## 各类梁反力、剪力、弯矩和挠度计算公式一览表

| -                 | 简支梁的反力、剪力、弯矩、挠度    |  |        |   |   |  |  |  |  |  |
|-------------------|--------------------|--|--------|---|---|--|--|--|--|--|
| 序次                | 图类                 | 图示   | 项目     | H                                       | h jöl   | 第  | 太阳 法和  |  |  |  |
|                   | 荷载                 | $A \downarrow^F B$   | 反力     | $R_A = R_B = \frac{F}{2}$               | 区区  | apin on  | - 建杏   |  |  |  |
| 1                 |                    | 1/2 1/2  | 剪力     | $V_{A} = R_{A};  V_{B} = -$             | $R_{\mathrm{B}}$  |  | ×  |  |  |  |
|                   | 弯矩                 |  | 弯矩     | $M_{\text{max}} = \frac{1}{4} Fl$       |   | - Annie  | 1 2 2  |  |  |  |
|                   | 剪力                 | DUTUDIAL THE TANK  | 挠度     | $W_{\text{max}} = \frac{F1^3}{48EI}$    | e Air   | Dia .  |  |  |  |  |
|                   |                    | 19 = 11.   | 反力     | $R_A = \frac{b}{l} F$ ; $R$             | R = −   |  | 202  |  |  |  |
|                   | 荷载                 | $ \begin{array}{c c} A & B \\ A & A \\ A & A \end{array} $ | 剪力     | $V_A = R_A$ ; $V_B = -$                 | $R_{\rm B}$   |  |  |  |  |  |
| 2                 |                    | I Para a lazy a  | 弯矩     | $M_{\text{max}} = \frac{Fab}{l}$        |   |  | 服器   |  |  |  |
|                   | 弯矩                 |  | 挠度     | $	au > b$ 时,在 $x = \sqrt{\frac{1}{2}}$  | $\sqrt{\frac{a}{3}}$  | (9+2b) 处,  | 大 美  |  |  |  |
|                   | 剪力                 |  |        | $W_{\text{max}} = \frac{Fb}{9EII}$      | / <u>(a</u>   | 3  |  |  |  |  |
|                   | 荷载                 | $A \downarrow^F \qquad F \downarrow B$                     | 反力     | $R_A = R_B = F$                         | de a  | 4 4 4  | 本格   |  |  |  |
| 3                 |                    |  | 剪力     | $V_A = R_{A}$ ; $V_B = -$               | $R_{\mathrm{B}}$  | NATION OF THE PARTY OF THE PART | - 8  |  |  |  |
|                   | 弯矩                 |  | 弯矩     | $M_{\text{max}} = Fa$                   |   |  | 1.長寒   |  |  |  |
|                   | 剪力                 |  | 挠度     | $W_{\text{max}} = \frac{Fa}{24El}$      | (312 -  | - 4a²)   | 4.79   |  |  |  |
|                   | 荷载                 | A  | 反力     | $R_{A} = R_{B} = \frac{3}{2} F$         | E I   | e print  |  |  |  |  |
|                   |                    | 1/4,1/4,1/4,1/4  | 剪力     | $V_A = R_{A}$ ; $V_B = -$               | $R_{\mathrm{B}}$  | 1-1-1-1  |  |  |  |  |
| 4                 | 弯矩                 |  | 弯矩     | $M_{\max} = \frac{1}{2} Fl$             |   |  |  |  |  |  |
|                   | 剪力                 |  | <br>挠度 | $W_{\text{max}} = \frac{19Fl^3}{384EI}$ |   | ingpiakilli  | da   |  |  |  |
|                   |                    |  | 反力     | $R_{A} = R_{B} = \frac{1}{2} qt$        |   | A mind A   | 30.7%  |  |  |  |
|                   | 荷载                 |  | 剪力     | $V_A = R_A$ ; $V_B = -1$                | R   |  |  |  |  |  |
| 5                 | 弯矩                 |  | 弯矩     | $M_{\text{max}} = \frac{1}{2} q l^2$    | В   |  | 张孝 191   |  |  |  |
|                   | 剪力                 |  | - 挠度   | $W_{\text{max}} = \frac{5ql^4}{384EI}$  | igi i   | TOUR STANKE  | the court of the control of the control of the court of t |  |  |  |
| distribution or a | 1-CHIRO-MARINOPHAN |  | DG DZ  | 384 <i>EI</i>                           | distribution of the second of | and the state of the  |  |  |  |  |

