Comprehensive Research on Magnetic Music and Sound Generation

Saba Machavariani

july 23, 2024

Abstract

This comprehensive research investigates the fundamental physics, theoretical models, and applications of magnetic music and sound generation. The study explores how magnetic fields interact with vibrating systems to create, modify, and manipulate audible vibrations. Two primary mechanisms are examined: electromagnetic induction, which underlies modern audio technology including speakers and electromagnetic pickups, and magnetostriction, which produces characteristic acoustic signatures in electrical equipment. The research develops detailed mathematical models for magnetically-damped oscillating systems, including vibrating strings and pendulums, deriving frequency equations and amplitude decay characteristics. Experimental methods for measuring sound frequency, amplitude, and damping parameters are presented, along with error analysis techniques. Applications range from electric guitar pickups and electromagnetic sustainers to industrial magnetostrictive transducers and experimental sound art. The theoretical framework establishes that magnetic damping causes exponential amplitude decay proportional to B^2 and can either increase or decrease oscillation frequency depending on field configuration. This interdisciplinary work combines acoustics, electromagnetism, materials science, and musical aesthetics.

Contents

1 Introduction to Magnetic Music

1.1 What is Magnetic Music?

Magnetic music and **magnetically-generated sound** refer to the use of magnetic fields to create, record, or manipulate audible vibrations. While historically associated with magnetic storage media (cassette tapes, magnetic tape), the term in experimental contexts describes the direct generation or modification of sound through physical phenomena where magnetic fields interact with matter to produce or alter sound waves.

At its core, magnetically generated sound is the conversion of electromagnetic energy into mechanical energy (vibration) that the human ear perceives as sound. This conversion occurs through two primary methods:

1.1.1 Primary Mechanisms

Electromagnetic Induction (Dominant Method) This is the fundamental principle behind 99% of modern audio technology (speakers, headphones, electromagnetic pickups). A varying electrical signal creates a fluctuating magnetic field in a coil of wire. This field pushes and pulls against a fixed permanent magnet, causing the coil and an attached diaphragm to vibrate and move air, creating sound waves.

Magnetostriction (The "Hum" Method) This occurs when ferromagnetic materials (like iron cores in transformers) physically change shape—expanding and contracting microscopically—when exposed to a changing magnetic field. This creates the characteristic "hum" or "whine" heard in heavy electrical equipment and power transformers.

In experimental contexts, "Magnetic Music" refers to instruments that use magnetic fields to induce vibration in strings, tines, or other resonating objects without physical contact, or to modify the acoustic properties of vibrating systems.

1.2 Historical Background

The history of magnetic sound represents a journey from crude steel wires to high-fidelity tape recording and modern electromagnetic instruments.

1.2.1 Key Milestones

1888: Theoretical Birth American engineer **Oberlin Smith** first published the concept that sound could be recorded by magnetizing a steel wire, though he never built a working prototype.

1898: The Telegraphone Danish engineer **Valdemar Poulsen** created the first working magnetic recorder. It used a steel wire wrapped around a drum. Despite being noisy and low-fidelity, it proved the concept was viable.

1928: Magnetic Tape Invented **Fritz Pfleumer** in Germany invented magnetic tape by coating paper with iron oxide powder. This was much lighter and cheaper than steel wire, making practical magnetic recording feasible.

1935: The Magnetophon German company AEG released the Magnetophon, the first practical tape recorder. During World War II, the discovery of **AC Biasing**—adding an inaudible high-frequency signal to the recording—drastically improved sound quality, allowing the Germans to broadcast pre-recorded music that sounded "live," confusing Allied intelligence.

Post-WWII Era The technology was brought to the United States, notably by Bing Crosby, who wanted to pre-record his radio shows. This led to the standard reel-to-reel tape, the 8-track cartridge, and eventually the ubiquitous **Compact Cassette** introduced by Philips in 1963.

1.3 Physical Principles

Two main laws of physics govern how magnets create or influence sound:

1.3.1 The Lorentz Force (Electromagnetic Induction)

This is the principle behind loudspeakers, dynamic microphones, and electromagnetic pickups. The law states that a current-carrying wire placed in a magnetic field experiences a force perpendicular to both the field and the current, described by:

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}) \tag{1}$$

where $\mathbf{F}$ is the force on the charge, q is the electric charge, $\mathbf{v}$ is the velocity of the charge, and $\mathbf{B}$ is the magnetic field.

Application in Speakers: In a loudspeaker, the "voice coil" is suspended in a strong magnetic field. When an AC audio signal (current) flows through the coil, the Lorentz force pushes the coil out and pulls it in rapidly. This moves the speaker cone, compressing air to create sound waves.

Application in Pickups: In an electromagnetic pickup (such as in electric guitars), the process works in reverse. A permanent magnet wrapped in wire creates a stable field. When a steel string vibrates in this field, it disturbs the magnetic flux, inducing a tiny electrical current in the wire (Faraday's Law) which is then amplified.

1.3.2 Magnetostriction

This is the principle behind the "mains hum" of electrical substations and transformers.

The Phenomenon: Ferromagnetic materials contain microscopic "domains" (tiny built-in magnets). When an external magnetic field is applied, these domains rotate to align with the field. This re-alignment causes the material to physically deform (stretch or shrink) by a tiny fraction—typically on the order of micrometers.

The Sound: Because the magnetic field in AC power cycles 50 or 60 times a second (Hz), the iron core of a transformer stretches and shrinks 100 or 120 times a second (twice per cycle). This physical vibration of the metal core creates pressure waves in the surrounding air, producing a constant low-frequency drone.

1.4 Applications

Magnetic sound generation and manipulation has numerous applications:

Everyday Audio Technology:

- Loudspeakers and headphones (electromagnetic transduction)
- Electric guitar pickups (magnetic field disturbance detection)
- Dynamic microphones (reverse speaker principle)
- Magnetic recording media (tape, hard drives)

Experimental and Creative Instruments:

- EBow (Electronic Bow): electromagnetic sustainer for guitars
- Theremin: electromagnetic field sensing (capacitive)
- Magnetovore: recycling electromagnetic fields from obsolete technology
- Electromagnetic pianos: magnetic excitation of piano strings

Industrial and Scientific:

- Magnetostrictive transducers (sonar, ultrasonic cleaning)
- Active noise cancellation systems
- Magnetic actuators for bone conduction hearing aids
- Surface Acoustic Wave (SAW) devices in telecommunications

2 Physics of String Vibration and Sound

2.1 The Wave Equation for Vibrating Strings

The motion of a vibrating string is governed by the one-dimensional wave equation. For a string stretched along the x-axis with tension T and linear mass density μ (mass per unit length), the vertical displacement $y(x,t)$ follows this partial differential equation:

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2} \tag{2}$$

where v is the wave propagation speed along the string, given by:

$$v = \sqrt{\frac{T}{\mu}} \tag{3}$$

For a string fixed at both ends (like on a guitar or violin) of length L, the boundary conditions are:

- $y(0, t) = 0$ (fixed at one end)
- $y(L, t) = 0$ (fixed at other end)

The solutions are **standing waves** of the form:

$$y_n(x, t) = A_n \sin\left(\frac{n\pi x}{L}\right) \cos(\omega_n t + \phi_n) \tag{4}$$

where n is an integer $(1, 2, 3 \ldots)$ representing the harmonic mode number.

