

## L2SWBM and Copula Mini-Workshop

University of Michigan  
School for Environment and Sustainability

July 2022



# Outline

- 1 Introduction
- 2 Day 1 (Monday) - L2SWBM Technical Updates
  - L2SWBM background and technical overview
  - Recent and ongoing research projects
  - Version control and code repositories
  - Reality check (under the hood)
- 3 Day 2 (Tuesday) - Copulas and Federal Perspectives
  - Using Copulas
  - Federal Agency Partners
- 4 Day 3 (Wednesday) - Beyond the (Laurentian) Great Lakes
  - African Great Lakes
  - Mono Lake
  - NSF large lakes proposal (and other ideas)

# Outline

## 1 Introduction

## 2 Day 1 (Monday) - L2SWBM Technical Updates

- L2SWBM background and technical overview
- Recent and ongoing research projects
- Version control and code repositories
- Reality check (under the hood)

## 3 Day 2 (Tuesday) - Copulas and Federal Perspectives

- Using Copulas
- Federal Agency Partners

## 4 Day 3 (Wednesday) - Beyond the (Laurentian) Great Lakes

- African Great Lakes
- Mono Lake
- NSF large lakes proposal (and other ideas)



# University of Michigan L2SWBM Research Team



Drew Gronewold,  
Principal Investigator  
Joseph Smith

Hong Do, Ph.D

Dani Cohen, MS  
MS

Ryan Armbrustmacher,  
MS  
Sarah Katz, Ph.D  
candidate

Alex Vandeweghe

Jennani Jayaram



Yifan Luo, MS

Maegan Muir, MS

Justin Huber, MS

Hannah Paulson, MS



# GREAT LAKES ST. LAWRENCE GOVERNORS & PREMIERS



# Outline

## 1 Introduction

## 2 Day 1 (Monday) - L2SWBM Technical Updates

- L2SWBM background and technical overview
- Recent and ongoing research projects
- Version control and code repositories
- Reality check (under the hood)

## 3 Day 2 (Tuesday) - Copulas and Federal Perspectives

- Using Copulas
- Federal Agency Partners

## 4 Day 3 (Wednesday) - Beyond the (Laurentian) Great Lakes

- African Great Lakes
- Mono Lake
- NSF large lakes proposal (and other ideas)

# Outline

## 1 Introduction

## 2 Day 1 (Monday) - L2SWBM Technical Updates

- L2SWBM background and technical overview
- Recent and ongoing research projects
- Version control and code repositories
- Reality check (under the hood)

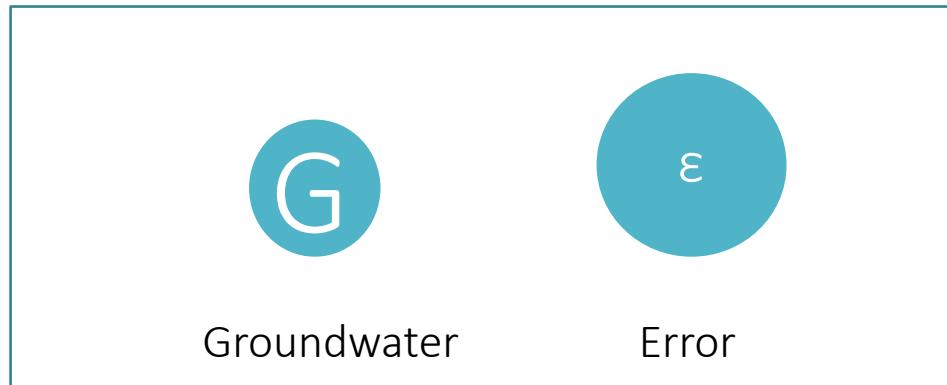
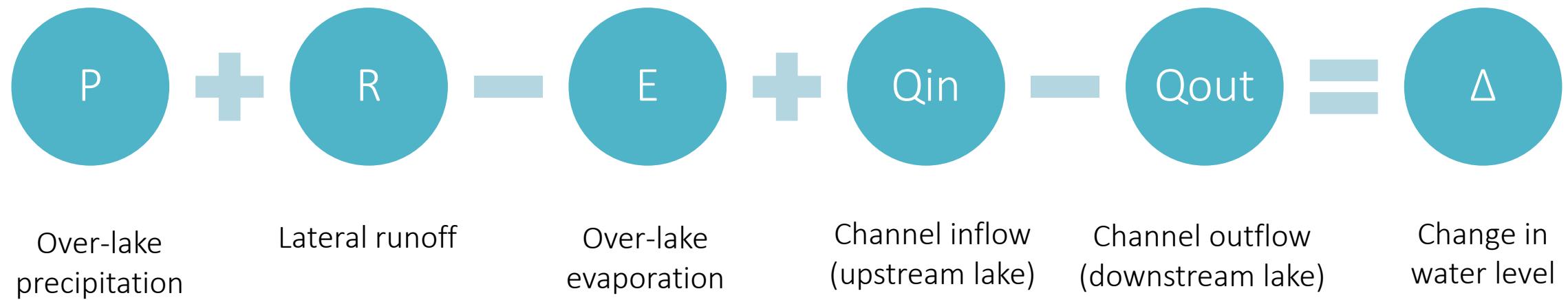
## 3 Day 2 (Tuesday) - Copulas and Federal Perspectives