| 字次 | 图类      | 图示             | 项目       | 计严 第 式 2 3 2 3 2 5   |
|----|---------|----------------|----------|--|
|    | 荷载      | 4 4 B          | 反力       | $R_A = R_B = qa$   |
|    |         | 1              | 剪力       | $V_A = R_A$ ; $V_B = -R_B$   |
| 6  | 弯矩      |                | 弯矩       | $M_{\text{max}} = \frac{1}{2} q a^2$   |
|    | 剪力      |                | 挠度       | $W_{\text{max}} = \frac{q a^2}{48EI} (3\dot{I}^2 - 2a^2)$  |
| .  | 荷载      | AATTI B        | 反力       | $R_{A} = \frac{qa}{2} \left( 2 - \frac{a}{l} \right) ;  R_{B} = \frac{qa^{2}}{2l}$   |
|    | 1-3 42  |                | 剪力       | $V_A = R_A$ ; $V_B = -R_B$   |
| 7  | مين جيد |                | who have | $\stackrel{\underline{\omega}}{=} x = a - \frac{a^2}{2L} \mathbb{H}^4,$  |
|    | 弯矩      |                | 弯矩       | $M_{\text{max}} = \frac{q a^2}{8l} \left( 4b + \frac{a^2}{l} \right) = \frac{q a^2}{8} \left( 2 - \frac{a}{l} \right)^2$                                 |
| 1  | 剪力      | , x t          | 挠度       | $W_{\text{max}} = \frac{q a^2 l^2}{24 E I} \left[ \left( 2 - \frac{a^2}{l^2} - \frac{2 x^2}{l^2} \right) \frac{x}{l} + \frac{(x-b)^4}{a^2 l^2} \right].$ |
|    | 荷载      |                | 反力       | $R_{\rm A} = \frac{qb^2}{2l}$ ; $R_{\rm B} = \frac{qb}{2} \left(2 - \frac{b}{l}\right)$  |
|    | 17, 454 | +              | 剪力       | $V_A = R_A$ ; $V_B = -R_B$   |
| 8  |         |                | गोड अन   | $\stackrel{\text{dif}}{=} x = a + \frac{b^2}{2l} \text{ [t]};$   |
|    | 弯矩      |                | 弯矩       | $M_{\text{max}} = \frac{qb^2}{8l} \left( 4a + \frac{b^2}{l} \right) = \frac{qb^2}{8} \left( 2 - \frac{b}{l} \right)^2$                                   |
|    | 剪力      | † ×            | 挠度       | $W_{\text{max}} = \frac{qb^2l^2}{24EI} \left[ \left( 2 - \frac{b^2}{l^2} - \frac{2x^2}{l^2} \right) \frac{x}{l} + \frac{(x-a)^4}{b^2l^2} \right]$        |
|    | dt 4t-  | A              | 反力       | $R_{\rm A} = R_{\rm B} = \frac{qb}{2}$   |
| 0  | 荷载      |                | 剪力       | $V_A = R_A$ ; $V_B = -R_B$   |
| 9  | 弯矩      |                | 弯矩       | $M_{\text{nax}} = \frac{qbl}{8} \left( 2 - \frac{b}{l} \right)$  |
|    | 剪力      |                | 挠度       | $W_{\text{max}} = \frac{qbl^3}{384EI} \left(8 - \frac{4b^2}{I^2} + \frac{b^3}{I^3}\right)$   |
|    | 荷载      | A B B          | 反力       | $R_{A} = \frac{qa_{2}b}{l},  R_{B} = \frac{qa_{1}b}{l}$  |
|    | 17月 年又  | a <sub>1</sub> | 剪力       | $V_A = R_A$ ; $V_B = -R_B$   |
| 10 | 弯矩      |                | 弯矩       | $M_{\text{max}} = \frac{qba_2}{l} \left( a + \frac{ba_2}{2l} \right)$  |
|    |         |                |          | $W_{\text{max}} = \frac{aba_2}{24EI} \left[ \left( 4l - 4\frac{a_2^2}{l} - \frac{b^2}{l} \right) x - 4\frac{x^3}{l} + \frac{(x-a)^4}{6a^3} \right]$      |
|    | 剪力      |                | 挠度       | 式中: $x = a + \frac{ba_2}{l}$   |

| 序次 | 图类    | 图示      | 项目 | 日 第 计 一 4 算 出 式 、 类 图 太  |
|----|-------|---------|----|--|
|    | 荷载    | 1 9 9 B | 反力 | $R_{A} = R_{B} = qb$   |
| 11 | 11.47 |         | 剪力 | $V_A = R_A$ ; $V_B = -R_B$   |
|    | 弯矩    |         | 弯矩 | $M_{\text{max}} = qba_1$   |
| j. | 剪力    |         | 挠度 | $W_{\text{max}} = \frac{qba_1}{2EI} \left( \frac{l^2}{4} - \frac{a^2}{3} - \frac{b^2}{12} \right)$ |

## 2.悬臂梁

| Out ON The Command out of |     |            | 悬臂梁的反力、剪:                             | 力、弯矩和挠 | 度 附表 15-4-16   |
|---------------------------|-----|------------|---------------------------------------|--------|--|
| 序次                        | 图   | 类          | 图示                                    | 项目     | 计算式  |
|                           | 荷   | 载          | A EB                                  | 反 力    | $R_{\rm B} = F$  |
|                           |     |            | 1_1_                                  | 剪力     | $\dot{V}_{\rm B} = -R_{\rm B}$   |
| 1                         | 弯   | 矩          |                                       | 弯 矩    | $M_{x} = -F_{x}$ , $M_{max} = M_{B} = -F1$                                       |
|                           | 剪   | 力          |                                       | 挠度     | $W_{\text{ora} \times} = W_{\text{A}} = \frac{FI^3}{3EI}$                        |
|                           | 荷   | A4.        | A F                                   | 反力     | $R_{\rm B} = F$  |
| 2                         | 1만  | 载          | A   B   B   B   B   B   B   B   B   B | 剪力     | $V_{\rm B} = -R_{\rm B}$   |
| 2                         | 弯   | 矩          |                                       | 弯矩     | $M_{x} = -F(x-a)$ ; $M_{max} = M_{B} = -Fb$                                      |
|                           | 剪   | カ          |                                       | 挠度     | $W_{\text{max}} = W_{\text{A}} = \frac{Fb^2l}{6EI} \left(3 - \frac{b}{l}\right)$ |
|                           | 荷   | 载          | nF B                                  | 反 力    | $R_B = nF$   |
|                           | 143 | <b>華</b> X | l=na                                  | 剪力     | $V_{B} = -R_{3}$   |
| 3                         | 弯   | 矩          |                                       | 弯. 矩   | $M_{\text{max}} = M_{\text{B}} = -\frac{n+1}{2} FI$                              |
|                           | 剪   | カ          |                                       | 挠 度    | $W_{\text{max}} = W_{\text{A}} = \frac{3n^2 + 4n + 1}{24nEI} F I^3$              |
|                           | 荷   | 载          | A $B$                                 | 反 力    | $R_{\rm B} = qI$   |
| ,5 La                     |     |            | +/                                    | 剪力     | $V_{\rm B} = -R_{\rm B}$   |
| 4                         | 弯   | 矩          |                                       | 弯矩     | $M_{\text{max}} = M_{\text{B}} = -\frac{cl^2}{2}$                                |
|                           | 剪   | カ          |                                       | 挠  度   | $W_{\text{max}} = W_{\text{A}} = \frac{qI^4}{8EI}$                               |