2.2 Mersenne's Law: Fundamental Frequency Equation

By solving the wave equation with the boundary conditions, we derive the frequency equation. The wavelength λ_n for the n-th harmonic is restricted by the string length L:

$$\lambda_n = \frac{2L}{n} \tag{5}$$

Since frequency $f = v/\lambda$, substituting the wave speed $v = \sqrt{T/\mu}$ yields the fundamental equation for the frequency of the n-th harmonic, known as **Mersenne's Law**:

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}} \tag{6}$$

For the **fundamental frequency** ($n = 1$), which determines the perceived pitch:

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \tag{7}$$

This is one of the most important equations in musical acoustics, relating string properties to sound.

2.3 How String Properties Affect Sound

2.3.1 String Length (L)

Relationship: Frequency is **inversely proportional** to length.

$$f \propto \frac{1}{L} \tag{8}$$

Physics: A longer string allows for a longer wavelength ($\lambda = 2L$). Since the wave speed v remains constant (assuming constant tension and density), a longer wavelength results in a lower frequency.

Practical Application:

- Fretting a guitar string shortens the effective vibrating length L, increasing the pitch
- Bass instruments require longer strings to achieve low frequencies
- Piano bass strings are longer than treble strings

Effect on Amplitude: For a given input energy, a longer string generally allows for larger spatial amplitude (displacement) because it is less stiff for the same tension.

2.3.2 Tension (T)

Relationship: Frequency is proportional to the **square root of tension**.

$$f \propto \sqrt{T} \tag{9}$$

Physics: Higher tension increases the restoring force on displaced string segments, causing them to snap back faster. This increases the wave speed v, and consequently the frequency.

Practical Application:

- Tuning pegs work by changing tension
- To raise the pitch by one octave (double frequency), you must quadruple the tension
- Excessive tension can damage the instrument

Effect on Amplitude: Higher tension increases the restoring force (stiffness). For a fixed plucking force, higher tension results in lower amplitude ($A \propto 1/T$).

2.3.3 Linear Mass Density / Thickness (μ)

Relationship: Frequency is **inversely proportional to the square root of linear mass density**.

$$f \propto \frac{1}{\sqrt{\mu}} \tag{10}$$

Physics: μ represents inertia. A heavier (thicker) string is harder to accelerate. This creates a slower wave speed v, resulting in a lower frequency for a given length and tension.

Geometry: Since $\mu = \rho \cdot A$ (density times cross-sectional area) and Area $\propto r^2$ (radius squared), frequency is roughly inversely proportional to the string's diameter/thickness.

Amplitude Effect: Thicker strings generally require more energy to displace but sustain vibration longer due to higher momentum. They may have lower amplitude for the same input energy compared to a lighter string, but this depends heavily on the driving force.

2.4 Harmonic Frequencies and Overtones

A real string rarely vibrates at just the fundamental frequency ($n = 1$). It vibrates in a complex superposition of multiple modes simultaneously.

Fundamental (1st Harmonic, $n = 1$): The string vibrates as one single arc. This is the loudest component and defines the pitch we "hear."

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \tag{11}$$

Overtones ($n > 1$): **2nd Harmonic ($n = 2$):**

- Frequency is $2f_1$
- The string has a node in the exact center
- Sounds one octave higher than fundamental

3rd Harmonic ($n = 3$):

- Frequency is $3f_1$
- Two nodes divide the string into thirds
- Sounds an octave and a fifth higher than fundamental

Timbre: The relative amplitude of these harmonics gives an instrument its unique "color" or timbre. A plucked string near the bridge excites many higher harmonics (bright sound), while plucking near the center excites mostly the fundamental (mellow sound).

2.5 Sound Generation Mechanics

It is important to note that a vibrating string moves very little air on its own because it is so thin—it "slices" through the air with minimal displacement.

Energy Transfer Process:

1. **Coupling:** The string's vibrational energy is transferred physically to a **resonant body** (guitar body, violin belly, piano soundboard) via the bridge.
2. **Impedance Matching:** The large surface area of the soundboard vibrates against the air much more efficiently than the string could alone. This matches the acoustic impedance between the string and the air.
3. **Radiation:** The soundboard compresses and rarefies air molecules, creating longitudinal pressure waves (sound) that travel to the ear.

Without this coupling mechanism, a vibrating string produces very little audible sound, which is why solid-body electric guitars are nearly silent acoustically.

3 Magnetic Force Effects on Vibrating Systems

3.1 The Lorentz Force: The Fundamental Mechanism

The interaction between magnetic fields and vibrating conducting systems begins with the Lorentz force. When a conductor moves through a magnetic field, the free charge carriers (electrons) inside the conductor move with it. A charge q moving with velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$ experiences a force:

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}) \tag{12}$$

Physical Explanation: If a metallic object vibrates, the atoms (and the crystal lattice) move back and forth. The free electrons share this macroscopic velocity. The magnetic field exerts a transverse force on these electrons, causing them to drift. This drift of electrons constitutes an electric current.

3.2 Eddy Currents in Moving Conductors

As the conductor moves through a non-uniform magnetic field, or moves such that the magnetic flux through it changes, electromotive forces (EMF) are induced according to Faraday's Law of Induction.

3.2.1 Motional EMF

For a conductor of length L moving at velocity v perpendicular to a field B, the induced voltage is:

$$\mathcal{E} = vBL \tag{13}$$

In a bulk conductor (like a pendulum bob or a thick wire), these induced voltages create circulating loops of current known as **Eddy Currents**.

3.2.2 Current Density

Using Ohm's Law in local form ($\mathbf{J} = \sigma \mathbf{E}_{effective}$), where σ is the electrical conductivity:

$$\mathbf{J} = \sigma(\mathbf{v} \times \mathbf{B}) \tag{14}$$

(Assuming no external electrostatic field)

These eddy currents flow in closed loops within the conductor, perpendicular to the magnetic field lines.

3.3 Electromagnetic Damping

Eddy currents dissipate energy. As currents circulate through the metal's internal resistance, energy is lost as heat (Joule heating).

3.3.1 Energy Perspective

The power dissipated as heat is:

$$P_{loss} = \int \frac{J^2}{\sigma} dV \tag{15}$$

This energy comes from the kinetic energy of the vibrating system. Consequently, the mechanical amplitude of the vibration decreases over time—the system is damped.

3.3.2 Force Perspective (Lenz's Law)

The induced eddy currents create their own magnetic field. By Lenz's Law, this induced field opposes the change that created it. More directly, the induced current $\mathbf{J}$ itself interacts with the external field $\mathbf{B}$ to create a Lorentz force on the conductor:

$$\mathbf{F}_{mag} = \int (\mathbf{J} \times \mathbf{B}) dV \tag{16}$$

Substituting $\mathbf{J} \approx \sigma(\mathbf{v} \times \mathbf{B})$:

$$\mathbf{F}_{mag} \approx \int \sigma(\mathbf{v} \times \mathbf{B}) \times \mathbf{B} \, dV \tag{17}$$

For velocity $\mathbf{v}$ perpendicular to $\mathbf{B}$, this simplifies to a drag force proportional to velocity:

$$\mathbf{F}_{drag} \propto -\sigma B^2 \mathbf{v} \tag{18}$$

Key Point: Because the force opposes velocity ($\mathbf{F} \propto -\mathbf{v}$), it acts as a viscous damper, similar to moving through a thick fluid.

3.4 Dynamics of Oscillating Systems

3.4.1 Vibrating Metallic Strings

In a steel string vibrating over a pickup, the string is magnetized. As it moves, the magnetic flux through the pickup coil changes, inducing a signal. Conversely, if the string is non-magnetic but conductive (like copper), moving it through a strong static field induces eddy currents in the string itself.