- Using Copulas
- Federal Agency Partners

## 4 Day 3 (Wednesday) - Beyond the (Laurentian) Great Lakes

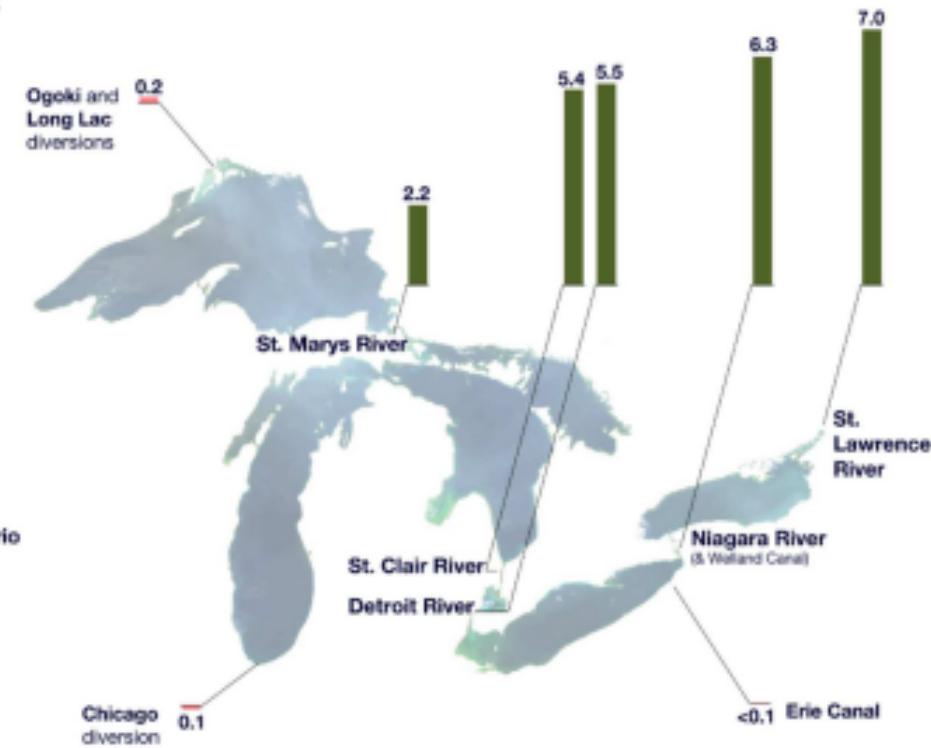
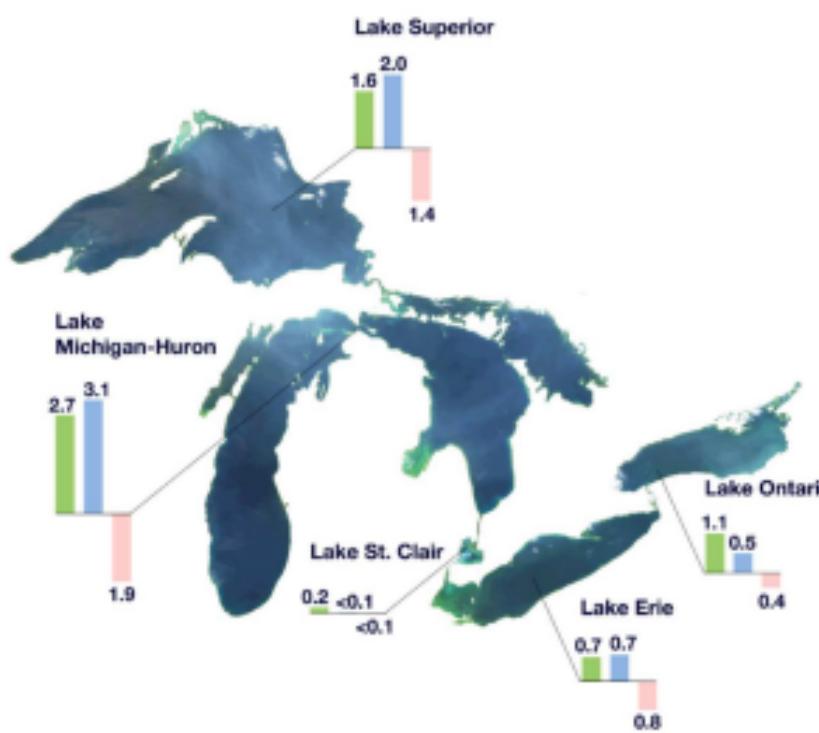
- African Great Lakes
- Mono Lake
- NSF large lakes proposal (and other ideas)