|     |           | 6-4-4-1-1-1-1-1-1-1-1-1-1-1-1-1-1-1-1-1-      | }   | NAME AND ADDRESS OF | MINISTER OF THE PERSON NAMED IN | Product assessed Biologic Cambridge Acut Recipies Auditorians de Baserium, de addresses ameny not aux difficilités de l'encour factor, de l'encour de l'encour   |
|-----|-----------|---|-----|---------------------|---------------------------------|--|
| 序次  | 图类        | 图》示   |     | 项                   | 目                               | 日本 算 式   |
|     | 荷 载       |   |     |                     | カ<br>—                          | $R_{\rm B} = qa$   |
|     |           |   | _ - | 剪                   | カー                              | $V_{\rm B} = -R_{\rm B}$   |
| - 5 | 弯 矩       |   |     | 弯                   | 矩                               | $M_{\text{max}} = M_{\text{B}} = -\frac{gat}{2}(2-a)$  |
|     | 剪力        |   |     | 挠                   | 度                               | $W_{\text{max}} = W_{\text{A}} = \frac{-qI^4}{24EI} \left(3 - 4\frac{b^3}{I^3} + \frac{b^4}{I^4}\right)$   |
|     | alle alle | A TUIL B                                      |     | 反                   | 力                               | $R_{\rm B} = qb$   |
|     | 荷 载       |   |     | 剪                   | 力                               | $V_{\rm B} = -R_{\rm B}$   |
| 6   | 弯 矩       | <b>東國</b>                                     | 175 | 弯                   | 矩                               | $M_{\rm B} = -\frac{qb^2}{2}$  |
| -   | 剪力        | -   |     | 挠                   | 度                               | $W_{A} = \frac{qb^{2}l}{24EI} \left( 4 - \frac{b}{l} \right)$  |
| -41 |           | A   |     | 反                   | カ                               | $R_{\rm B} = qc$   |
|     | 荷载        | a+c+b   |     | 剪                   | 力                               | $V_{\rm B} = -R_{\rm B}$   |
| 7   | 弯 矩       |   |     | 弯                   | 矩                               | $M_{\rm B} = -qcb$   |
|     | 剪 力       |   |     | 挠                   | 度                               | $W_{A} = \frac{qc}{24EI} (12b^{2}I - 4b^{3} + ac^{2})$   |
|     | 3.一端      | 简支另一端固定梁                                      |     | 1 0                 |                                 | The second secon |
|     |           | 一端简支另一端固                                      | 定等  | 2的)                 | 反力                              | 1、剪力、弯矩和挠度 附表 15-4-17  |
| 序次  | 图类        | 图示项   | 目   | i i i i             |                                 | 计 算 式  |
|     |           | A F 反   | 力   | R                   | A =                             | $\frac{5}{16}F$ ; $R_{\rm B} = \frac{11}{16}F$   |
|     | 荷载        | A C   | 力   | ŭ.                  | 1                               | $R_{\rm A}$ ; $V_{\rm B} = -R_{\rm B}$   |
| 1   | 弯矩        | 弯   | 矩   | M                   | T o =                           | $=\frac{5}{32}Fl_3M_{\rm B}=-\frac{3}{16}Fl$   |
|     | 剪力        | turnul in in its                              | 度   | 当                   | x =                             | $0.4471$ 时。 $W_{\text{max}} = 0.00932 \frac{FI^3}{EI}$   |
|     | 荷载        | $A \longrightarrow B$ $\overline{\mathbb{R}}$ | カ   | R                   | A =                             | $\frac{Fb^{2}}{2l^{2}}\left(3-\frac{b}{l}\right); R_{B}=\frac{Fa}{2l}\left(3-\frac{a^{2}}{l^{2}}\right)$   |
|     | 1, 3, 454 | 剪   | 力   | V                   | A =                             | $R_{\rm A}; V_{\rm B} = -R_{\rm B}$  |
| . 2 | 弯矩        |   | 短   | 当                   | x =                             | $a \mathbb{H}, M_{\max} = \frac{Fab^2}{2l^2} \left(3 - \frac{b}{l}\right)$   |
|     | 剪力        | TITT BY                                       | 達度  |                     | B段<br>Vx=                       | $= \frac{1}{6EI} \left[ R_{A} (3l^{2}x - x^{3}) - 3Fb^{2}x + F(x - a)^{2} \right]$   |