The magnetic drag force acts to suppress the vibration, typically affecting higher frequency harmonics more distinctly because the skin depth of the eddy currents varies with frequency.

3.4.2 Magnetic Pendulums (Waltenhofen's Pendulum)

Consider a conductive plate swinging between the poles of a magnet. As the plate enters the field, the flux increases, inducing currents that try to repel the plate from the field. As it leaves, currents reverse to try to attract it back. Both effects retard the motion.

- **Solid Plate:** Large eddy current loops form → Strong damping (possibly overdamped)
- **Slotted Plate:** Slots break the large current loops → Weak damping (underdamped)

This is a classic physics demonstration of electromagnetic induction and Lenz's Law.

3.5 Mathematical Models of Magnetically-Damped Oscillations

We model magnetically damped systems as Damped Harmonic Oscillators.

3.5.1 Newton's Second Law

$$m\mathbf{a} = \mathbf{F}_{restoring} + \mathbf{F}_{damping} \tag{19}$$

For a simple spring-mass system or small-angle pendulum:

1. **Restoring Force:** $F_{spring} = -kx$
2. **Magnetic Damping Force:** $F_{mag} = -\alpha v = -\alpha \dot{x}$

The damping coefficient α for a simple geometry (like a wire of length L and resistance R in field B) is derived from $F = ILB$ and $I = \frac{BLv}{R}$:

$$\alpha = \frac{B^2 L^2}{R} \tag{20}$$

3.5.2 The Differential Equation

$$m\ddot{x} + \alpha\dot{x} + kx = 0 \tag{21}$$

Dividing by mass m:

$$\ddot{x} + 2\zeta\omega_0\dot{x} + \omega_0^2 x = 0 \tag{22}$$

where $\omega_0 = \sqrt{k/m}$ is the natural frequency and $\zeta = \frac{\alpha}{2\sqrt{mk}}$ is the damping ratio.

3.5.3 Solution (Underdamped Case, $\zeta < 1$)

The system oscillates with exponentially decaying amplitude:

$$x(t) = Ae^{-\gamma t}\cos(\omega_d t + \phi) \tag{23}$$

where:

- **Decay Rate (γ):** $\frac{\alpha}{2m} \propto \frac{\sigma B^2}{m}$
- **Damped Frequency (ω_d):** $\sqrt{\omega_0^2 - \gamma^2}$

Key Takeaway: The decay rate γ is proportional to the square of the magnetic field strength (B^2) and the conductivity (σ). Stronger magnets or better conductors stop the vibration faster.

4 Theoretical Models

4.1 Mathematical Model for a Simple Vibrating String

This model describes a taut, flexible string fixed at both ends—the canonical problem in wave mechanics.

4.1.1 The Wave Equation

Consider a string of length L, linear mass density μ (kg/m), and tension T (N). Let $y(x, t)$ be the vertical displacement of the string at position x and time t.

Newton's second law applied to a small segment dx yields the classical one-dimensional wave equation:

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} \tag{24}$$

where c is the wave speed (phase velocity):

$$c = \sqrt{\frac{T}{\mu}} \tag{25}$$

4.1.2 Boundary Conditions

For a string fixed at both ends ($x = 0$ and $x = L$):

1. $y(0, t) = 0$
2. $y(L, t) = 0$

4.1.3 General Solution and Frequency

Using separation of variables $y(x, t) = X(x)\Theta(t)$, we obtain:

- Spatial solutions: $X(x) = \sin(kx)$
- Temporal solutions: $\Theta(t) = \cos(\omega t)$

Applying boundary condition $y(L, t) = 0 \implies \sin(kL) = 0$.
Therefore, $k_n L = n\pi$ for $n = 1, 2, 3, \ldots$ (integers).
The angular frequency for the n-th harmonic is:

$$\omega_n = ck_n = \sqrt{\frac{T}{\mu}} \left(\frac{n\pi}{L}\right) \tag{26}$$

The frequency f_n (Hz) is:

$$f_n = \frac{\omega_n}{2\pi} = \frac{n}{2L}\sqrt{\frac{T}{\mu}} \tag{27}$$

4.2 Mathematical Model for a Pendulum with a String

This treats the system as a simple pendulum where the mass of the string is negligible compared to the bob, or the string acts as a rigid rod.

4.2.1 Equation of Motion

A mass m hangs from a string of length L. The restoring force is gravity (mg). For a small angular displacement θ:

$$F_{restoring} = -mg\sin(\theta) \approx -mg\theta \tag{28}$$

Using Newton's second law for rotation ($\tau = I\alpha$), where moment of inertia $I = mL^2$ and $\alpha = \frac{d^2\theta}{dt^2}$:

$$-mgL\theta = mL^2\frac{d^2\theta}{dt^2} \tag{29}$$

Simplifying:

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0 \tag{30}$$

This is the equation for Simple Harmonic Motion (SHM) with natural frequency:

$$\omega_0 = \sqrt{\frac{g}{L}} \tag{31}$$

The period of oscillation is:

$$T = 2\pi\sqrt{\frac{L}{g}} \tag{32}$$

4.3 Incorporating Magnetic Damping

Magnetic damping typically occurs when a conductive object (the string or the pendulum bob) moves through a magnetic field B. This induces an electromotive force (EMF), creating eddy currents (in bulk material) or current flow (in a wire circuit), which generates a Lorentz force opposing the motion.

4.3.1 The Damping Force

The damping force F_d is generally proportional to velocity v for standard electromagnetic braking:

$$F_d = -bv \tag{33}$$

where b is the magnetic damping coefficient.

- For a wire of length l in a circuit with resistance R: $b \approx \frac{B^2l^2}{R}$
- For a bulk conductor (eddy currents): b depends on conductivity, volume, and field geometry

4.3.2 Modified Equations of Motion

A. Damped Vibrating String We add a damping term proportional to transverse velocity $\frac{\partial y}{\partial t}$ to the wave equation:

$$\mu\frac{\partial^2 y}{\partial t^2} + \beta\frac{\partial y}{\partial t} = T\frac{\partial^2 y}{\partial x^2} \tag{34}$$

Dividing by μ:

$$\frac{\partial^2 y}{\partial t^2} + 2\gamma\frac{\partial y}{\partial t} = c^2\frac{\partial^2 y}{\partial x^2} \tag{35}$$

where $2\gamma = \frac{\beta}{\mu}$ is the damping term per unit mass.

B. Damped Pendulum We add a damping torque $\tau_d = -bL^2\dot{\theta}$ (assuming force acts on the bob):

$$mL^2\ddot{\theta} + bL^2\dot{\theta} + mgL\theta = 0 \tag{36}$$

Dividing by mL^2:

$$\ddot{\theta} + \frac{b}{m}\dot{\theta} + \frac{g}{L}\theta = 0 \tag{37}$$

Or:

$$\ddot{\theta} + 2\gamma\dot{\theta} + \omega_0^2\theta = 0 \tag{38}$$

where $\gamma = \frac{b}{2m}$ is the damping parameter.

4.4 Frequency Equations With and Without Magnetic Fields

4.4.1 Without Magnetic Field ($b = 0$, $\gamma = 0$)

The system oscillates at its **Natural Frequency** (ω_0).