# Historical Conventional Water Balance Model

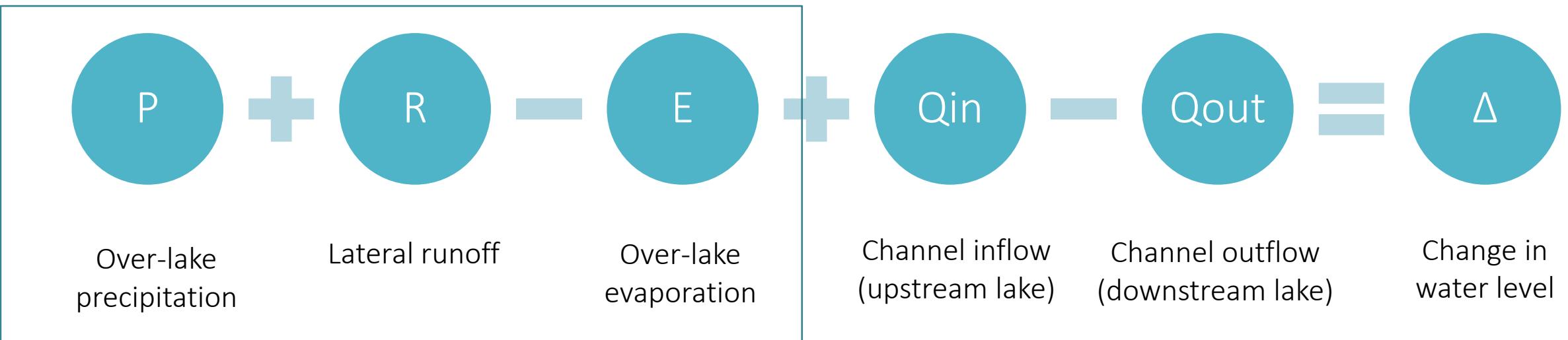




All values are averaged over the period 1950-2010 and are in thousands of cubic meters per second.