| rik W      | Alc [32]  | 图  | 18i E  | 计是 第 式 类图 五孔   |
|------------|-----------|--|--------|--|
| 序次         | 图类        | The second secon | 项目     | PI ST PV   |
|            | 荷载        | $A_{A}$ $C$ $D_{A}$ $B$  | 反力     | $R_{A} = \frac{F}{2} \left( 2 - 3 - \frac{a}{l} + 3 - \frac{a^{2}}{l^{2}} \right), R_{A} = \frac{F}{2} \left( 2 + 3 - \frac{a}{l} - 3 - \frac{a^{2}}{l^{2}} \right)$ |
|            |           | 1 1  | 剪力     | $V_A = R_A$ ; $V_B = -R_B$   |
| 3          | 弯矩        |  | 弯矩     | $M_{\text{max}} = M_{\text{C}} = R_{\text{A}} a;  M_{\text{B}} = -\frac{3Fa}{2} \left( 1 - \frac{a}{l} \right)$  |
|            | 剪力        |  | 挠度     | $W_x = \frac{1}{6EI} [R_A(3l^2x - x^3) - 3F(l^2 - 2al + 2a^2)x + F(x - a)^3]$  |
|            | 荷载        | A B  | 反力     | $R_{\rm A} = \frac{3}{8} qt,  R_{\rm B} = \frac{5}{8} qt$  |
| 1 1        | - 3319    | ALCOLOUGH AND  | 剪力     | $V_A = R_A$ ; $V_B = -R_B$   |
| 4          | 弯矩        |  | 弯矩     | 当 $x = \frac{3}{8}l$ 时, $M_{max} = \frac{9ql^4}{128}$  |
|            | 剪力        |  | 挠度     | 当 $x = 0.4221$ 时, $W_{\text{max}} = 0.00542 \frac{qI^4}{EI}$   |
|            | alls also | A C b B  | 反力     | $R_{\rm A} = \frac{qa}{8} (8 - ba + a^3), R_{\rm B} = \frac{qa^2}{8l} (bl - a^2)$  |
|            | 何软        | 荷载 a C b   |        | $V_A = R_A$ ; $V_B = -R_B$   |
|            | 弯矩        |  | 弯矩     |  |
| 120        |           | - 0.3  | = n W  | AC段:   |
| 5          |           |  | 15     | $W_{x} = \frac{1}{24EI} \left[ 4R_{A} (3l^{2}x - x^{3}) - 4qa(3bl + a^{2})x + qx^{4} \right]$  |
|            | 剪力        |  | 挠度     | BC段:   |
|            |           | animum.  | LONGE! | $W_{x} = \frac{1}{24 EI} [4R_{A}(3l^{2}x - x^{3}) - qa(a^{3} + 12blx + bax^{2} - 4x^{3}]$ $\underset{x}{} x = a \underline{b} \underline{f},$                        |
|            | Aş .      | $\left  \begin{array}{c} 1 \\ \end{array} \right  = \frac{1}{4} \int_{\mathbb{R}^{N}} dt  dt  \left  \frac{1}{2} \int_{\mathbb{R}^{N}} dt  dt  \left  \frac{1}{2} \int_{\mathbb{R}^{N}} dt  dt  dt  \right   dt  dt  dt  dt  dt  dt  dt  d$  | 1) =   | $W_{a} = \frac{1}{24 EI} [4aR_{A}(3l^{2} - a^{2}) - 3qa^{2}(4lb + a^{2})]$   |
|            | -jit-sht  | A6 TIMER   | 反力     | $R_{A} = \frac{qb^{3}}{8l^{3}} \left( 4 - \frac{b}{l} \right) ; R_{B} = \frac{qb}{8} \left( 8 - 4 \frac{b^{2}}{l^{2}} + \frac{b^{3}}{l^{3}} \right)$                 |
|            | 荷载        |  | 剪力     | $V_A = R_A$ ; $V_B = -R_B$   |
|            | 弯矩        |  | 弯矩     | $x = a + \frac{R_A}{q}$   $M_{\text{max}} = R_A \left( a + \frac{R_A}{2q} \right)$   |
| - W        | 1010      | 7  |        | AC段:   |
| 6          |           | Tre  | Z.     | $W_{x} = \frac{1}{6EI} [R_{A}(3I^{2}x - x^{3}) - qb^{3}x]$   |
|            | 剪力        |  | 挠度     | BC段:   |
|            |           |  |        | $W_z = \frac{1}{24EI} [4R_A(3l^2x - x^3) - 4qb^3x + q(x - a)^4]$<br>当 $x = a$ 时:   |
|            |           |  | 14     | $W_{z} = \frac{1}{6EI} [aR_{A}(3I^{2} - a^{2}) - qb^{3}]$  |
| M CHANGING |           | Dent i in all 100 material processor 32 materials of 300 materials in order materials of the control of the Allin  | -      | OLD I  |

| 序次  | 图类  | 图示   | 项目          |  | 计 算 式   |  |  |
|-----|-----|--|-------------|--|---|--|--|
| (-  | 荷载  |  | 反力          | $R_{A} = \frac{q b_{1}}{8l^{3}} (12b^{2}l - 4b^{3} + ab_{1}^{2})$ $R_{B} = qb_{1} - R_{A}$ |   |  |  |
| 8   |     | 1-1-1-1  | 剪力          | V A =  | $=R_{A}$ , $V_{B}=-R_{B}$   |  |  |
| 5   | 弯矩  |  | 弯矩          | 当x:  | $= a_1 + \frac{R_A}{q} \mathbb{H}_1  M_{\text{max}} = R_A \left( a_1 + \frac{R_A}{2q} \right)$  |  |  |
| 1.0 | -23 | F 3 C C C - 202 - 17 5 2   |             | AC   | Q: (8)  |  |  |
|     | 剪力  |  | 挠度          | CD   | $= \frac{1}{24EI} [4R_{A}(3l^{2}x - x^{3})  qb_{1}(12b^{2} + b^{2}_{1})x]$ $= \frac{1}{24EI} [4R_{A}(3l^{2}x - x^{3}) - qb_{1}(12b^{2} + b^{2}_{1})x + q(x - a_{1})^{4}]$ |  |  |
| 1   |     | 89   |             |  |   |  |  |
|     | 4.两 | 端固定梁   |             | , Lines,   | Cardo Sa Villiano - Ta Cardo  |  |  |
|     |     | 两单   | <b>尚固定</b>  | 梁的反:   | 力、剪力、弯矩和挠度 附表 15-4-18<br>   |  |  |
| 序边  | 图数  | 图. 示   | 9.4         | 项目   | 计 算 式   |  |  |
|     |     | F  | F           |  | $R_{A} = R_{B} = \frac{1}{2} F$   |  |  |
|     | 荷素  | 1数 . 1/2 - 1 | 1           | 剪力   | $V_A = R_A$ ; $V_B = -R_B$  |  |  |
| 1   | 弯矢  | і шт   |             | 弯矩   | $M_{\text{max}} = \frac{1}{8} FI$   |  |  |
|     | 剪プ  |  | V 1 V 2 V 7 | 挠度   | $W_{\text{max}} = \frac{FI^3}{192EJ}$   |  |  |
| 7.  |     | 1 2 2 2 2 2  |             | 反力   | $R_{A} = \frac{Fb^{2}}{l^{2}} \left(1 + \frac{2a}{l}\right),  R_{B} = \frac{Fa^{2}}{l^{2}} \left(1 + \frac{2b}{l}\right)$   |  |  |
|     | 荷载  |  | +           | 剪力   | $V_A = R_A$ ; $V_B = -R_B$  |  |  |
| 2   | 弯角  |  | Ī           | 弯矩   | $M_{\text{max}} = M_{\text{C}} = \frac{2Fa^2b^2}{l^3}$  |  |  |
|     | 剪フ  |  |             | 挠度   | 若a>b, 当 $x = \frac{2al}{3a+b}$ 时。 $W_{\text{max}} = \frac{2F}{3E!} \times \frac{a^2b^2}{(3a+b)^2}$  |  |  |
| •   |     |  | TL:         | 反力   | $R_{A} \Rightarrow R_{B} = \frac{q!}{2}$  |  |  |
|     | 荷毒  | A  | 1 - 1 - 1 B |  | $V_A = R_A$ ; $V_B = -R_B$  |  |  |
| 3   | 弯角  |  | 1           | 弯矩   | $M_{\text{max}} = \frac{qt^2}{24}$  |  |  |
|     | 剪力  |  |             | 挠度   | $W_{\text{max}} = \frac{qI^4}{384EI}$   |  |  |