- **String** ($n = 1$)**:** $\omega_0 = \frac{\pi}{L}\sqrt{\frac{T}{\mu}}$
- **Pendulum:** $\omega_0 = \sqrt{\frac{g}{L}}$

4.4.2 With Magnetic Field ($\gamma > 0$)

The system oscillates at the **Damped Frequency** (ω_d).

The solution takes the form of a damped sinusoid. The system will oscillate only if it is underdamped ($\omega_0^2 > \gamma^2$).

The general frequency equation is:

$$\omega_d = \sqrt{\omega_0^2 - \gamma^2} \tag{39}$$

Or in terms of frequency (Hz):

$$f_d = \frac{1}{2\pi}\sqrt{\omega_0^2 - \gamma^2} \tag{40}$$

4.5 Effect of Magnetic Field on Frequency and Amplitude

4.5.1 Amplitude Effect (Exponential Decay)

The magnetic field extracts energy from the system. The amplitude $A(t)$ decays exponentially over time:

$$A(t) = A_0 e^{-\gamma t} \tag{41}$$

Since $\gamma \propto B^2$ (usually), a stronger magnetic field causes much faster decay of the vibration.

The energy of the system decays as:

$$E(t) = E_0 e^{-2\gamma t} \tag{42}$$

4.5.2 Frequency Effect (Frequency Shift)

The magnetic field **lowers** the frequency of oscillation:

$$\omega_d < \omega_0 \tag{43}$$

However, for many practical systems, $\gamma \ll \omega_0$, meaning the frequency shift is often very small (second-order effect).

- If B increases $\rightarrow \gamma$ increases $\rightarrow \omega_d$ decreases
- If B is strong enough such that $\gamma \geq \omega_0$, the system becomes critically damped or overdamped and stops oscillating entirely

4.6 Derivation of Modified Frequency Formulas

Let us solve the differential equation for the Damped Harmonic Oscillator (which applies to both the pendulum and specific modes of the string).

4.6.1 Step 1: The Differential Equation

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = 0 \tag{44}$$

4.6.2 Step 2: Characteristic Equation

Assume a solution of the form $x(t) = e^{rt}$. Substituting derivatives:

$$r^2 e^{rt} + 2\gamma r e^{rt} + \omega_0^2 e^{rt} = 0 \tag{45}$$

$$r^2 + 2\gamma r + \omega_0^2 = 0 \tag{46}$$

4.6.3 Step 3: Solve for Roots (Quadratic Formula)

$$r = \frac{-2\gamma \pm \sqrt{4\gamma^2 - 4\omega_0^2}}{2} \tag{47}$$

$$r = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2} \tag{48}$$

4.6.4 Step 4: Underdamped Case ($\omega_0 > \gamma$)

Since we are looking for vibrations, the term under the square root must be negative ($\gamma^2 - \omega_0^2 < 0$). We factor out $\sqrt{-1} = i$:

$$r = -\gamma \pm i\sqrt{\omega_0^2 - \gamma^2} \tag{49}$$

Define the damped frequency ω_d:

$$\omega_d \equiv \sqrt{\omega_0^2 - \gamma^2} \tag{50}$$

So the roots are $r = -\gamma \pm i\omega_d$.

4.6.5 Step 5: Final Solution and Frequency

The general solution is:

$$x(t) = e^{-\gamma t}(C_1 e^{i\omega_d t} + C_2 e^{-i\omega_d t}) \tag{51}$$

Using Euler's formula, this simplifies to:

$$x(t) = A_0 e^{-\gamma t}\cos(\omega_d t + \phi) \tag{52}$$

4.6.6 Conclusion

The modified frequency in the presence of the magnetic field is:

$$\omega_d = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2} \tag{53}$$

Where b is the coefficient dependent on the magnetic field strength (B). If we assume $b = kB^2$ for some constant k, the explicit dependence is:

$$\omega_d(B) = \sqrt{\omega_0^2 - \left(\frac{kB^2}{2m}\right)^2} \tag{54}$$

This shows that increasing magnetic field strength decreases the oscillation frequency, though the effect is typically small unless damping is very strong.

5 Experimental Methods

5.1 Techniques for Measuring Sound Frequency and Amplitude

To quantify the effect of magnetism on a vibrating system, precise transducers and analysis tools are required.

5.1.1 Transducers (Sensors)

Electromagnetic Pickup (Guitar Pickup):

- Best for ferromagnetic strings (steel/nickel)
- Directly converts string velocity into voltage
- Insensitive to background acoustic noise
- Works on principle of Faraday's Law

Piezoelectric Sensor:

- Contact microphone
- Good for non-ferromagnetic strings (nylon, copper)
- Attaches to the bridge or soundboard
- Converts mechanical stress to electrical signal

Condenser Microphone:

- Measures acoustic sound pressure
- Requires quiet environment (anechoic chamber ideal)
- Sensitive to ambient noise
- Captures the actual acoustic output

Laser Doppler Vibrometer (LDV):

- "Gold standard" for non-contact measurement
- Measures exact velocity and displacement using laser interferometry
- Expensive but highly accurate
- Can measure vibration at specific points on the string

5.1.2 Analysis Tools

Oscilloscope:

- Essential for viewing the waveform (shape of the vibration)
- Measures peak-to-peak voltage (V_{pp}), which correlates to amplitude
- Can capture transient behavior and decay envelopes
- Digital storage oscilloscopes can record and analyze decay patterns

Spectrum Analyzer / FFT (Fast Fourier Transform):

- Converts time-domain signal into frequency domain
- Allows visualization of fundamental frequency (f_0) and all harmonics ($f_1, f_2, \ldots$)
- Can measure relative harmonic amplitudes
- Essential for analyzing timbre and spectral content

Frequency Counter:

- Dedicated hardware device for accurate frequency measurement
- Provides highly accurate readout of dominant frequency
- Useful for precision measurements of frequency shifts

5.2 Experimental Setup Design

A modified **Sonometer** setup was used for this research to allow for precise control over tension, length, and magnetic field application.

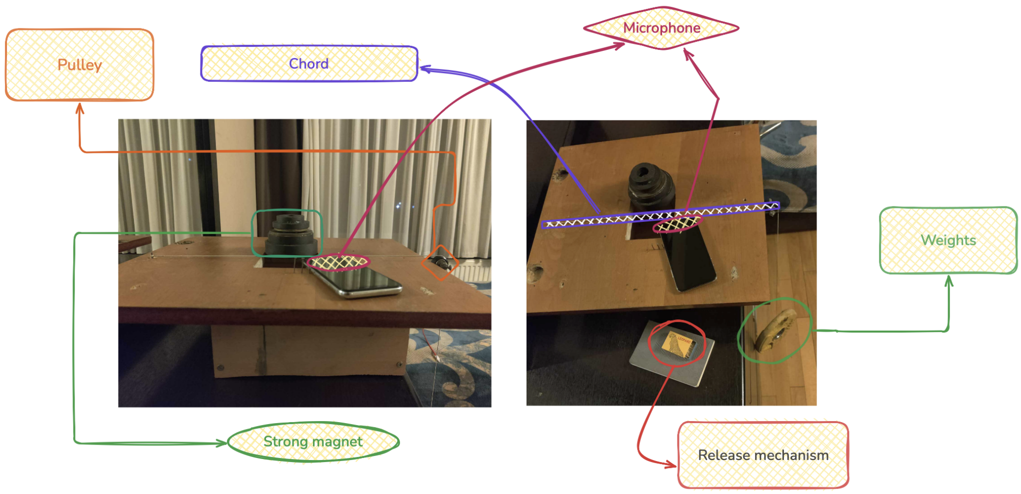


Figure 1: Experimental setup illustrating the pulley system, chord/string tensioning, microphone placement, and the magnetic release mechanism used to initiate consistent vibration.