# Historical Conventional Water Balance Model





Contents lists available at [ScienceDirect](#)

## Journal of Great Lakes Research

journal homepage: [www.elsevier.com/locate/jglr](http://www.elsevier.com/locate/jglr)



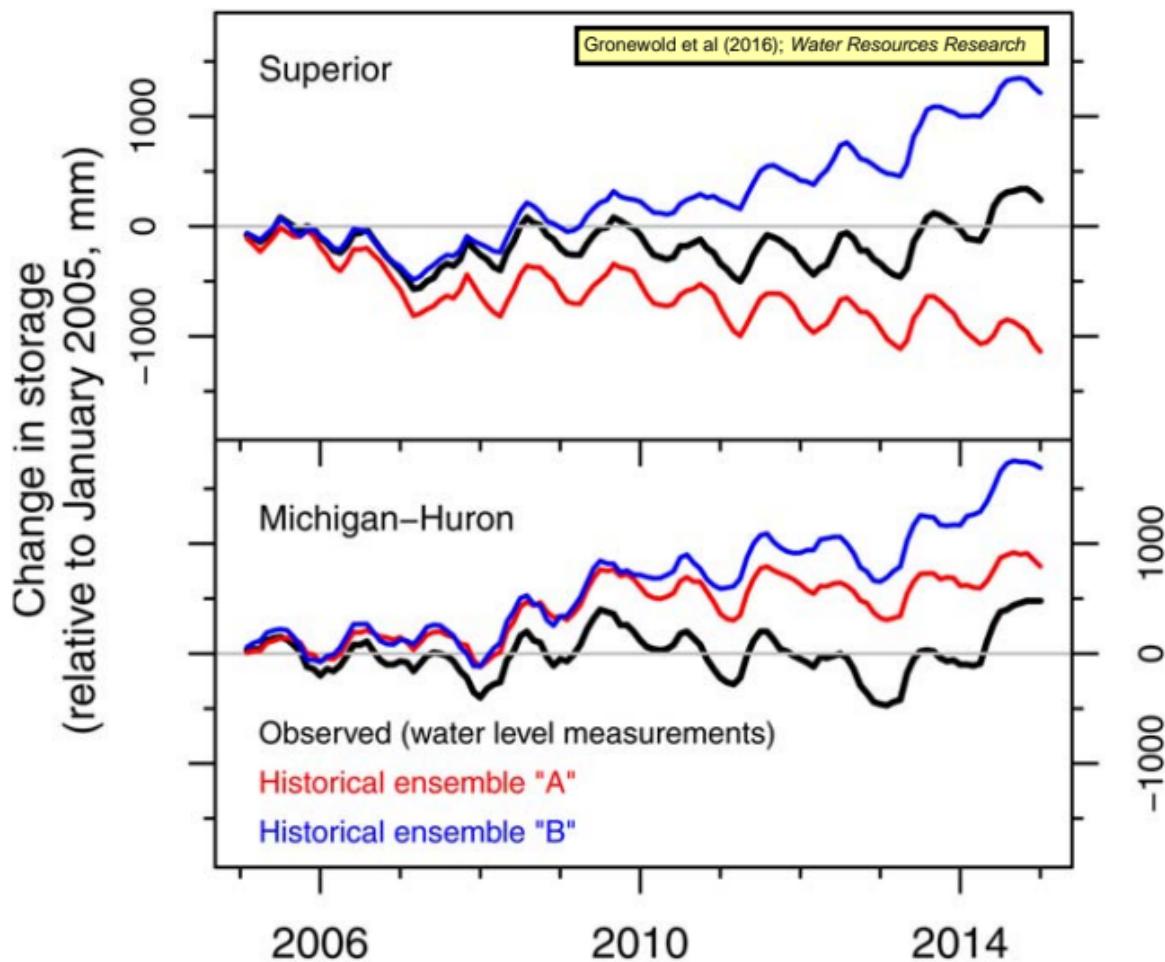
# Development and application of a North American Great Lakes hydrometeorological database – Part I: Precipitation, evaporation, runoff, and air temperature

