odd(s(0)).

odd(s(s(X))) ← odd(X).

Least Herbrand Model

Th. 2.12. If J' is a model of P ~~and~~ $\{G\}$ then $J = \{A \in B_P \mid J' \models A\}$ is a Herbrand model of $P \cup \{G\}$.

Th. 2.14. Let M be a family of Herbrand models of P. Then $\cap M$ is a Herbrand model of P.

M_P is the least Herbrand Model of P.

Th. 2.16. $M_P = \{A \in B_P \mid P \models A\}$

Def. $T_P(I) = \left\{ A_0 \;\middle|\; \begin{array}{l} A_0 \leftarrow A_1 \cdots A_m \in \text{ground}(P) \text{ and} \\ \{A_1 \cdots A_m\} \subseteq I \end{array} \right\}$

Th 2.20. $M_P = T_P \uparrow \omega$ and M_P is the lfp. of T_P

Least Herbrand Model

Herbrand Universe ^{U_A}: All ground terms constructible from the alphabet A with at least one constant.

Herbrand base ^{B_A}: All ground atoms ....

$$\left.\begin{array}{l} U_P: \\ B_P: \end{array}\right\} \begin{array}{l} \text{functors are those in } P \\ \text{predicates} \end{array}$$

Herbrand interpretation: $\mathfrak{I}$ is a Herbrand interpretation if

$$\left.\begin{array}{l} * \text{ Domain of } \mathfrak{I} \text{ is } U_P \\ * \; f_J(x_1 \cdots x_n) = f(x_1 \cdots x_n) \end{array}\right\} \text{pre-defined}$$

* P_J is a subset of B_P^n

Herbrand model: A Herbrand interpretation that is a model

Definite Programs

Program:

child(tom, john)

child(ann, tom)

child(john, mark)

child(alice, john)

$\forall X \forall Y\,(\,grandchild(X,Y) \leftarrow \exists Z\,(child(X,Z) \land child(Z,Y))$

$\forall X \forall Y \forall Z\,(\,grandchild(X,Y) \leftarrow (child(X,Z) \land child(Z,Y))$

Queries: $\leftarrow child(X, john)$ $\qquad \exists X.\, child(X, john$

$\leftarrow grandchild(X, john)$

Definite Programs

* Definite clauses: clauses with at most one positive literal.

Rules: $\forall(A_0 \vee \neg A_1 \vee \neg A_2 \cdots \vee \neg A_m)$

$\forall(A_1 \wedge A_2 \cdots \wedge A_m \rightarrow A_0)$

$\forall(A_0 \leftarrow A_1 \wedge A_2 \cdots \wedge A_m)$

$\underset{\text{Head}}{A_0} \leftarrow \underbrace{A_1 \wedge A_2 \cdots \wedge A_m}_{\text{Body}}$

Facts: No body.

$A_0 \leftarrow \blacksquare$

Goals: No head

$\square \leftarrow A_1 \wedge \cdots \wedge A_m$

Empty clause: $\square \leftarrow \blacksquare$

Substitution

Application: $f(t_1 \cdots t_m)\theta \triangleq f(t_1\theta, \cdots t_m\theta)$

Thus, a substitution induces a mapping from terms to terms.

Composition: Let $\theta = \{x_1/s_1, \dots x_m/s_m\}$

$\sigma = \{y_1/t_1, \cdots y_n/t_n\}$

$$\theta\sigma = \{x_1/s_1\sigma, \dots x_m/s_m\sigma\} \cup \sigma \restriction (\{y_1 \cdots y_n\} \setminus \{x_1 \cdots x_m\})$$

Functional Composition

Restriction:

$$\eta \restriction \nu = \{\, x_i/t_i \in \eta \mid x_i \in \nu \,\}$$

Idempotence: θ is idempotent iff $\theta = \theta\theta$

Substitution

Def. A substition θ is a finite set of pairs of terms $\{x_1/t_1, \dots x_n/t_n\}$ with x_i a variable, t_i a term, $x_i \neq t_i$ and $\forall i \neq j.\ x_i \neq x_j$.

<u>Domain</u> $\quad dom(\theta) = \{x_1, \dots x_n\}$

<u>Range</u> $\quad Range(\theta) = \bigcup_{i=1}^{n} vars(t_i)$

where $vars(t_i)$ is the set of variables in t_i

A substitution θ is an (almost identity) function from Vars to Terms

Empty substitution is ε (identity)

<u>Application</u> $\quad x\theta \triangleq \begin{cases} t & \text{if } x/t \in \theta \\ x & \text{otherwise} \end{cases}$