Title

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Hydrodynamic force

One-body limit:

$$\boldsymbol{F}_i^{\rm self} = -6\pi\mu a(\boldsymbol{v}_i - \boldsymbol{U}^{\infty}(\boldsymbol{r}_i)) \tag{1}$$

$$\boldsymbol{T}_i^{\rm self} = -8\pi\mu a^3(\boldsymbol{\omega}_i - \boldsymbol{\Omega}^{\infty}) \tag{2}$$

Melrose and Ball [2004] uses the leading squeeze mode of lubrication force:

$$\boldsymbol{F}_{ij}^{\rm pair} = -\alpha_{\rm n}(h_{ij})\big\{(\boldsymbol{v}_i - \boldsymbol{v}_j)\cdot\boldsymbol{n}_{ij}\big\}\boldsymbol{n}_{ij} \tag{3}$$

$$\boldsymbol{T}_{ij}^{\rm pair} = \boldsymbol{0}, \tag{4}$$

where

$$\alpha_{\rm n}(h_{ij}) = \frac{3\pi\mu a^2}{2h_{ij}} \tag{5}$$

Hydrodynamic interaction acting on particle i is given by

$$\boldsymbol{F}_i^{\rm H} = \boldsymbol{F}_i^{\rm self} + \sum_j \boldsymbol{F}_{ij}^{\rm pair} \tag{6}$$

$$\boldsymbol{T}_i^{\rm H} = \boldsymbol{T}_i^{\rm self} \tag{7}$$

Dimensionless equations

The unit of length, velocity and force are given: $L_0 = a$, $v_0 = a\dot{\gamma}$, and $F_0 \equiv 6\pi\mu a v_0$.

The dimensionless variables are introduced: $\tilde{h}_{ij} = h_{ij}/L_0$, $\tilde{\boldsymbol{v}}_i = \boldsymbol{v}_i/U_0$, and $\tilde{\boldsymbol{F}}_{ij}^{\mathrm{pair}} = \boldsymbol{F}_{ij}^{\mathrm{pair}}/F_0$.

The relations are written as follows:

$$\tilde{\boldsymbol{F}}_i^{\mathrm{self}} = -(\tilde{\boldsymbol{v}}_i - \tilde{\boldsymbol{U}}^{\infty}(\tilde{\boldsymbol{r}}_i)) \tag{8}$$

$$\tilde{\boldsymbol{F}}_{ij}^{\mathrm{pair}} = -\frac{1}{4\tilde{h}_{ij}}\big\{(\tilde{\boldsymbol{v}}_i - \tilde{\boldsymbol{v}}_i)\cdot\boldsymbol{n}_{ij}\big\}\boldsymbol{n}_{ij} \tag{9}$$

$$\boldsymbol{U}^{\infty}(\boldsymbol{r}) = \dot{\gamma} z \boldsymbol{e}_x \tag{10}$$

$$\Longrightarrow \tilde{\boldsymbol{U}}^{\infty}(\tilde{\boldsymbol{r}}) = \dot{\gamma} z \boldsymbol{e}_x / U_0 = \dot{\gamma} z \boldsymbol{e}_x / (\dot{\gamma} a) = \tilde{z}\boldsymbol{e}_x \tag{11}$$

Matrix form

$$\boldsymbol{F}_{ij}^{\mathrm{pair}} = -\frac{1}{4h_{ij}}\big\{(\boldsymbol{v}_i - \boldsymbol{v}_j)\cdot\boldsymbol{n}_{ij}\big\}\boldsymbol{n}_{ij} \tag{12}$$

$$= -\frac{1}{4h_{ij}}\Big[\big\{(\boldsymbol{v}_i - \boldsymbol{U}_i^{\infty}) - (\boldsymbol{v}_j - \boldsymbol{U}_j^{\infty}) + \boldsymbol{U}_i^{\infty} - \boldsymbol{U}_j^{\infty}\big\}\cdot\boldsymbol{n}_{ij}\Big]\boldsymbol{n}_{ij} \tag{13}$$

