

- The framework we offer is something others can 'plug-in' to and add to.

- Software developed

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1. What does it resemble from the past?
2. What past events are most relevant for our objectives?
3. Can we incorporate past events in a systematic, principled manner?

Example (Weekend of March 6th - 8th, 2020)

🏷 Coronavirus   🏷 Health   🏷 Executive Order

🏷 Public Safety

MARCH 7, 2020 | Albany, NY

At Novel Coronavirus Briefing, Governor Cuomo Declares State of Emergency to Contain Spread of Virus

Oil nose-dives as Saudi Arabia and Russia set off 'scorched earth' price war

PUBLISHED SUN, MAR 8 2020·9:01 AM EDT | UPDATED MON, MAR 9 2020·5:33 PM EDT

Oil crashes by most since 1991 as Saudi Arabia launches price war

By Matt Egan, CNN Business

🕐 3 minute read · Updated 3:21 PM EDT, Mon March 9, 2020

Punchline

- Credible forecasting is possible under news shocks, so long as we incorporate external information to account for the nonzero errors.

Outline

Premise: There is breaking news at some fractional lag t-ϵ

- Market (i.e. crowd-sourced) mechanisms for evaluating the news are either offline, too thin, or otherwise unavailable. They could also be simply unreliable.
- The qualitative aspects of the news provide a basis upon which to
  - match to past news shocks
  - match in a p-dimensional covariate space

Model Setup

For $t = 1, \ldots, T_i$ and $i = 1, \ldots, n+1$, the model $\mathcal{M}_1$ is defined as

$$\mathcal{M}_1 : \begin{aligned} y_{i,t} &= F(\mathcal{F}_{i,t-1}) + \alpha_{i,t} + \epsilon_{i,t} \\ \alpha_{i,t} &= \mathbf{x}_{i,t}^T \lambda_{i,t} \\ \mathbf{x}_{i,t}^T &= (1, x_{i,t}^1, ..., x_{i,t}^p) \text{ (observable and deterministic w.r.t to } \mathcal{F}_{i,t-1}) \\ \lambda_{i,t}^T &= (u_{i,t}, \lambda_t^1, ..., \lambda_t^p) \text{ (unobservable and potentially random)}, \end{aligned} \tag{1}$$

$$\lambda_{i,t} \sim \mathcal{F}_\lambda \text{ with } \mathbb{E}_{\mathcal{F}_\lambda}(\lambda) = \mu_{\lambda_t}, \mathrm{Var}_{\mathcal{F}_\lambda}(\lambda) = \Sigma_{\lambda_t},$$

and time-invariant error structure

$$\epsilon_{i,t} \overset{iid}{\sim} \mathcal{F}_\epsilon \text{ with } \mathbb{E}_{\mathcal{F}_\epsilon}(\epsilon) = 0, \mathrm{Var}_{\mathcal{F}_\epsilon}(\epsilon) = \sigma_\epsilon^2,$$

Model Details

Note that $\lambda_{i,t}^T = (u_{i,t}, \lambda_t^1, ..., \lambda_t^p)$

- includes the term $u_{i,t}$ that is not shared among donors.
- is time-varying allows us to capture that most time points are without news shocks (in which case $\alpha_{i,t}$ is of negligible effect), but conditional upon information arriving between T_1^* and $T_1^* + 1$, we may know with near certainty that λ_{1,T_1^*+h} will be nonzero in norm for some $h > 0$.

Significance of the Covariates $x_{i,t}$

Covariates chosen for inclusion may be any $\mathcal{F}_t$-measurable function, for example

- levels
- differences in levels
- log returns
- percentage returns
- measurable transformations of the above

Key criterion for inclusion: how plausible is the covariate as a proxy for risk conditions for the time series under study?

Forecasting

We now present two one-step-ahead forecasts. First is the unadjusted forecast. The second is the adjusted forecast, which differs by the predicted correction term:

$$\text{Forecast 1: } \hat{y}_{unadjusted, T_1^* + 1} = \hat{\mathbb{E}}[y_{1, T_1^* + 1} | \mathcal{F}_{T_1^*}]$$

$$\text{Forecast 2: } \hat{y}_{adjusted, T_1^* + 1} = \hat{\mathbb{E}}[y_{1, T_1^* + 1} | \mathcal{F}_{T_1^*}] + \hat{\alpha}_{T_1^* + 1} \ .$$

Distance-based Weighting in Action

(For ease of exposition, we omit time indices)

- Observe the pair $(\{\hat{\alpha}_i\}_{i=2}^{n+1}, \{\mathbf{x}_i\}_{i=2}^{n+1})$.