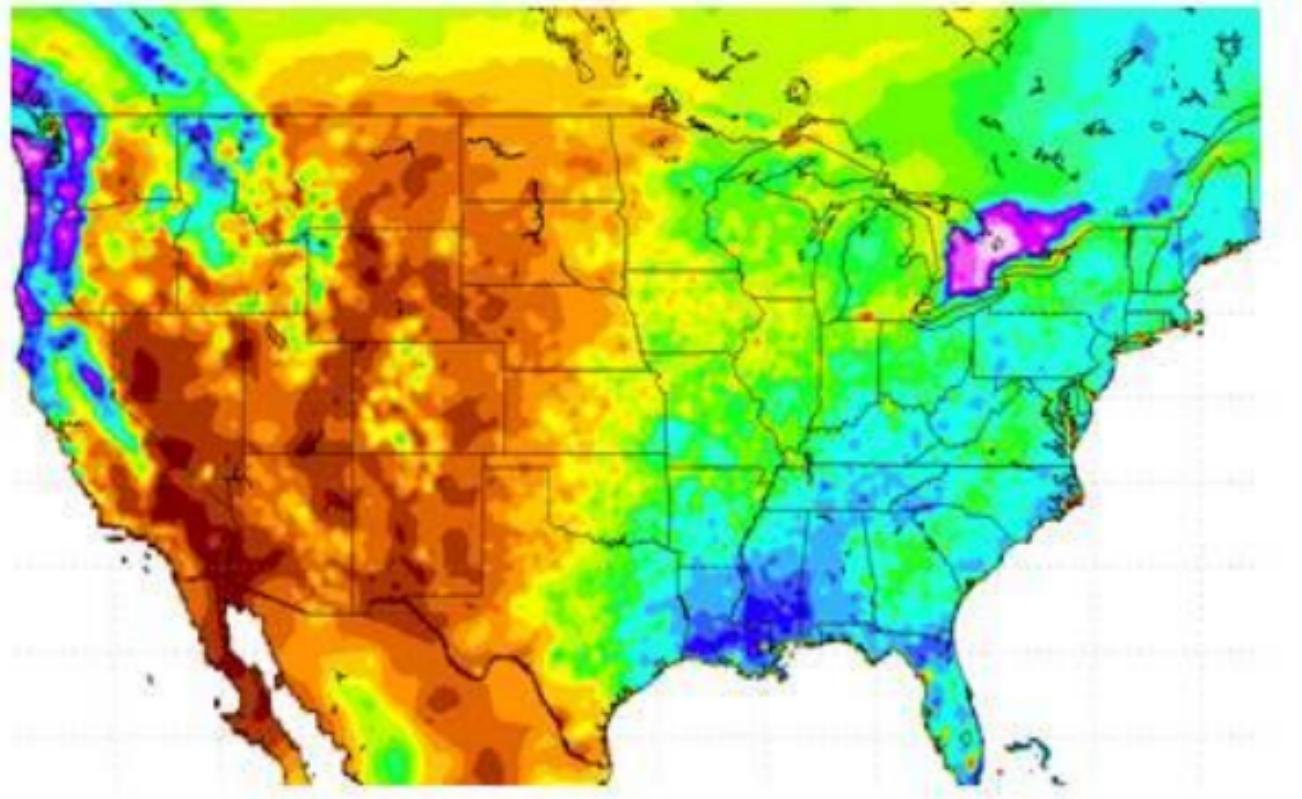


Timothy S. Hunter <sup>a</sup>, Anne H. Clites <sup>a</sup>, Kent B. Campbell <sup>a,b</sup>, Andrew D. Gronewold <sup>a,\*</sup>

<sup>a</sup> Great Lakes Environmental Research Laboratory, National Oceanic and Atmospheric Administration, Ann Arbor, MI 48108, USA