$\Delta\boldsymbol{v} \equiv (\boldsymbol{v} - \boldsymbol{U}^{\infty})$

$$\Delta\boldsymbol{v}\cdot\boldsymbol{n}\boldsymbol{n} = (\Delta v_x n_x + \Delta v_y n_y + \Delta v_z n_z)n_x\boldsymbol{i} \tag{14}$$

$$+ (\Delta v_x n_x + \Delta v_y n_y + \Delta v_z n_z)n_y\boldsymbol{j} \tag{15}$$

$$+ (\Delta v_x n_x + \Delta v_y n_y + \Delta v_z n_z)n_z\boldsymbol{k} \tag{16}$$

$$= \begin{pmatrix} n_x n_x & n_y n_x & n_z n_x \\ n_x n_y & n_y n_y & n_z n_y \\ n_x n_z & n_y n_z & n_z n_z \end{pmatrix}\begin{pmatrix} \Delta v_x \\ \Delta v_y \\ \Delta v_z \end{pmatrix} \tag{17}$$

$$= \boldsymbol{n}\otimes\boldsymbol{n}\Delta\boldsymbol{v} \tag{18}$$

$$\boldsymbol{F}_i = -\Delta\boldsymbol{v}_i - \sum_j\left(\frac{1}{4h_{ij}}\boldsymbol{n}_{ij}\otimes\boldsymbol{n}_{ij}\Delta\boldsymbol{v}_i - \frac{1}{4h_{ij}}\boldsymbol{n}_{ij}\otimes\boldsymbol{n}_{ij}\Delta\boldsymbol{v}_j + \frac{1}{4h_{ij}}(\boldsymbol{U}_i^{\infty} - \boldsymbol{U}_j^{\infty})\cdot\boldsymbol{n}_{ij}\boldsymbol{n}_{ij}\right) \tag{19}$$

$$\begin{pmatrix} \vdots \\ \boldsymbol{F}_i \\ \vdots \\ \boldsymbol{F}_j \\ \vdots \end{pmatrix} = \begin{pmatrix} \vdots \\ \boldsymbol{F}_i^{\mathrm{self}} + \boldsymbol{F}_{ij}^{\mathrm{pair}} \\ \vdots \\ \boldsymbol{F}_j^{\mathrm{self}} + \boldsymbol{F}_{ji}^{\mathrm{pair}} \\ \vdots \end{pmatrix} \tag{20}$$

$$= -\begin{pmatrix} \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & 1 + \frac{1}{4h_{ij}}\boldsymbol{n}_{ij}\otimes\boldsymbol{n}_{ij} & \cdots & -\frac{1}{4h_{ij}}\boldsymbol{n}_{ij}\otimes\boldsymbol{n}_{ij} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & -\frac{1}{4h_{ji}}\boldsymbol{n}_{ji}\otimes\boldsymbol{n}_{ji} & \cdots & 1 + \frac{1}{4h_{ji}}\boldsymbol{n}_{ji}\otimes\boldsymbol{n}_{ji} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix}\begin{pmatrix} \vdots \\ \Delta\boldsymbol{v}_i \\ \vdots \\ \Delta\boldsymbol{v}_j \\ \vdots \end{pmatrix} \tag{21}$$

$$+ \begin{pmatrix} \vdots \\ -\frac{1}{4h_{ij}}(\boldsymbol{U}_i^{\infty} - \boldsymbol{U}_j^{\infty})\cdot\boldsymbol{n}_{ij}\boldsymbol{n}_{ij} \\ \vdots \\ -\frac{1}{4h_{ji}}(\boldsymbol{U}_j^{\infty} - \boldsymbol{U}_i^{\infty})\cdot\boldsymbol{n}_{ji}\boldsymbol{n}_{ji} \\ \vdots \end{pmatrix} \tag{22}$$

References

J. R. Melrose and R. C. Ball. Continuous shear thickening transitions in model concentrated colloids—the role of interparticle forces. *J. Rheol.*, 48:937–960, 2004.