5.2.1 Core Components

1. The Base:

- Heavy, rigid wooden or metal beam (Sonometer)
- Prevents body resonances that could interfere with measurements
- Provides stable platform for mounting components

2. The String:

- **Ferromagnetic wire** (e.g., piano wire, electric guitar string) for studying magnetic attraction/stiffness effects
- **Conductive non-ferromagnetic wire** (e.g., copper, aluminum) for studying Lorentz force or eddy current damping
- Different gauges for studying thickness effects

3. The Mounts:

- **Fixed End:** A sturdy clamp or bridge
- **Movable Bridge:** A prism-shaped wedge to define the vibrating length (L)
- **Pulley and Masses:** At the free end to set precise tension ($T = mg$)

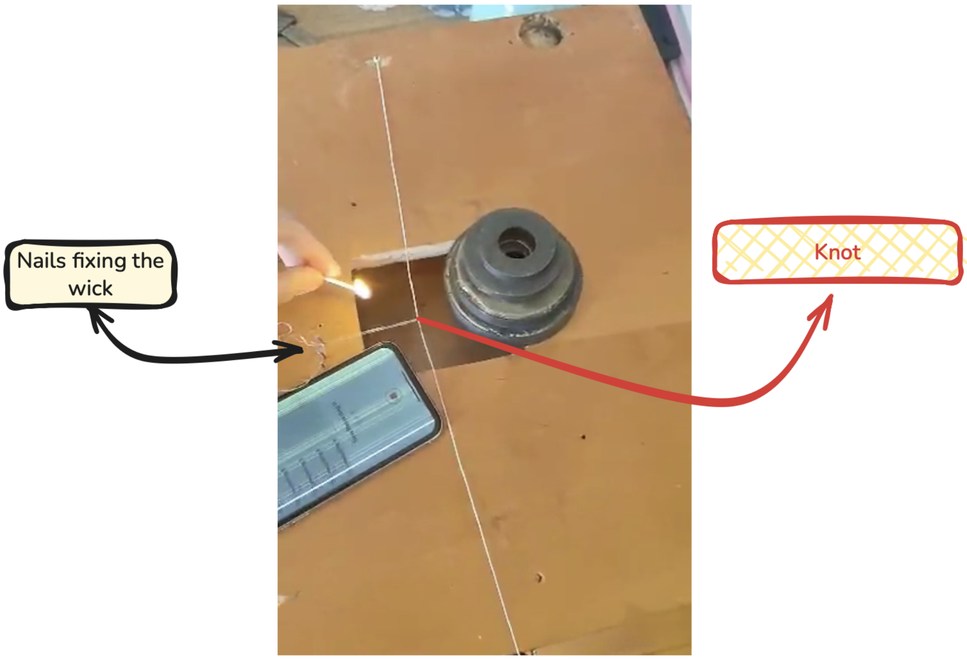


Figure 2: Detail of the knot and wick fixation method. Nails are used to fix the wick, ensuring the string length remains constant and tension is applied uniformly.

4. Magnetic Field Source:

- **Helmholtz Coils:** Two large circular coils placed on either side of the string, creating a uniform magnetic field over a large region
- **Neodymium Magnets:** For high-intensity, localized fields; mount on micrometer stage to vary distance (and thus field strength) precisely
- **Electromagnets:** Allow variable field strength by adjusting current

5. Exciter (Driver):

- **Mechanical:** A mechanical wave driver (piston) attached to the string near the fixed end
- **Electromagnetic:** Pass an AC current through the string itself while it sits in a permanent magnetic field; the Lorentz Force ($F = IL \times B$) drives the wire at the AC frequency

5.3 Measuring the Effect of Different Parameters

To isolate effects, change **one variable at a time** (control variables) while keeping others constant.

5.3.1 String Length (L)

- **Method:** Move the bridge position
- **Measurement:** Measure L with a meter stick or digital caliper (± 1 mm precision)
- **Theory:** Frequency $f \propto 1/L$
- **Expected Result:** Doubling length halves frequency (one octave lower)

5.3.2 String Thickness/Radius (r)

- **Method:** Use different gauge wires
- **Measurement:** Measure diameter with micrometer screw gauge at 3 different points along the wire and average
- **Theory:** Mass per unit length $\mu \propto r^2$, so $f \propto 1/r$
- **Expected Result:** Thicker strings produce lower frequencies

5.3.3 Load Mass / Tension (T)

- **Method:** Add slotted masses (m) to the hanger
- **Calculation:** Tension $T = mg$
- **Theory:** $f \propto \sqrt{T}$
- **Expected Result:** Quadrupling mass doubles frequency (one octave higher)

5.3.4 Magnetic Field Strength (B)

Method A (Electromagnets/Coils):

- Vary the current (I) supplied to the electromagnet/Helmholtz coils
- B is proportional to I for electromagnets

Method B (Permanent Magnets):

- Change the distance (d) between magnet and string
- B decreases approximately as $1/d^3$ for dipole field

Measurement:

- Use Hall Effect Probe (Gaussmeter) placed exactly where the string sits
- Measure actual B-field in Tesla or Gauss
- Map field strength at different positions

5.4 Data Collection Methods and Best Practices

5.4.1 Resonance Sweeping

Don't just measure "a" frequency. Use this method:

1. Drive the system with a variable frequency source
2. Slowly sweep the driving frequency from low to high
3. Monitor amplitude on oscilloscope or spectrum analyzer
4. Look for **amplitude peak** indicating resonance
5. The frequency at maximum amplitude is the resonant frequency

This method is more accurate than free vibration analysis.

5.4.2 Damping Measurement (Ring-down Method)

1. Drive the string to resonance at constant amplitude
2. Cut the power to the driver instantly
3. Record the "decay envelope" on a storage oscilloscope
4. The time it takes for amplitude to drop to $1/e$ (approximately 37%) is the decay time constant (τ)
5. Calculate damping coefficient: $\gamma = 1/\tau$

This directly quantifies magnetic damping strength.

5.4.3 Hysteresis Check

Ferromagnetic materials may show hysteresis:

1. Measure frequency vs. magnetic field by increasing the field from zero to maximum
2. Then measure again by decreasing the field from maximum to zero
3. Plot both curves
4. If the "up" path differs from the "down" path, hysteresis is present

5.5 Error Analysis and Uncertainty

5.5.1 Random Errors

- Repeat every measurement 3–5 times
- Calculate mean and standard deviation
- The spread of values gives statistical uncertainty
- Use error bars on graphs

5.5.2 Systematic Errors

End Corrections:

- The string doesn't vibrate exactly from the sharp edge of the bridge
- String has some stiffness and finite thickness
- This makes the "effective length" slightly different from measured length
- Can be corrected theoretically or empirically

Mass Calibration:

- Ensure lab weights are accurate
- Check calibration of electronic balances
- Account for pulley friction

Magnetic Field Non-uniformity:

- Map the magnetic field carefully
- Ensure the vibrating portion of the string is in a uniform field region
- Or account for field gradients in analysis

5.5.3 Calculating Uncertainty

If $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$, the fractional uncertainty is:

$$\frac{\delta f}{f} = \sqrt{\left(\frac{\delta L}{L}\right)^2 + \left(\frac{1}{2}\frac{\delta T}{T}\right)^2 + \left(\frac{1}{2}\frac{\delta \mu}{\mu}\right)^2} \tag{55}$$

Compare experimental error bars to this theoretical uncertainty to validate the model.

