Shear rate dependence of no-Brownian suspension

Tue 5th Mar, 2013 10:16am

Units

Our first target is rheology of non-Brownian suspensions under simple shear flows (shear rates, $\dot{\gamma}$). It is convenient to use units of length, velocity and force as follows:

$$L_0 = a_0, \tag{1}$$
$$U_0 = L_0 \dot{\gamma}, \tag{2}$$
$$F_0 = 6\pi\eta_0 a U_0 = 6\pi\eta_0 a^2 \dot{\gamma}. \tag{3}$$

a_0 is the radius of particles (smaller one, if it is bidisperse, i.e., $a_0 < a_1$).

Non-dimensional variables are denoted by hat ($\hat{}$):

$$\hat{F} = F/F_0, \tag{4}$$
$$\hat{T} = T/(F_0 L_0), \tag{5}$$
$$\hat{S} = S/(F_0 L_0), \tag{6}$$

and $\hat{U} = U/U_0$, $\hat{\Omega} = \Omega$, and $\hat{E} = EL_0/U_0$.

Due to the linearity of Stokes regime, hydrodynamic interaction can be written as non-dimensional model, where the shear rate does not appear in the model. We can scale all non-dimensional results by the shear rate.

The finite lubrication

Our working hypothesis is *to use finite lubrication.* For monodisperse system,

$$\boldsymbol{F}_{\mathrm{lub}}^{(i,j)} = -\alpha(h)(\boldsymbol{v}^{(i)} - \boldsymbol{v}^{(j)}) \cdot \boldsymbol{n}^{(i,j)}\boldsymbol{n}^{(i,j)}. \tag{7}$$

$$\alpha(h) = \begin{cases} 3\pi\eta_0 a^2/2(h+c) & h > 0 \\ 3\pi\eta_0 a^2/2c & h \leq 0 \end{cases} \tag{8}$$

This way to introduce the finiteness is not obvious. We might imagine smaller spheres with a radius $b < a$. We may consider them as hydrodynamic radii.

Each particle have both geometrical and hydrodynamic radii: a and b. When the gap is geometrically $h \equiv r - 2a$, the hydrodynamic gap is

$$h' = r - 2b = r - 2a + 2(a - b) = h + c. \tag{9}$$

where $c \equiv 2(a-b)$. So, the eqs. (7) and (8) for $h > 0$ is considered as a solution of Stokes equations for two spheres of radius b.

The non-dimensional form is

$$\hat{\boldsymbol{F}}_{\mathrm{lub}}^{(i,j)} = \hat{\alpha}(\hat{h})(\hat{\boldsymbol{v}}^{(i)} - \hat{\boldsymbol{v}}^{(j)}) \cdot \boldsymbol{n}^{(i,j)}\boldsymbol{n}^{(i,j)}. \tag{10}$$

$$\hat{\alpha}(\hat{h}) = \begin{cases} 1/4(\hat{h} + \hat{c}) & \hat{h} > 0 \\ 1/4\hat{c} & \hat{h} \leq 0 \end{cases} \tag{11}$$

If the above interpretation is ok to have the finiteness in the lubrication model, the shear-rate independence of the Stokes regime remains in the lubrication model. The trajectories of particles before the first contact never depend on shear rates. Just this finiteness allows particles to come into contact.

Contact force model

Contact forces act between two particles i and j, when the distance r is smaller than $r_0 = a_i + a_j$.

Rigid sphere model Typical hydrodynamic forces are much smaller than the force which can deform particles. Young's modulus of particles (1–100 GPa) is usually large enough to consider the particles are rigid body in sheared suspensions. In the rigid-body limit of elastic body, any force cannot cause finite deformation. In simulations (such as DEM), we need to use a finite potential function to mimic this behavior. As mentioned above, we can model the hydrodynamic interaction part as shear-rate independent. To combine models of hydrodynamic and contact interactions, there are two possibilities. The one is shear-rate independent contact force model, and the second is shear-rate scaling contact force model.

Since contact force model is unrelated to shear flows, the fist one seems reasonable. If so, in the non-denationalized simulation, the contact force looks to be scaled by shear rates:

$$\hat{F}_{\mathrm{c}}(\hat{r}) = \frac{F_{\mathrm{c}}(r)}{F_0} = \frac{F_{\mathrm{c}}(\hat{r}L_0)}{6\pi\eta_0 L_0^2}\frac{1}{\dot{\gamma}}, \tag{12}$$

where we use the same $F_{\mathrm{c}}(r)$. Thus, the hydrodynamic parts are equivalent for different shear rates, while the shear rate changes the contact force model in the non-denationalized simulation. We need to check this model can mimic the rigid spheres for a wide range of shear rates.

In the rigid-body limit, we may expect no shear-rate dependence in the non-Brownian+finite-lubrication suspension rheology. If so, we should use a shear-rate independent dimensionless simulation model by introducing shear-rate scaling contact model. We can just use the same contact model $\hat{F}_c(\hat{r})$ in the non-denationalized simulation. It means that we consider different contact models for different shear rates;

$$F_{\mathrm{c}}(r) = \hat{F}_{\mathrm{c}}(\hat{r}) F_0 = 6\pi\eta L_0^2 \dot{\gamma} \hat{F}_{\mathrm{c}}(r/L_0). \tag{13}$$

Since the absence of the shear rate in the code, it guarantees to have shear-rate independent results, which is expected in the rigid-bory limit. If friction is the origin to yield shear-rate dependence, it is better to use this shear-rate independent form.

Friction model Friction laws can be matter. The well-known formula

$$F_{\parallel} = \mu F_{\perp} \tag{14}$$

is a linear relation. In the non-dimensional form, shear-rate dependence does not appear:

$$\hat{F}_{\parallel} F_0 = \mu \hat{F}_{\perp} F_0 \quad \to \quad \hat{F}_{\parallel} = \mu \hat{F}_{\perp}. \tag{15}$$

Cohesion term is also expected:

$$F_{\parallel} = \mu F_{\perp} + \sigma A \tag{16}$$

The textbook of Israelachvili says

> At low loads the friction force is dominated by the adhesion contribution, but at high loads, where $A \propto F_{\perp}^{2/3}$, it is dominated by the load-dependent contribution $\mu F_{\perp}$.

$$\hat{F}_{\parallel} F_0 = \mu \hat{F}_{\perp} F_0 + \sigma (\hat{F}_{\perp} F_0)^{2/3} \tag{17}$$

$$\to \quad \hat{F}_{\parallel} = \mu \hat{F}_{\perp} + (\sigma F_0^{-1/3}) \hat{F}_{\perp}^{2/3} \tag{18}$$

This means that the cohesion term is less and less important at higher shear rate.