| -         |    | 1        |   |          |  |
|-----------|----|----------|---|----------|--|
| 序         | 次  | 图类       | <b>发图</b> 示                             | 项目       | 算 式 2  |
| and a     |    | 荷载       | A MINIMA IN A PARTIE B                  | 反力       | $R_{A} = R_{B} = qa$   |
|           |    | 刊软       |   | 剪力       | $V_{A} = R_{A}; V_{B} = -R_{B}$  |
|           | 4  | 弯矩       |   | 弯矩       | $\underline{\underline{\mathbf{H}}} \qquad M_{\mathrm{max}} = \frac{q a^3}{3l}$  |
|           |    | 剪力       |   | 挠度       | $\mathcal{E} \qquad W_{\text{max}} = \frac{qa^3l}{24EI} \left(1 - \frac{a}{l}\right)$  |
| -         |    |          | A TIME B                                | 反力       | $R_{A} = \frac{qa}{2}(2-2a^{2}+a^{3});  R_{B} = \frac{qa^{3}}{2l^{2}}(2-a)$  |
| 30        |    | 荷载       | a C b                                   | 剪力       | $V_{A} = R_{A}; V_{B} = -R_{B}$  |
|           | 5  | 弯矩       |   | 弯角       | E $M_{A} = -\frac{qa^{2}}{12}(6-8a+3a^{2})$  |
|           |    |          | 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1   |          | ACQ: $W_x = \frac{1}{6EI} \left[ -R_A x^3 - 3M_A x^2 + \frac{qx^4}{4} \right]$   |
|           |    | 剪力       |   | 挠胆       | 度 $BC$ 段: $W_x = \frac{19}{6EI} \left[ -R_A x^3 - 3M_A x^2 + \frac{qa}{4} (a^3 - 4a^2 x) \right]$  |
|           |    |          | 1 1 1 1 1                               |          | $+6ax^2-4x^3$  |
|           |    | 一一一一     | A B                                     | 反        | $h \mid R_{\rm A} = R_{\rm B} = \frac{qb}{2}$  |
| ÷         |    | 173 424  |   | 剪之       | $V_{A} = R_{A}; V_{B} = -R_{B}$  |
|           | 6  | 弯矩       |   | 弯        | $E M_{\text{max}} = \frac{qbl}{24} \left( 3 - 3 \frac{b}{l} + \frac{b^2}{l^2} \right)$   |
|           |    | 剪力       |   | 挠        | $W_{\text{max}} = \frac{qbl^3}{384EI} \left(2 - 2\frac{b^2}{l^2} + \frac{b^3}{l^3}\right)$   |
| e.<br>101 |    | c 拱1     | <b>悬臂的梁(外伸梁)</b>                        | 3 4 1 10 |  |
|           |    | л цг • С | 1 | 验的反力     | (、剪力、弯矩和挠度 附表 15-4-19  |
|           | 序次 | 图类       |   | 页目       | 计 算 式 大脚   |
| -         | 次  | 国民       | 1888.0 = 0.5.48                         | (CEL ES) | 70+0-12 1/2  |
|           |    | 荷载       | $C \triangle_A $ $\triangle_B $         |          | $R_{\rm A} = \left(1 + \frac{a}{l}\right) F$ , $R_{\rm B} = -\frac{a}{l} F$  |
|           |    |          |   | 剪力       | $V_{\rm C} = -F$ ; $V_{\rm B} = -R_{\rm B} = \frac{a}{l}F$   |
|           | 1  | 弯矩       | 2                                       | 弯矩 1     | $M_{\text{max}} = M_{\text{A}} = -Fa$  |
| - 12      |    | 剪力       |   | 这座       | $W_{C} = \frac{Fa^{2}l}{3EI} \left(1 + \frac{a}{l}\right)$ $\frac{dt}{dt} x = a + 0.5l \text{ ft},  W_{\text{min}} = -0.0642 \frac{Fal^{2}}{EI}$ |