5.6 Analyzing Frequency-Amplitude Relationships

6 Experimental Results

This section presents the data collected using the setup described above. The results are divided into non-magnetic baseline validations and measurements under magnetic influence.

6.1 Non-Magnetic System Validation

To ensure the accuracy of the experimental apparatus, baseline measurements were taken without the presence of strong external magnetic fields. The goal was to verify Mersenne's laws regarding mass (tension) and string length.

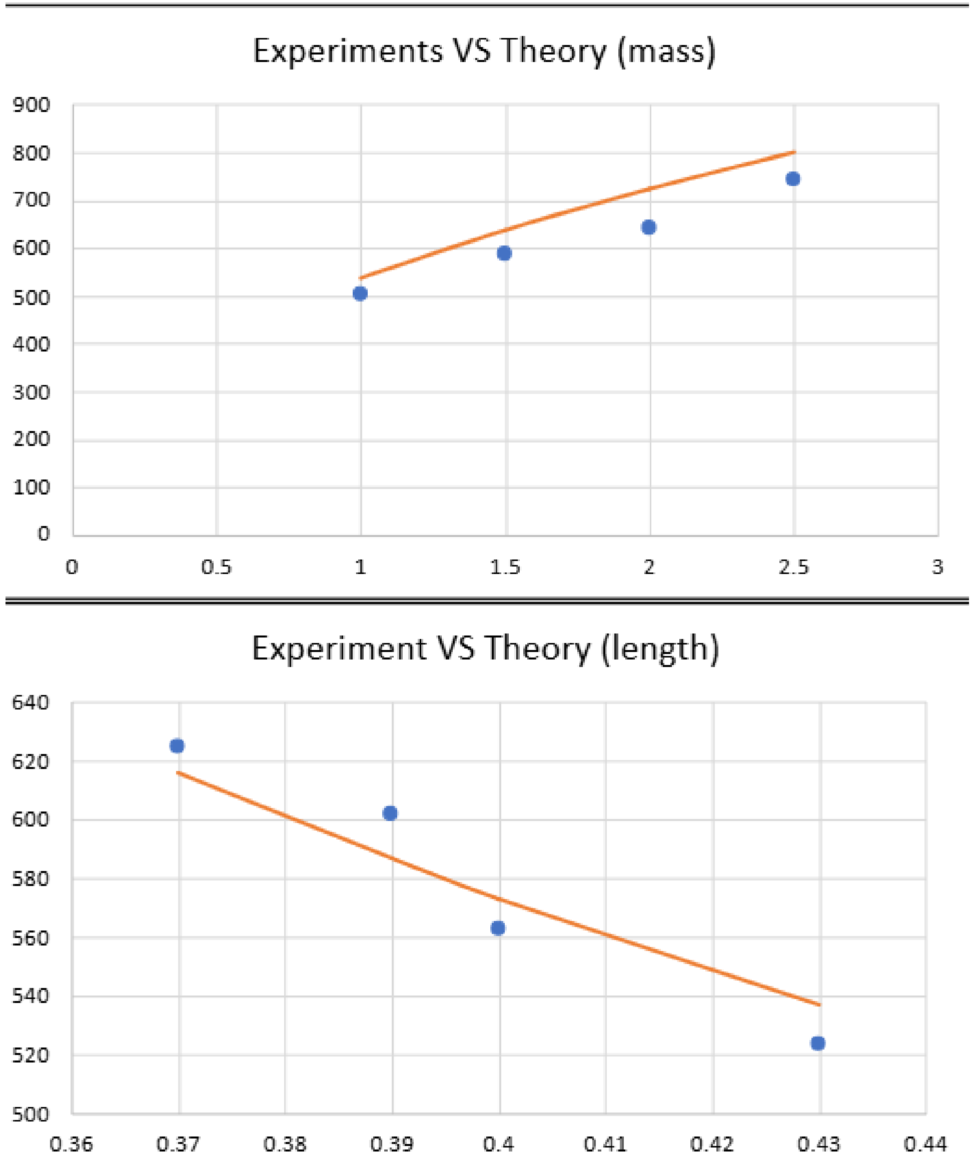
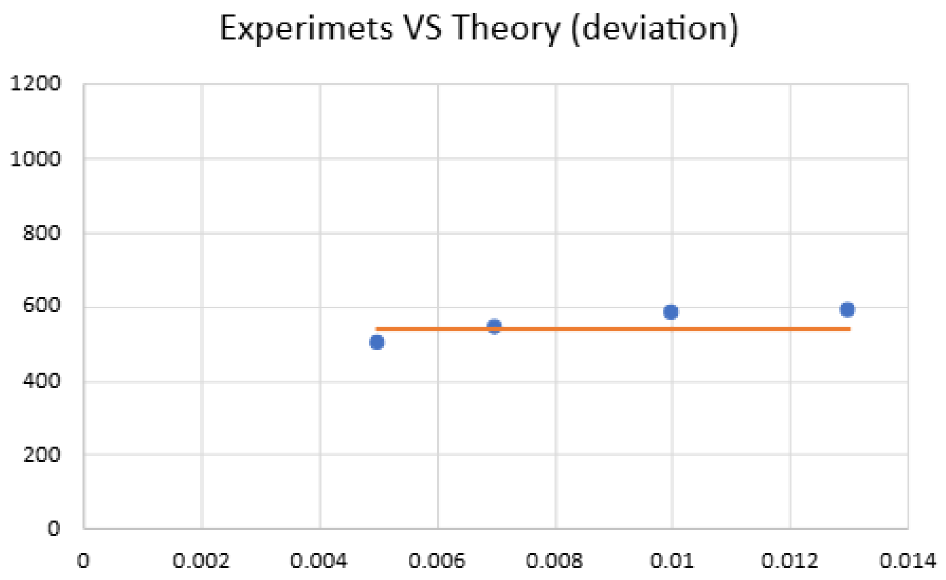


Figure 3: Baseline experimental results for the non-magnetic system compared to theoretical predictions. **Top:** Relationship between mass (tension) and frequency. **Bottom:** Inverse relationship between string length and frequency. The orange lines represent the theoretical model derived from Eq. 6, while blue dots represent experimental data.

As shown in Figure 3 (Top), the frequency increases with the square root of the mass (tension), following the theoretical prediction closely. The slight deviation at higher masses may be attributed to wire stretching or non-ideal boundary conditions at the bridge. Figure 3 (Bottom) confirms the inverse relationship between string length and frequency ($f \propto 1/L$).

6.2 Magnetic System Dynamics

Following baseline validation, the magnetic field was introduced to the system to observe damping and frequency shifts.



