Unification

Unifier

- Let E be a set of equations
  $\{s_1 \cong t_1, s_2 \cong t_2, \ldots, s_n \cong t_n\}$
  where s_i and t_i are terms
- A substitution θ is a ***unifier*** of E iff
  $s_k\theta \equiv t_k\theta$ for all k in [1..n]

Most General Unifier

- Let θ and σ be substitutions. θ is said to be more general than σ, denoted $\sigma \leq \theta$, if there is a substitution ω such that $\sigma = \theta\omega$.
- A substitution θ is a **most general unifier** (m.g.u.) of E iff θ is more general than any other unifier of E.
- Renaming substitution ρ is a permutation of V. Each renaming substitution ρ has an inverse ρ^{-1} such that $\rho\rho^{-1}=\varepsilon$
- Let o be a syntactic object (term, atom, equation, substitution and so on). Then $o\rho$ is called a variant of o.

Most General Unifiers

- **Fact:** If a set E of equations has a unifier then it has a most general unifier.
- **Fact:** The m.g.u.s of E are variant of each other. That is, E has a unique m.g.u. modulo renaming.
- **Fact:** All unifiers of E are instances of a m.g.u. of E.
- **Notation:** The m.g.u. of E is denoted mgu(E). The m.g.u. of $\{s \cong t\}$ is denoted mgu(s,t).

Solved form

- A set of equations $\{x_1 \cong t_1, x_2 \cong t_2, \ldots, x_n \cong t_n\}$ is said to be in ***solved form*** iff $x_1, x_2, \ldots, x_n$ are variables and none of $x_1, x_2, \ldots, x_n$ occur in any of $t_1, t_2, \ldots, t_n$
- ***Fact:*** Let $E = \{x_1 \cong t_1, x_2 \cong t_2, \ldots, x_n \cong t_n\}$ be in solved form then $\theta = \{x_1/t_1, x_2/t_2, \ldots, x_n/t_n\}$ is a m.g.u. of E.

Equivalent sets of equations

- Two sets of equations are ***equivalent*** iff they have the same set of unifiers.
- Solving a set of equations reduces to transform a set of equations into an equivalent set of equations in solved form.
- Algorithm in Figure 3.2 terminates. If E is unifiable that it returns a m.g.u. Otherwise, it returns ***failure***.

Unification Algorithm

Input: A set $\mathcal{E}$ of equations.
Output: An equivalent set of equations in solved form or **failure**.

repeat
    select an arbitrary $s \doteq t \in \mathcal{E}$;
    case $s \doteq t$ **of**
        $f(s_1, \ldots, s_n) \doteq f(t_1, \ldots, t_n)$ where $n \geq 0 \Rightarrow$
            replace equation by $s_1 \doteq t_1, \ldots, s_n \doteq t_n$;   % *case 1*
        $f(s_1, \ldots, s_m) \doteq g(t_1, \ldots, t_n)$ where $f/m \neq g/n \Rightarrow$
            halt with **failure**;   % *case 2*
        $X \doteq X \Rightarrow$
            remove the equation;   % *case 3*
        $t \doteq X$ where t is not a variable $\Rightarrow$
            replace equation by $X \doteq t$;   % *case 4*
        $X \doteq t$ where $X \neq t$ and X has more than one occurrence in $\mathcal{E} \Rightarrow$
            if X is a proper subterm of t **then**
                halt with **failure**   % *case 5a*
            else
                replace all other occurrences of X by t;   % *case 5b*
    esac
until no action is possible on any equation in $\mathcal{E}$;
halt with $\mathcal{E}$;

Figure 3.2: Solved form algorithm

Prolog

- Prolog is a programming language.

Prolog = Definite Programs + …