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- Following **abadie2003economic**, **abadie2010synthetic**, let $\| \cdot \|_{\mathbf{s}}$ denote any semi-norm on $\mathbb{R}^p$, and define

$$\{\pi\}_{i=2}^{n+1} = \arg\min_{\pi} \|\mathbf{v}_{1,T^*} - \mathbf{X}_{T^*}\pi\|_{\mathbf{s}} \, .$$

Visuals That Tell The Story
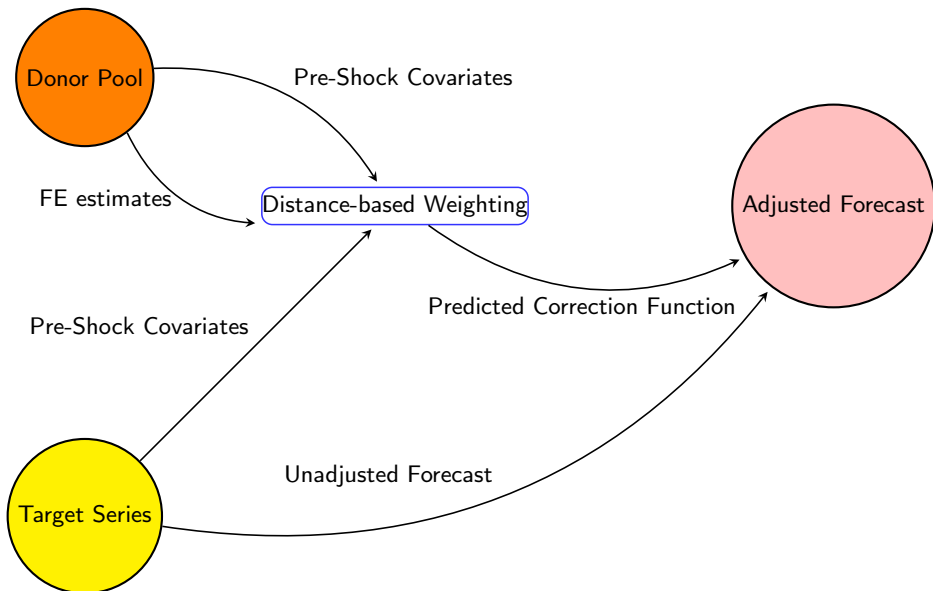
Visuals That Tell The Story

The forecaster's decision tree
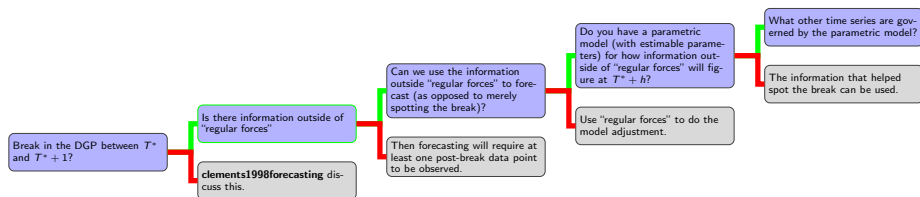
Figure: Forecast Model Adjustment: A Decision Tree
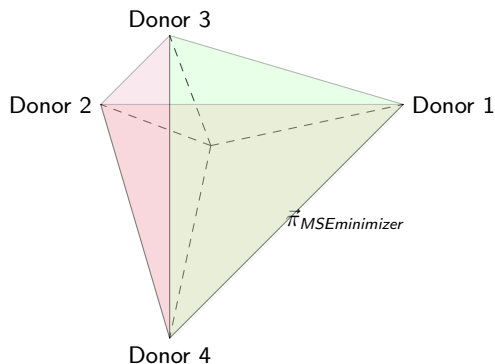
Visuals That Tell The Story

Figure: The 3-Simplex, Δ^3, where hypothetical minimizer is a convex combination of Donors 1 and 4.

Three slides about theory

Table: A 2x2 Schema of Forecast Information, With Examples

	Conventional Econometric Models	Outside Conventional Econometric Models
internal	Lags of the series itself; past shocks	Polynomial expansion of the feature space and other transformations without solid theoretical motivation
external	Macro variables like interest rates, commodity prices; weather-related variables	Google Trends, high-frequency data like prediction markets; past shocks under similar conditions

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An oft-abused term in applied math: similarity

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To give a name to what we're doing, we can call it distance-based weighting, which is a special case of an approximation-based n-ary similarity metric.