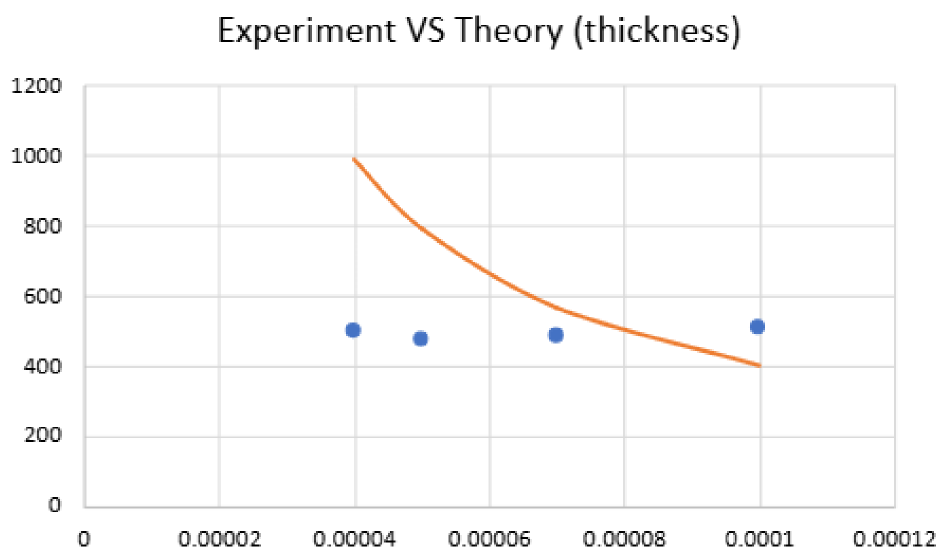


Figure 4: Experimental results under magnetic influence. **Top:** Deviation of experimental frequency from theoretical predictions as a function of magnetic interaction. **Bottom:** The effect of string thickness on frequency in the presence of a magnetic field. The orange line represents the theoretical decay model, while blue dots indicate measured values.

Figure 4 illustrates the complex interaction between the magnetic field and the string properties. The deviation plot (Top) suggests a constant offset or threshold behavior in the presence of the field. The thickness plot (Bottom) is particularly significant, showing that thicker strings (higher linear mass density) generally exhibit lower frequencies, but the curve fit suggests that the magnetic damping or stiffness effects (depending on whether the string is ferromagnetic) modify the standard $1/\sqrt{\mu}$ relationship. The rapid decay in frequency with increasing thickness in the lower plot aligns with the theoretical understanding that thicker conductors may experience stronger eddy current damping effects relative to their stored energy.

7 Analysis of Parameter Effects

This section provides detailed theoretical and practical analysis of how each parameter affects the sound properties of a vibrating string system.

7.1 Fundamental Physics: Mersenne's Law

The fundamental frequency (f_0) of a vibrating string is governed by Mersenne's law equation:

$$f_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \tag{56}$$

where f_0 is the frequency (Hz), L is the speaking length of the string (m), T is the tension (N), and μ is the linear mass density (kg/m).

All parameter effects can be understood through this fundamental relationship.

7.2 String Length (L)

7.2.1 Theoretical Effect

Frequency:

- Frequency is **inversely proportional** to length: $f \propto \frac{1}{L}$
- Doubling the length halves the frequency (drops by one octave)
- Halving the length doubles the frequency (raises by one octave)

Mathematical Relationship:

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \tag{57}$$

Amplitude:

- For a given input energy, longer strings allow larger spatial amplitude (displacement)
- Longer strings are less stiff ($k \propto 1/L$) for the same tension
- However, energy per unit length decreases

7.2.2 Practical Applications

Musical Instruments:

- Fretting a guitar shortens effective vibrating length, raising pitch
- Piano bass strings are much longer than treble strings
- String instruments use stopping (finger placement) to change length

Sound Quality:

- Longer strings at lower tension can produce warmer tone
- Shorter strings at higher tension produce brighter tone

7.3 String Thickness / Linear Mass Density (μ)

7.3.1 Theoretical Effect

Frequency:

- Frequency is **inversely proportional to the square root** of linear mass density
- $f \propto \frac{1}{\sqrt{\mu}}$

Derivation:

- $\mu = \rho \cdot A = \rho \cdot \pi r^2$
- Where ρ is material density and r is radius
- Therefore, $f \propto \frac{1}{r}$
- Doubling the diameter halves the frequency

Amplitude:

- Thicker strings require more energy to drive to same amplitude (higher inertia)
- Once vibrating, they sustain longer (maintain amplitude)
- Store more kinetic energy relative to air resistance losses
- Higher momentum means slower energy dissipation

7.4 Load Mass / Tension (T)

7.4.1 Theoretical Effect

Frequency:

- Frequency is **proportional to the square root** of tension
- $f \propto \sqrt{T}$
- To double frequency (one octave up), need $4\times$ the tension
- To halve frequency (one octave down), reduce tension to $1/4$

Amplitude:

- Higher tension increases restoring force (stiffness)
- For fixed plucking force, higher tension results in **lower amplitude**
- $A \propto 1/T$ for constant input force
- However, wave propagates faster, affecting energy distribution

7.4.2 Practical Applications

Tuning:

- Increasing tension brightens tone and raises pitch
- Tuning pegs work by changing tension
- Must balance desired pitch with structural stress

"Load Mass" Method:

- Hanging weight creates constant tension: $T = mg$
- Experimental advantage: precise, reproducible tension
- Eliminates tuning peg backlash and friction

7.5 Magnetic Field Strength (B)

This is the most complex parameter, with multiple competing effects.

7.5.1 Effect on Amplitude (Damping)

For a conductive string moving through magnetic field:

Eddy Current Damping:

- Motion induces eddy currents in the string
- Currents create Lorentz force opposing motion (Lenz's Law)
- Damping force: $F_d \propto B^2 \cdot v$

Result:

- Stronger magnetic fields cause faster amplitude decay
- Note sustains for shorter time
- Exponential decay: $A(t) = A_0 e^{-\gamma t}$ where $\gamma \propto B^2$

Energy Dissipation:

- Kinetic energy converted to heat in the string
- Power dissipated: $P \propto B^2 v^2$
- Stronger field means faster energy loss

7.5.2 Effect on Frequency (Complex)

The magnetic field can either increase or decrease frequency depending on configuration.

Case A: Frequency Decrease ("Negative Spring" Effect) **Mechanism:**

- Ferromagnetic string vibrating above permanent magnet
- As string moves closer to magnet, attractive force gets stronger
- This force pulls string further from its equilibrium position
- Effectively opposes the mechanical tension's restoring force

Physics:

- Magnetic force gradient creates position-dependent force
- Acts in opposite direction to tension restoring force
- Reduces effective stiffness of the system

Formula:

$$k_{eff} = k_{tension} - k_{magnetic} \tag{58}$$

Result:

- System becomes "softer" (less stiff)
- Lower stiffness → **Lower Frequency**
- $f_{new} = \frac{1}{2\pi}\sqrt{\frac{k_{eff}}{m}} < f_0$

Case B: Frequency Increase (Tension and "Positive Spring" Effects) Mechanism 1: Static Tension Increase (Geometric)

If magnetic field is very strong:

1. Pulls string into permanently deflected "bow" shape
2. Deformation increases path length of string
3. If string ends are fixed, this stretches the string
4. Significantly increases static tension: $T_0 \to T_{new}$

Result:

- If ΔT dominates "negative spring" effect
- Frequency **increases**: $f \propto \sqrt{T}$
- Net stiffening effect

8 Pendulum Dynamics with Magnetic Damping

Understanding pendulum behavior in magnetic fields is crucial for the experimental setup.

8.1 Equation of Motion

8.1.1 The Undamped System

For a simple pendulum of length L and mass m, the restoring force is gravity. For small angles (θ), the equation of motion is:

$$\frac{d^2\theta}{dt^2} + \omega_0^2\theta = 0 \tag{59}$$

where the natural angular frequency is:

$$\omega_0 = \sqrt{\frac{g}{L}} \tag{60}$$

The period of oscillation is:

$$T = 2\pi\sqrt{\frac{L}{g}} \approx 2\pi\sqrt{\frac{L}{9.8}} \tag{61}$$

This shows that period depends only on length and gravity (for small angles).

