HW6

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November 25, 2015

Exercise 1

a

True. In this case, the null space of A has one dimension. Suppose $x_1 \in null(A)$. x_0 is an element of P. Any elements of P is of the form $x_0 + cx_1, c \in \mathbb{R}$. Then P is constrained in a line and can't have 2 basic feasible point.

b

False. Consider minimize c, c is a constant, subject to $x \geq 0$, the optimal solution is $x \in [0, +\infty)$ is unbounded.

c

False. Consider the example of (b), any feasible solution is optimal.

d

True. If x_1 and x_2, are optimal, any convex combination of them are optimal solution.'

e

False. Consider min x_1 subject to $x_1 = 0, x_2 \geq 0$ the optimal solution is $\{0\} \times [0, \infty)$ is infinitely many, but only has one optimal BFS.

f

False. Consider max $|x_1 - 0.5| = max\{x_1 - 0.5x_3, -x_1 + 0.5x_3\}$ subject to $x_3 = 1, x_1 + x_2 = 1, x_1, x_2, x_3 \geq 0$. The unique optimal solution is (0.5, 0.5, 1), but is not basic solution.

Exercise 2

a

False, A has full rank, then BFS is non-degenrated. When x_j entered, the cost is strickly decreased since the solution is moving along a feasible direction chosen is based on $c_j < 0$, So the cost must change.

b

Because a variable can be entered only if the reduced cost is negative. A variable is leaving iff the coresponding reduced cost is nonnegative. So in next iteration, the variable will not reenter.

c

False. Consider the problem min $-x_1 - 2x_2$ such that $x_1 + x_2 \leq 1$, and $x_1, x_2 \geq 0$. The transformed problem is $x_1 + x_2 + x_3 = 1$, and $x_1, x_2, x_3 \geq 0$. If we start with x_3 and (0,0,1) with cost function equal to 0. Then in next interation the argorithm moves to (1,0,0), and the cost function is -1. In the very next interation, the argorithm moves to (0,1,0) with cost function -2. This is a counterexample is the statement.

d

False. Consider the min x_2 with the same constraint in (b). The optomal solution is $x_2 = 0$ and $x_1 \in [0, 1]$. But it has two linear independent BFS (1,0,0) and (0,0,1).

e

True. Directly follows Thm 2.4 of Bertsemas and Tistsiklis, *Introduction to Linear Optimization* (1997)