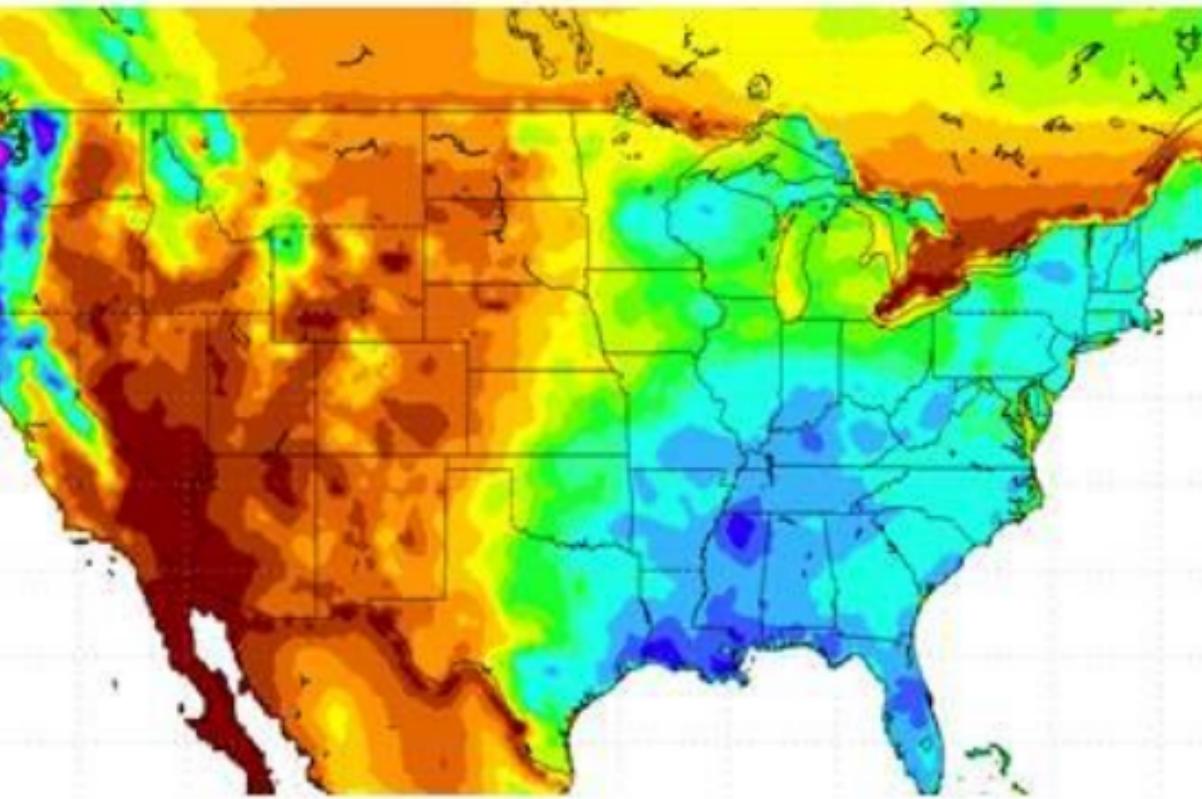
<sup>b</sup> Cooperative Institute for Limnology and Ecosystems Research, University of Michigan, Ann Arbor, MI 48109, USA





[in]