8.1.2 The Magnetically Damped System

When a conductive bob swings through a magnetic field, it experiences a retarding force proportional to its velocity (v).

Since linear velocity is related to angular velocity by $v = L(d\theta/dt)$, the damping force is:

$$F_d = -cv = -cL\frac{d\theta}{dt} \tag{62}$$

where c is a damping constant.

Applying Newton's Second Law for rotation ($\tau = I\alpha$):

$$-mgL\sin\theta - cL\left(L\frac{d\theta}{dt}\right) = mL^2\frac{d^2\theta}{dt^2} \tag{63}$$

Dividing by mL^2 and using small-angle approximation ($\sin\theta \approx \theta$):

$$\frac{d^2\theta}{dt^2} + 2\zeta\omega_0\frac{d\theta}{dt} + \omega_0^2\theta = 0 \tag{64}$$

This is the **Damped Harmonic Oscillator Equation**, where ζ (zeta) is the damping ratio, the term $2\zeta\omega_0$ is often written as γ (damping coefficient), and $\gamma = \frac{c}{m}$.

8.2 Interaction of Conductive Bob with Magnetic Fields

The interaction is governed by **Faraday's Law of Induction** and **Lenz's Law**.

8.2.1 Physical Process

1. Relative Motion:

- Conductive pendulum bob (copper, aluminum, conducting material) swings
- Moves through stationary magnetic field lines
- Creates relative motion between conductor and field

2. Electromotive Force (EMF):

- Relative motion causes magnetic flux through regions of bob to change
- Induces EMF driven by Lorentz force: $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$
- Free electrons in metal experience this force

3. Eddy Currents:

- Since material is conductive, EMF drives circulating current loops
- These are called **Eddy Currents**
- Flow within volume of the bob
- Pattern depends on bob geometry and field configuration

4. Opposing Force:

- According to Lenz's Law, induced currents create their own magnetic fields
- These fields oppose the change in flux
- Results in drag force always opposite to velocity direction
- Braking effect on pendulum motion

8.3 Eddy Current Damping Mechanism

Eddy current damping is a form of non-contact electromagnetic braking.

8.3.1 Energy Dissipation

Power Loss:

- Bob has finite electrical resistance (R)
- As eddy currents (I_{eddy}) flow, energy dissipated as heat
- Power dissipation rate: $P = I_{eddy}^2 R$ (Joule heating)

Energy Source:

- Energy comes directly from kinetic energy of pendulum
- Causes amplitude of oscillation to decrease with every swing
- Transforms mechanical energy to thermal energy

Velocity Dependence:

- Magnitude of induced current proportional to bob speed: $I_{eddy} \propto v$
- Therefore, damping force also proportional to velocity: $F_{damping} \propto v$
- Characteristic of "viscous" damping

8.3.2 Damping Coefficient

The damping coefficient γ roughly scales as:

$$\gamma \propto \frac{\sigma B^2 V}{m} \tag{65}$$

where σ is the electrical conductivity of bob material, B is the magnetic field strength, V is the volume of bob (larger volume $\rightarrow$ more eddy current paths), and m is the mass of bob.

8.4 Experimental Observations

8.4.1 Exponential Decay

If you plot amplitude (A) vs. time (t), it follows an envelope:

$$A(t) = A_0 e^{-\gamma t/2} \tag{66}$$

Taking the logarithm:

$$\ln(A) = \ln(A_0) - \frac{\gamma t}{2} \tag{67}$$

This gives a straight line on semi-log plot, with slope $-\gamma/2$.

The **logarithmic decrement** δ is constant:

$$\delta = \ln\left(\frac{A_n}{A_{n+1}}\right) = \frac{\gamma T_d}{2} \tag{68}$$

where T_d is the damped period.

8.4.2 Waltenhofen's Effect

If magnetic field is extremely strong or bob is highly conductive, the system becomes **overdamped**.

Criteria:

- Overdamped when $\zeta > 1$, or $\gamma > \omega_0$
- Critically damped when $\zeta = 1$, or $\gamma = \omega_0$

Behavior:

- Pendulum does not oscillate
- Slowly returns to equilibrium without crossing it
- "Oozes" back to vertical position
- Exponential approach to equilibrium

Mathematical form (overdamped):

$$\theta(t) = (A + Bt)e^{-\gamma t} \tag{69}$$

No sinusoidal component—pure exponential decay.

9 Conclusion

This comprehensive research document covers the fundamental physics, theoretical models, experimental methods, and applications of magnetic music and sound generation. The interaction between magnetic fields and vibrating systems involves complex phenomena including:

1. **Electromagnetic induction** — fundamental mechanism for sound transduction
2. **Eddy current damping** — primary magnetic effect on vibrating conductors
3. **Frequency modulation** — magnetic fields can increase or decrease oscillation frequency
4. **Amplitude decay** — magnetic damping causes exponential energy dissipation
5. **Parametric dependencies** — system behavior depends on string properties, magnetic field strength, and geometry

The experimental investigation of these phenomena requires careful control of variables, precise measurement techniques, and thorough understanding of the underlying physics. Applications range from musical instruments (electric guitar pickups, electromagnetic sustainers) to industrial uses (magnetic braking, ultrasonic transducers) to experimental sound art.

The theoretical models presented provide a framework for predicting and understanding the behavior of magnetically-influenced vibrating systems, while the experimental methods enable quantitative verification of these predictions. This interdisciplinary field combines acoustics, electromagnetism, materials science, and musical aesthetics to create new possibilities for sound generation and manipulation.

References

[1] Mersenne, M. (1636). *Harmonie Universelle*. Paris.

[2] Rayleigh, Lord. (1877). *The Theory of Sound*. London: Macmillan.

[3] Faraday, M. (1832). "Experimental Researches in Electricity." *Philosophical Transactions of the Royal Society of London*, 122, 125–162.

[4] Jackson, J. D. (1999). *Classical Electrodynamics* (3rd ed.). New York: Wiley.

[5] Griffiths, D. J. (2017). *Introduction to Electrodynamics* (4th ed.). Cambridge: Cambridge University Press.

[6] Fletcher, N. H., & Rossing, T. D. (1998). *The Physics of Musical Instruments* (2nd ed.). New York: Springer-Verlag.

[7] Benade, A. H. (1976). *Fundamentals of Musical Acoustics*. Oxford: Oxford University Press.

[8] Hall, D. E. (2001). *Musical Acoustics* (3rd ed.). Pacific Grove, CA: Brooks/Cole.

[9] Kinsler, L. E., Frey, A. R., Coppens, A. B., & Sanders, J. V. (2000). *Fundamentals of Acoustics* (4th ed.). New York: Wiley.

[10] Rossing, T. D. (1990). *The Science of Sound* (2nd ed.). Reading, MA: Addison-Wesley.

[11] Berg, R. E., & Stork, D. G. (2004). *The Physics of Sound* (3rd ed.). San Francisco: Pearson.

[12] Begun, S. J. (1949). *Magnetic Recording*. New York: Murray Hill Books.

[13] Daniel, E. D., Mee, C. D., & Clark, M. H. (1998). *Magnetic Recording: The First 100 Years*. New York: IEEE Press.

[14] Roads, C. (1996). *The Computer Music Tutorial*. Cambridge, MA: MIT Press.

[15] Collins, N. (2009). *Handmade Electronic Music: The Art of Hardware Hacking* (2nd ed.). New York: Routledge.