Three slides about theory

The term $\mathbf{x}_{i,t}^T \lambda_{i,t}$ is deemed the *correction term* for donor i at time t, and the function

$$\xi \colon \mathcal{F}_{2, T_2^* + h} \times \ldots \times \mathcal{F}_{n+1, T_{n+1}^* + h} \to \mathcal{F}_{1, T_1^* + h}$$

that estimates or predicts $\alpha_{1, T_1^* + h}$ will be deemed the correction function.

The correction function maps donor information — be it observables or inferences — to the prediction function for the time series under study.

Global Overview

Abstracting away from particular models, what does our method require?

1. **Object-to-predict** Random object (indexed over time and possibly space, as well) obeying specification with additive errors

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5. **Reliable Correction Function Estimation** There must exist a correction function (presumably based on the correction term) that maps data from the donor pool to the *predicted* correction term in the time series under study based on similarity.

Do we have theoretical guarantees?

We do!

Our formal results fall into the following buckets:

1. convergence in distribution results
2. consistency results
3. asymptotic loss

Here we share only the first two, and for both GARCH and AR(p) models.

GARCH - Formal Results

Proposition

Assume

1. *For each i, $1 \leq i \leq n$, $\{a_{i,t}\}_{t=0,\dots,T_i}$ obeys a GARCH-X(m, s), with volatility shocks found in $\mathcal{M}_1$, where T_i is the length of the ith series.*

2. *For each i, $\{\omega_{i,t}^*\}_{t=0,\dots,T_i}$ is potentially non-zero at $\{T_i^* + 1, \dots, T_i^* + L_i^{vol}\}$, $\omega_{i,T_i^*+1}^* \equiv \dots \equiv \omega_{i,T_i^*+L_i^{vol}}^*$, and zero otherwise, where the arrival of T_i^* is governed by a time-invariant distribution on $\{a_{i,t}\}_{t=0,\dots,T_i-1}$, and both the arrival and conclusion of the shock is observable by the researcher.*

3. *The conditions in Assumption 0 of **han2014asymptotic** hold.*

Then for any $i, 1 \leq i \leq n + 1$, and for any $r, 1 \leq r \leq L_i^{vol}$, $\hat{\omega}_{i,T_i^*+r}^* \xrightarrow{p} \omega_{i,T_i^*+r}^*$ as $t \to \infty$.

Additionally, $\hat{\omega}_{i,*}^* \xrightarrow{d} \omega_{i,T_i^*+r}^*$ as $t \to \infty$, and if for all $i, 1 \leq i \leq n + 1$, $u_{i,t} \equiv 0$ on $\{T_i^* + 1, \dots, T_i^* + L_i^{vol}\}$, then $\hat{\omega}_{i,T_i^*+r}^* \xrightarrow{p} \omega_{i,T_i^*+r}^*$.

In plain language, this result gives us distributional results about our correction terms as we estimate over arbitrarily long horizons.

GARCH - Formal Results

Having established the consistency of the estimators $\hat{\omega}^*_{i, T^*_i + r}$, we extend that result to prove asymptotic properties of the conditional forecast function itself.

Proposition

Assume

1. *All conditions listed in the preceding proposition* **??**.
2. *There exist weights $\{\pi_i\}_{i=2}^{n+1}$ such that $\mathbf{v}_{1, T^*_1} = \sum_{i=2}^{n+1} \pi_i \mathbf{v}_{i, T^*_i}$.*

Then for any r, $1 \le r \le L^{vol}_1$, $\hat{\sigma}^2_{adjusted, T^*_1 + r} \xrightarrow{d} \sigma^2_{1, T^*_1 + r}$ as $t \to \infty$ in the donor pool, and if for all $i, 1 \le i \le n+1$, $u_{i,t} \equiv 0$ on $\{T^*_i + 1, ..., T^*_i + L^{vol}_i\}$, then $\hat{\sigma}^2_{adjusted, T^*_1 + r} \xrightarrow{p} \sigma^2_{1, T^*_1 + r}$.

In plain language, our volatility prediction have a known distribution, by virtue of our correction terms have a known distribution.

AR(p) - Formal Results

We now present a similar AR(p) result here

An inventory of models

- GARCH (paper is under review at the *International Journal of Forecasting*)

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- HAR (to be submitted in the coming weeks as part of second paper)
- Exponential Shocks (to be submitted in the coming weeks as part of second paper)

GARCH Volatility Forecasts

The least you need to know here: GARCH is the premier volatility modeling family for daily data.

We demonstrate our method's usefulness with an easy-to-approach example.

Why apply our method to the 2016 US Election?

- You can win the US Presidency without a majority.
- No incumbent candidate
- Donald J. Trump espoused unorthodox, populist positions on healthcare, trade, foreign policy
- Donald J. Trump had no record to assess or criticize
- It was not predicted – hence it delivered news.