| 序次 | 图类    | <b>美图</b> 示   | 项目     | 算   |
|----|-------|---|--------|---|
| C  |       | A THINTING B  | 反力     | $R_{A} = R_{B} = qa$  |
|    | 荷载    |   | 剪力     | $V_A = R_A$ ; $V_B = -R_B$  |
| 4  | 弯矩    |   | 弯矩     | $M_{\text{max}} = \frac{q a^3}{3l}$   |
|    | 剪力    |   | 挠度     | $W_{\text{max}} = \frac{q  a^3  l}{24  E  l}  \left( 1 - \frac{a}{l} \right)$   |
|    | 荷载    |   | 反力     | $R_{\rm A} = \frac{qa}{2}(2-2a^2+a^3);  R_{\rm B} = \frac{qa^3}{2l^2}(2-a)$   |
|    |       | 1   | 剪力     | $V_A = R_A$ ; $V_B = -R_B$  |
| 5  | 弯矩    |   | 弯矩     | $M_{A} = -\frac{q a^{2}}{12} (6 - 8a + 3a^{2})$ $\stackrel{\text{de}}{=} x = \frac{R_{A}}{q} \text{ H};  M_{\text{max}} = \frac{R^{2}_{A}}{2q} + M_{A}$ |
|    |       | $\frac{1}{2} \left( \frac{-2\pi}{4} \left( 1 + \frac{1}{2} \left( \Delta_{1} \right) \right) \right)$ |        | AC段: $W_x = \frac{1}{6EI} \left[ -R_A x^3 - 3M_A x^2 + \frac{qx^4}{4} \right]$  |
|    | 剪力    |   | 挠度     | $BC$ 段: $W_x = \frac{1}{6EI} \left[ -R_A x^3 - 3M_A x^2 + \frac{qa}{4} (a^3 - 4a^2) \right]$  |
|    |       |   | -1     | $+6ax^2-4x^3$   |
|    | 荷载    | A $B$ $B$   | 反力     | $R_{\rm A} = R_{\rm B} = \frac{qb}{2}$  |
|    |       | 4 4   | 剪力     | $V_A = R_A$ ; $V_B = -R_B$  |
| 6  | 弯矩    |   | 弯矩     | $M_{\text{max}} = \frac{qbl}{24} \left(3 - 3 \frac{b}{l} + \frac{b^2}{l^2}\right)$  |
|    | 剪力    |   | 挠度     | $W_{\text{max}} = \frac{qbl^{s}}{384EI} \left(2 - 2\frac{b^{2}}{l^{2}} + \frac{b^{3}}{l^{3}}\right)$  |
|    | - HI  | 目 日菜 44、30、 / 41、41、30、 )   | 100    |   |
|    | 5. 電流 | 悬臂的梁(外伸梁)   | 4 491  |   |
|    |       | 外伸梁田  | ]反刀、   | 剪力、弯矩和挠度 附表 15-4-19   |
| 序次 | 图类    | 图示示项目   | d 83 h | 计 第 式   |
|    | 荷载    | 反力 AB 反力  | R      | $r_{A} = \left(1 + \frac{a}{l}\right) F,  R_{B} = -\frac{a}{l} F$   |
|    | 刊架    | は は 対 対   |        | $C = -F$ ; $V_B = -R_B = \frac{a}{l}F$  |
| 1  | 弯矩    | 弯线  | E M    | $_{\text{max}} = M_{\text{A}} = -Fa$  |
|    | 剪力    | 提到<br>  | F      | $X_{\rm C} = \frac{F a^2 l}{3EI} \left(1 + \frac{a}{l}\right)$ $x = a + 0.5l$ H, $W_{\rm min} = -0.0642 \frac{F a l^2}{EI}$                             |
|    |       |   |        | ET  |

|  |         |  | 72       |  |
|--|---------|--|----------|--|
| 序次   | 图类      | 图示   | 项目       | 计 茶算 战式 张扬 200   |
|  | 荷载      | $F$ $C \stackrel{\Delta}{\longrightarrow} A \stackrel{B \stackrel{\Delta}{\longrightarrow}} D$ | 反力       | $R_A = R_B = F$  |
| 2.5  | 11.4%   | $C \stackrel{\triangle}{\longrightarrow} A \qquad B \stackrel{\triangle}{\longrightarrow} D$   | 剪力       | $V_A = -R_A$ ; $V_B = R_B$   |
| 2  | 弯矩      |  | 弯矩       | $M_A = M_B = -Fa$  |
|  |         |  | 18       | $W_{C} = W_{D} = \frac{Fa^{2}l}{6EI} \left(3 + 2\frac{a}{l}\right)$  |
|  | 剪力      |  | 挠度       | 当 $x = a + 0.51$ 时, $W_{\min} = -\frac{Fal^2}{8EI}$  |
| -  |         |  |          |  |
|  | 荷载      |  | 反力       | $R_{A} = \frac{ql}{2} \left(1 + \frac{a}{l}\right)^{2};  R_{B} = \frac{ql}{2} \left(1 - \frac{a}{l}\right)^{2}$  |
|  |         |  | 剪力       | $V_{A\pm} = -qa; V_{A\pm} = R_A - qa; V_B = -R_B$  |
| 3  |         |  |          | $M_{\rm A} = -\frac{1}{2} q a^2$   |
|  | 弯矩      |  | 弯矩       | 若 $l>a$ , 当 $x=\frac{1}{2}\left(1+\frac{a}{2}\right)^2$ 时, $M_{\text{max}}=\frac{ql^2}{8}\left(1-\frac{a^2}{l^2}\right)^2$   |
|  | 剪力      |  | 挠度       | $W_{C} = \frac{qal^{3}}{2AEI} \left( -1 + 4 \frac{a^{2}}{I^{2}} + 3 \frac{a^{3}}{I^{3}} \right)$   |
| - N  | 9377    |  | DEE      | 2101   |
| A THE  | 荷载      |  | 反力       | $R_{\rm A} = R_{\rm B} = \frac{ql}{2} \left( 1 + 2 \frac{a}{l} \right) = \frac{q}{2} (l + 2a)$   |
|  |         | $\begin{array}{c c} & & & & & & & & & & & & & & & & & & &$                                     | 剪力       | $V_{A\pm} = -qa$ ; $V_{A\pm} = \frac{1}{2}ql$ ; $V_{B\pm} = -\frac{1}{2}ql$ ; $V_{B\pm} = qa$  |
| 4  |         | 4 4  |          | $M_{\rm A} = M_{\rm B} = \frac{1}{2} qa^2; M_{\rm max} = \frac{qI^2}{8} \left(1 - 4\frac{a^2}{I^2}\right)$   |
|  | 弯矩      |  | 弯矩       | $M_{A} = M_{B} = \frac{1}{2} qa^{2}; M_{\text{max}} = \frac{1}{8} \left(1 - 4\frac{1}{l^{2}}\right)$   |
|  | 剪力      |  | 挠度       | $W_{\text{max}} = \frac{ql^4}{384EI} \left( 5 - 24 \frac{a^2}{l^2} \right)$  |
| -  | ,       |  | 反力       | $R_{\rm A} = \frac{qa}{2} \left( 2 + \frac{a}{l} \right) ; R_{\rm B} = -\frac{qa^2}{2l}$   |
|  | 荷载      |  | -3.78    | 20 54 1 406 1 2 2 20 11 20 1 2 2   |
| and the same of th |         |  | 剪力       | $V_{AE} = -qa; V_{AE} = V_{B} = -R_{B} = \frac{qa^{2}}{l}$   |
| 5  | 弯矩      |  | 弯矩       | $M_{\text{max}} = M_{\text{A}} = -\frac{q a^2}{2}$   |
| - 6  | 1 -4-21 | 35.51  | 3 152.03 | $W_{C} = \frac{qa^{3}I}{24EI}\left(4 + \frac{3a}{I}\right)$  |
|  | 剪力      |  | 挠度       |  |
| _  |         | <i>a a</i>   | 1 /      | The state of the s |
| -  | 荷载      |  | 反力       | $R_{A} = R_{B} = qa$ $V = P  V = P  P$   |
|  |         |  | 剪力       | $V_A = -R_A$ ; $V_B = R_B$   |
| 6  | 弯矩      |  | 弯矩       | $M_{\rm A} = M_{\rm B} = -\frac{1}{2} q a^2$   |
| 247700-  |         | <u> </u>   | 2111     | $W_{\rm C} = W_{\rm D} = \frac{qa^{8}l}{8EI} \left(2 + \frac{a}{l}\right)$   |
|  | 剪力      | W. In  | 挠度       |  |
|  |         | 15 T 5550,0 mm   |          | = 3 = 6 т 0.35 µу, и спп 16EI  |

