출력층

가중치 기울기: $\partial w_{jk} = \frac{\partial E}{\partial w_{jk}} = \frac{\partial E}{\partial u_k}\frac{\partial u_k}{\partial w_{jk}}$

$= \frac{\partial E}{\partial u_k} \mathrm{y_j}$

$= (\frac{\partial E}{\partial y_k}\frac{\partial y_k}{\partial u_k})\mathrm{y_j}$

$= \delta_\mathrm{k}\mathrm{y_j}$

Bias 기울기 : $\partial \mathrm{b_k} = \frac{\partial \mathrm{E}}{\partial \mathrm{b_k}} = \frac{\partial \mathrm{E}}{\partial \mathrm{u_k}}\frac{\partial \mathrm{u_k}}{\partial \mathrm{b_k}}$

$= \frac{\partial \mathrm{E}}{\partial \mathrm{b_k}} * 1$

$= \frac{\partial E}{\partial y_k}\frac{\partial y_k}{\partial u_k}$

$= \delta_\mathrm{k}$

입력 기울기: $\partial y_j = \frac{\partial E}{\partial y_j} = \sum_{r=1}^{n}\frac{\partial E}{\partial u_r}\frac{\partial u_r}{\partial y_j}$

$= \sum_{r=1}^{n}\frac{\partial E}{\partial u_r}\mathrm{w_{jr}}$

$= \sum_{r=1}^{n}\frac{\partial E}{\partial y_k}\frac{\partial y_k}{\partial u_r}\mathrm{w_{jr}}$

$= \sum_{r=1}^{n}\delta_\mathrm{r}\mathrm{w_{jr}}$

은닉층

가중치 기울기: $\partial w_{ij} = \frac{\partial E}{\partial wi_j} = \frac{\partial E}{\partial u_j}\frac{\partial u_j}{\partial wi_j}$

$= \frac{\partial E}{\partial u_j}\mathrm{y_i}$

$= (\frac{\partial E}{\partial y_j}\frac{\partial y_j}{\partial u_j})\mathrm{y_i}$

$= \delta_\mathrm{j}\mathrm{y_i}$

Bias 기울기 : $\partial \mathrm{b_j} = \frac{\partial \mathrm{E}}{\partial \mathrm{b_j}} = \frac{\partial \mathrm{E}}{\partial \mathrm{u_j}}\frac{\partial \mathrm{u_j}}{\partial \mathrm{b_j}}$

$= \frac{\partial \mathrm{E}}{\partial \mathrm{b_j}} * 1$

$= \frac{\partial E}{\partial y_j}\frac{\partial y_j}{\partial u_j}$

$= \delta_\mathrm{j}$

최종 정리

가중치 기울기: $\partial w_{jk} = \frac{\partial E}{\partial w_{jk}}$

Bias 기울기: $\partial \mathrm{b_k} = \frac{\partial E}{\partial b_k}$

입력 기울기: $\partial y_j = \frac{\partial E}{\partial y_j}$

$\frac{\partial u_k}{\partial w_{jk}} = \frac{\partial(\sum_{q=1}^{m} y_q w_{qk} + b_k)}{\partial w_{jk}}$

$= \frac{\partial}{\partial w_{jk}}(y_1 w_1 + \ldots + y_j w_{jk} + y_m w_{mk} + b_k)$

$= \frac{\partial y_j w_{jk}}{\partial w_{jk}} = \mathrm{y_j}$

$\frac{\partial u_k}{\partial b_k} = \frac{\partial(\sum_{q=1}^{m} y_q w_{qk} + b_k)}{\partial b_k}$

$= \frac{\partial}{\partial b_k}(y_1 w_1 + \ldots + y_j w_{jk} + y_m w_{mk} + b_k)$

$= \frac{\partial b_k}{\partial b_k} = 1$

$\delta_\mathrm{x} = \frac{\partial E}{\partial y_x}\frac{\partial y_x}{\partial u_x}$