IYG **iShares U.S. Financial Services ETF**

Figure: IYG includes JPM, BAC, WF, CITI, among other financial majors

1. **Model choice** GARCH(1,1) on the daily log return series of IYG in each donor
2. **Covariate Choice**
   - previous 30 log returns of IYG (large pre-treatment period, in the language of SC)
   - log return Crude Oil (CL.F)
   - VIX
   - log return of the VIX
   - log returns of the 3-month, 5-year, 10-year, and 30-year US Treasuries
   - return of the most recently available monthly spread between AAA and BAA corporate debt
   - log return in the trading volume of the ETF IYG itself
3. **Donor pool construction** US Elections from 2004, 2008, 2012
4. **Choice of estimator for volatility** Sum of 77 squared five-minute returns generated between 9:35am and 4pm on November 9th, 2016.

2016 Election
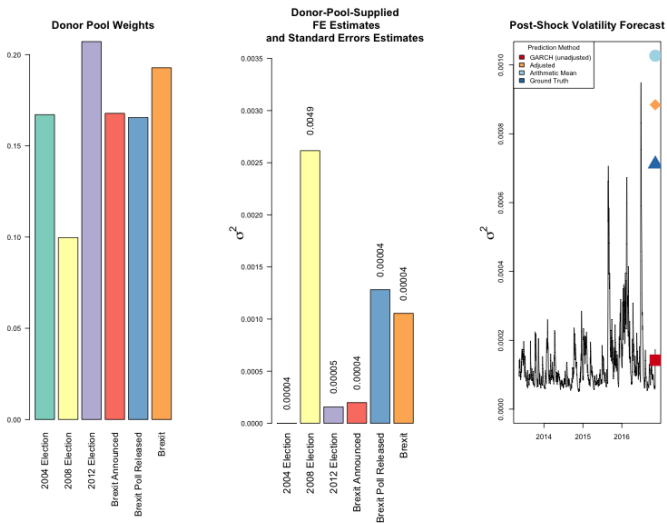
Figure: The volatility induced by the 2016 US election

Heterogeneous Autoregression (HAR)
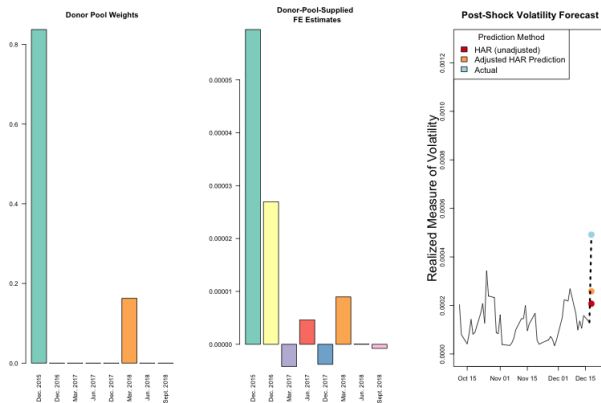
Figure: Click the image above to see a video of the R package's functionality.

Simulations: Parameter Correction Using An Aggregated Decay Parameter from Donors

Recall an $\mathcal{M}_1$ model on the volatility, which is characterized by an exogenous shock to the volatility equation generated by an affine function of the covariates.

Now consider, for $i = 1, 2, ..., n + 1$, the family of models

$$
\mathcal{M}_{exp}: \begin{aligned}
& y_{i,t} = \alpha_i + \eta[1 - e^{-\psi_i[t - T_i^*]}]\mathbf{1}_{t \geq T_i^* + 1} + \epsilon_{i,t}, \epsilon_{i,t} \\
& \psi_i = \mathbf{x}_{i,t}^T \lambda_{i,t} \\
& \mathbf{x}_{i,t}^T = (1, x_{i,t}^1, ..., x_{i,t}^p) \\
& \lambda_{i,t}^T = (u_{i,t}, \lambda_t^1, ..., \lambda_t^p),
\end{aligned}
\tag{2}
$$

with time-varying and observable covariate vector $\mathbf{x}_{i,t}^T$, time-varying, unobservable, and potentially random vector

$$
\lambda_{i,t} \sim \mathcal{F}_\lambda \text{ with } \mathbb{E}_{\mathcal{F}_\lambda}(\lambda) = \mu_{\lambda_t}, \text{Var}_{\mathcal{F}_\lambda}(\lambda) = \Sigma_{\lambda_t},
$$

and time-invariant error structure just as in $\mathcal{M}_1$.