| 序次                              | 图类        | 图示                                     | 项目    | 型   |
|---------------------------------|-----------|--|-------|---|
| 187                             | 荷载        | $C \downarrow A A \qquad \downarrow B$ | 反力 剪力 | $R_{A} = \frac{F}{2} \left( 2 + 3 \frac{a}{l} \right) , R_{B} = -\frac{3Fa}{2l}$ $V_{A} = -R_{A}, V_{B} = R_{B}$  |
| 7                               | 弯矩        |  | 弯矩    | $M_A = -Fa$ ; $M_B = \frac{Fa}{2}$  |
|                                 | 剪力        |  | 挠度    | $W_{C} = \frac{F a^{2} l}{12EI} \left( 3 + 4 \frac{a}{l} \right)$ $\stackrel{\text{M}}{=} x = a + \frac{1}{3} \text{ Hz},  W_{\text{min}} = -\frac{F a l^{2}}{27EI}$    |
|                                 | 荷载        |  | 反力    | $R_{A} = \frac{ql}{8} \left( 3 + 8 \frac{a}{l} + 6 \frac{a^{2}}{l^{2}} \right) , R_{B} = \frac{ql}{8} \left( 5 - 6 \frac{a^{2}}{l^{2}} \right)$                         |
| 8                               | 19 敦      | $\frac{C}{a}$                          | 剪力    | $V_{A\dot{\Xi}} = -qa; V_{A\dot{\Xi}} = R_{B}; V_{B} = -R_{B}$  |
| 0                               | 弯矩        |  | 弯矩    | $M_{\rm A} = -\frac{q a^2}{2}$ ; $M_{\rm B} = -\frac{q l^2}{8} \left(1 - 2\frac{a^2}{l^2}\right)$   |
| 388                             | 挠度        |  | 挠度    | $W_{C} = \frac{qat^{3}}{48EI} \left( -1 + 6\frac{a^{2}}{I^{2}} + 6\frac{a^{3}}{I^{3}} \right)$  |
| 5.0<br>5.4                      | 荷载        |  | 反力 剪力 | $R_{A} = \frac{qa}{4} \left( 4 + 3 - \frac{a}{l} \right) ; R_{B} = -\frac{3qa^{2}}{4l}$ $V_{A \pm} = -qa; V_{A \pm} = V_{B} = R_{B}$                                    |
| 9                               | 弯矩        |  | 弯矩    | $M_{\rm A} = -\frac{qa^2}{2}$ ; $M_{\rm B} = \frac{qa^2}{4}$  |
|                                 | 挠度        |  | 挠度    | $W_{\rm C} = \frac{qa^3l}{8EI} \left(1 + \frac{a}{l}\right)$  |
| -                               | 7 - A - F | 10 (0) /                               | 反力    | $R_{\rm A} = -\frac{3M}{2l};  R_{\rm B} = \frac{3M}{2l}$  |
| Dan 4                           | 荷载        |  | 剪力    | $V_A = -R_A$ ; $V_B = -R_B$   |
| 10                              | 弯矩        |  | 弯矩    | $M_{A} = M_{3}  M_{B} = -\frac{M}{2}$   |
| 816<br>827<br>827<br>816<br>817 | 剪力        | 21.0                                   | 挠度    | $W_{C} = \frac{Mal}{4EI} \left( \overline{1} + 2 \frac{a}{l} \right)$ $\stackrel{\text{def}}{=} x = a + \frac{1}{3} \text{ lef},  W_{\text{max}} = \frac{Ml^{2}}{27EI}$ |