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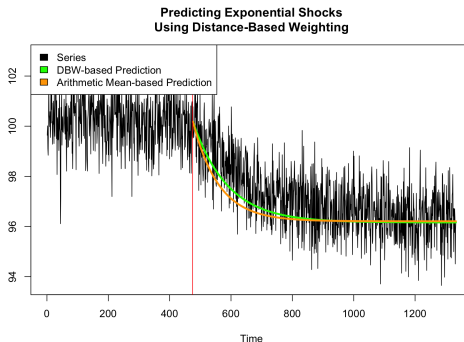
Figure: Simulated random walk centered at $\mu = 100$, subject to a shock at approximately time index 440. Shock of size -4 is not realized over a single index. Instead, shock is governed by a decay exponential decay parameter ψ_1, as are exponential shocks in $n = 10$ donor series. We estimate ψ_1 using a convex combination of the estimated decay parameters $\{\psi_i\}_{i=2}^{n+1}$, resulting in a distance-based weighting prediction of the shock. We also illustrate the arithmetic-mean-based prediction of the shock using the color orange. This prediction is based on an overestimate of ψ_1 and hence results in residuals that undershoot the time series under study.

Software

R package
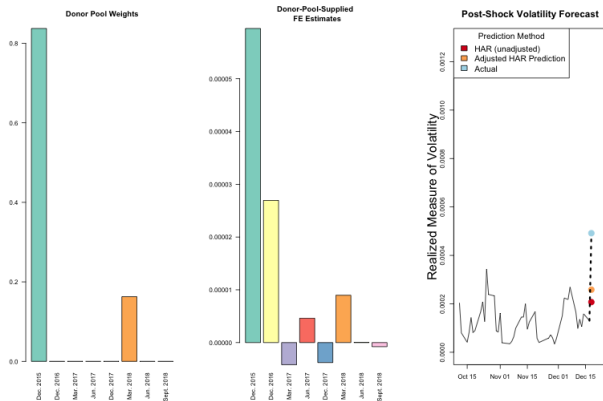
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LLMs

Use NLP to identify donors.

Robustness of the Approach

What assurances do we have that the method will be good?

Some ideas here:

1. multiverse
2. permute donors

A Multiverse Approach to Forecast Evaluation

QL Loss of the Adjusted Forecast	Omitted Covariate	Omitted
0.0001	Log Return of $^\rfloor IRX$	None
0.0007	NA (Average Adjusted Forecast)	NA (Average Adju
0.0007	Log Return of CL=F	2004-11
0.0008	Log Return of $^\rfloor IRX$	2004-11
⋮	⋮	⋮
0.0080	Log Return of CL=F	2008-11
0.0099	Log Return of $^\rfloor VIX$	2004-11
0.0099	NA (Median Adjusted Forecast)	NA (Median Adju
0.0115	Log Return of CL=F	2016-02
0.0127	None	2016-02
⋮	⋮	⋮
0.0216	Log Return of $^\rfloor IRX$	2016-06
0.0219	None	None
0.0222	Log Return of $^\rfloor FVX$	2016-02
⋮	⋮	⋮
2.3529	All	All

We shall group the extensions into five buckets:

- How much can we automate?
- Alternatives for fixed effect estimation
- Alternative estimators and estimands
- What can you do with a volatility forecast?
- Where else is distanced-based weighting useful?
- Can we extend the results of (**bodilsen2023exploiting**)

How much can we automate?

What if the covariates are difficult to specify?

Proposed solution:

Use shrinkage estimation to detect fleeting signals in the cross section of a_t^2 (**chinco2019sparse**).

Limitations of what we're currently doing

- Our real data examples cannot be scaled up due to the need to for human involvement in donor and covariate curation
- We assume that the exogenous variables (distinct from the covariates) are known at time T_i^*
- Heterogeneity of DGPs
- Forecast evaluation
- We could only truly explore a subset of the vast parameter space.
- The failure of distance-based weighting to extrapolate

New Frontiers in Distance-based Weighting

- Forecast conditional upon the shock *prior* to the shock's arrival
- Integrate lessons from literature on under/over reactions to information shocks (**jiang2017information**)
- Distance-based Weighting of Impulse Response Functions

Distance-based Weighting of Impulse Response Functions

Suppose

- We have a collection of p-variate time series of lengths $T_i, i = 1, 2, ... n + 1$.
- We are interested in the response of variable r to shocks in variable j, $1 \leq r \leq j \leq p$.

There many ways to estimate $IRF_1(r, j)$.

Can we somehow aggregate the estimates $\widehat{IRF}_i(r, j)$, $i = 2, 3, ..., n + 1$?
Additional research questions:

- What DGP would best motivate/justify such a method?
- Which method of IRF estimation would perform best?

References I