## 6.二跨连续梁

二跨等跨连续梁和二跨不等跨连续梁的内力和挠度的计算系数分别示于附表15-4-和附表15-4-21中。

|   | 二等跨梁的内力和挠度系数 附表 15-4-20   |                                 |       |        |        |                 |                |        |        |  |  |  |  |
|---|---|---------------------------------|-------|--------|--------|-----------------|----------------|--------|--------|--|--|--|--|
| 序 | 荷载图   | 跨内最                             | 大弯距   | 支座弯距   | 剪一力    |                 |                | 跨度中点挠度 |        |  |  |  |  |
| 次 | 荷载图   | M <sub>1</sub>   M <sub>2</sub> |       | М в    | VA VEL |                 | V <sub>c</sub> | -W 1   | W 2    |  |  |  |  |
| 1 | $AA \xrightarrow{B} l \xrightarrow{(M_2)}$  | 0.070                           | 0.070 | -0.125 | 0.375  | 0.625           | -0.375         | 0.521  | 0.521  |  |  |  |  |
| 2 | 4 B C   | 0.096                           |       | -0.063 | 0.437  | 0.063           | 0.063          | 0.912  | -0.391 |  |  |  |  |
| 3 | $ \begin{array}{c cccc} & & & & & & F \\ & & & & & & & A \\ \hline A & & & & & B & & C \\ & & & & & & & & & & & & & \\ & & & & &$   | 0.156                           | 0.156 | -0.188 | 0.312  | -0.688<br>0.688 | -0.312         | 0.911  | 0.911  |  |  |  |  |
| 4 | F A B C C   | 0.203                           |       | -0.094 | 0.406  | 0.094           | 0.094          | 1.497  | -0.583 |  |  |  |  |
| 5 |   | 0.222                           | 0.222 | -0,333 | 0.667  | 1.333           | -0.667         | 1.466  | 1.466  |  |  |  |  |
| 6 | F   F   A A B C   A B | 0.278                           |       | -0.167 | 0.833  | 0.167           | 0.167          | 2.508  | -1.042 |  |  |  |  |

注: 1.在均布荷载作用下: M=表中系数 $\times$   $ql^2$ ; V=表中系数 $\times$  ql; W=表中系数 $\times$   $ql^4$ 

2.在集中荷载作用下: M=表中系数×Fl; V=表中系数×F; W=表中系数× $\frac{Fl^3}{100EI}$ 。

## 二不等跨梁的内力和挠度系数

附表 15-4-21

| 荷载  | $AA \qquad AB \qquad AC \qquad \downarrow \qquad l_1 \qquad \downarrow \qquad l_2 = nl_1 \qquad \downarrow \qquad \qquad \downarrow$  |  |  |  |  | $ \begin{array}{c c}     \hline                                $   |  | $ \begin{array}{c cccc} A & B & C \\ \downarrow & l_1 & l_2 = nl_1 \\ \downarrow & l_1 & l_2 = nl_1 \end{array} $              |  |
|---|---|--|--|--|--|--|--|--|--|
| 数   | $\binom{M_{\rm B}}{(M_{\rm max})}M_{\rm BA}$  | $M_{BC} V_{A}$   | V B左<br>(V max)  | V B右 (V max)   | V <sub>c</sub>   | M AB (M max)   | VA<br>(Vmax)   | M <sub>BC</sub> (M <sub>max</sub> )  | (V C (V pr.a.x)  |
| 1.0<br>1.1<br>1.2<br>1.3<br>1.4<br>1.5<br>1.6<br>1.7<br>1.8<br>1.9<br>2.0<br>2.25 | -0.1250 0.0703<br>-0.1388 0.0653<br>-0.1550 0.0595<br>-0.1738 0.0532<br>-0.1950 0.0465<br>-0.2188 0.0396<br>-0.2450 0.0325<br>-0.2733 0.0256<br>-0.3050 0.0190<br>-0.3388 0.0130<br>-0.33750 0.0078<br>-0.4766 0.0003 | 0.0898 0.36<br>0.1108 0.34<br>0.1333 0.32<br>0.1572 0.30<br>0.1825 0.28<br>0.2092 0.255<br>0.2374 0.22<br>0.2669 0.19<br>0.2978 0.16<br>0.3301 0.122<br>0.4170 0.023 | 50 - 0.6250<br>- 0.6387<br>- 0.6550<br>63 - 0.6737<br>0 - 0.6950<br>13 - 0.7187<br>0 - 0.7450<br>13 - 0.7737<br>- 0.8350<br>0 - 0.8387<br>0 - 0.8350 | 0.6250<br>0.6761<br>0.7292<br>0.7836<br>0.8393<br>0.8953<br>1.0110<br>1.0694<br>1.1283<br>1.1875<br>1.3368 | - 0.3750<br>- 0.4239<br>- 0.4708<br>- 0.5164<br>- 0.5607<br>- 0.6042<br>- 0.6469<br>- 0.7306<br>- 0.7717<br>- 0.8125<br>- 0.9132<br>- 1.0125 | 0.0957<br>0.0970<br>0.0982<br>0.0993<br>0.1003<br>0.1013<br>0.1021<br>0.1029<br>0.1037<br>0.1044<br>0.1050<br>0.1065 | 0.4375<br>0.4405<br>0.4432<br>0.4457<br>0.4590<br>0.4590<br>0.4519<br>0.4554<br>0.4569<br>0.4583<br>0.4615<br>0.4643 | 0.0957<br>0.1142<br>0.1343<br>0.1558<br>0.1788<br>0.2032<br>0.2291<br>0.2564<br>0.2850<br>0.3155<br>0.3472<br>0.4327<br>0.5272 | - 0.4375<br>- 0.4780<br>- 0.5182<br>- 0.5582<br>- 0.5979<br>- 0.6375<br>- 0.6769<br>- 0.7162<br>- 0.7554<br>- 0.7944<br>- 0.8333<br>- 0.9303<br>- 1.0268 |

注:  $1.M = 表中系数 \times ql_1^2$ ;  $V = 表中系数 \times ql_1$ ;

 $2.(M_{\text{max}}),(V_{\text{max}})$ 表示它为相应跨内的最大内力。