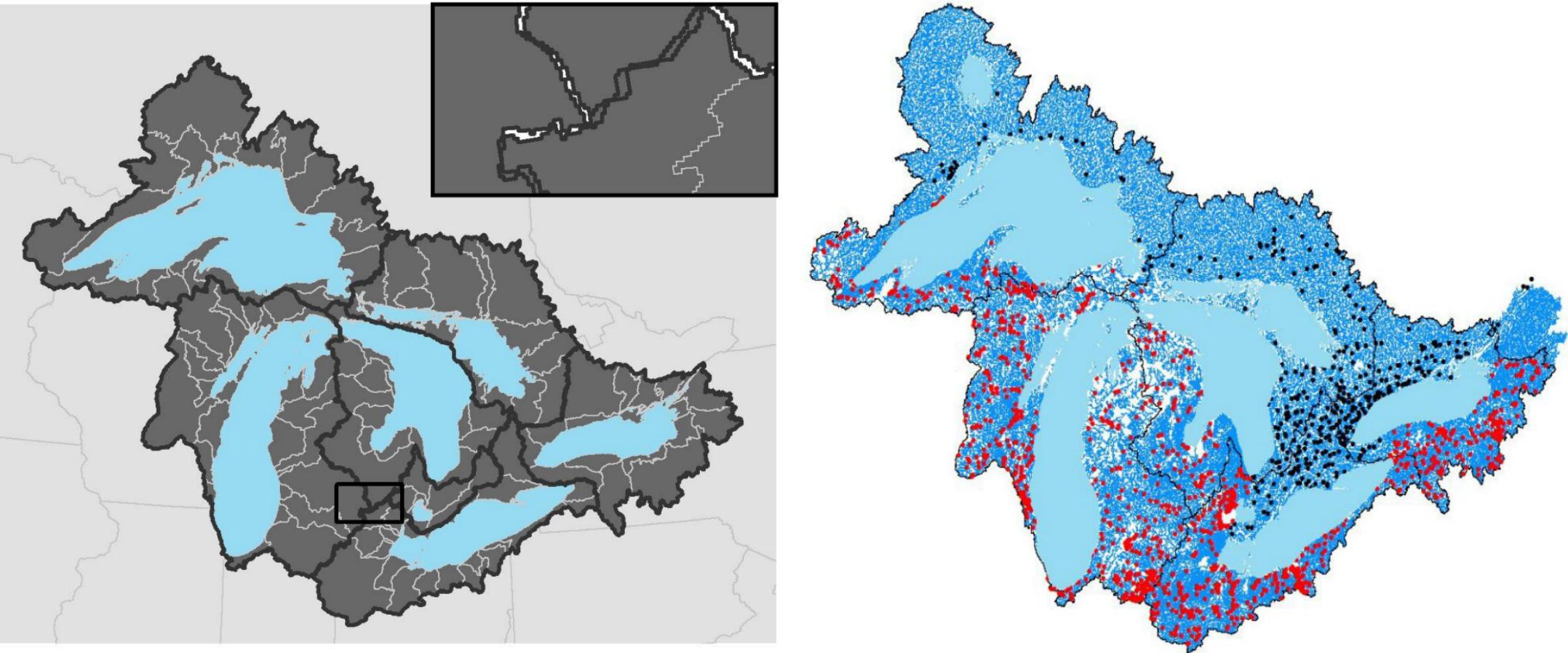
4 8 12 16 20 24 28 32 36 40 50 60 70 80 100 120 140 160



[in]

4 8 12 16 20 24 28 32 36 40 50 60 70 80 100 120 140 160

Gronewold et al (2018); BAMS



White Shoal Lighthouse: Lake Michigan  
Photo courtesy Dick Moehl (Lighthouse Keepers Association)





[dholtschlag-usgs / LakeSuperior](#) Public[Notifications](#)[Fork 0](#)[Star 0](#) ▾[Code](#) [Issues](#) [Pull requests](#) [Actions](#) [Projects](#) [Wiki](#) [Security](#) [Insights](#)[master](#) ▾[1 branch](#)[0 tags](#)[Go to file](#)[Code](#) ▾

dholtschlag-usgs Remove large data files

b25c53e on Mar 9, 2015 [17 commits](#)

Superior	Cumulative update	8 years ago
.gitignore	Remove large data files	8 years ago
.gitignore.bak	Remove large data files	8 years ago
LaplacesDemon_15.03.01.tar.gz	Cumulative update	8 years ago
README.md	Updated Readme with waterBudgetCondLakeSupBOMLevel.R	8 years ago
README.md.bak	Updated Readme with waterBudgetCondLakeSupBOMLevel.R	8 years ago
SupMergedCMS.txt	Update with Monthly Multiple Regression Models	8 years ago
SupPrecipCMS.txt	R markdown file and output for checkCompNBS	8 years ago
Superior.Rproj	R markdown file and output for checkCompNBS	8 years ago
junkCode.R	Catchup commit	8 years ago
maacs.Rda	Catchup commit	8 years ago
stdModel.R	Catchup commit	8 years ago

## About

**Water Budget for Lake Superior**[Readme](#)[0 stars](#)[1 watching](#)[0 forks](#)

## Releases

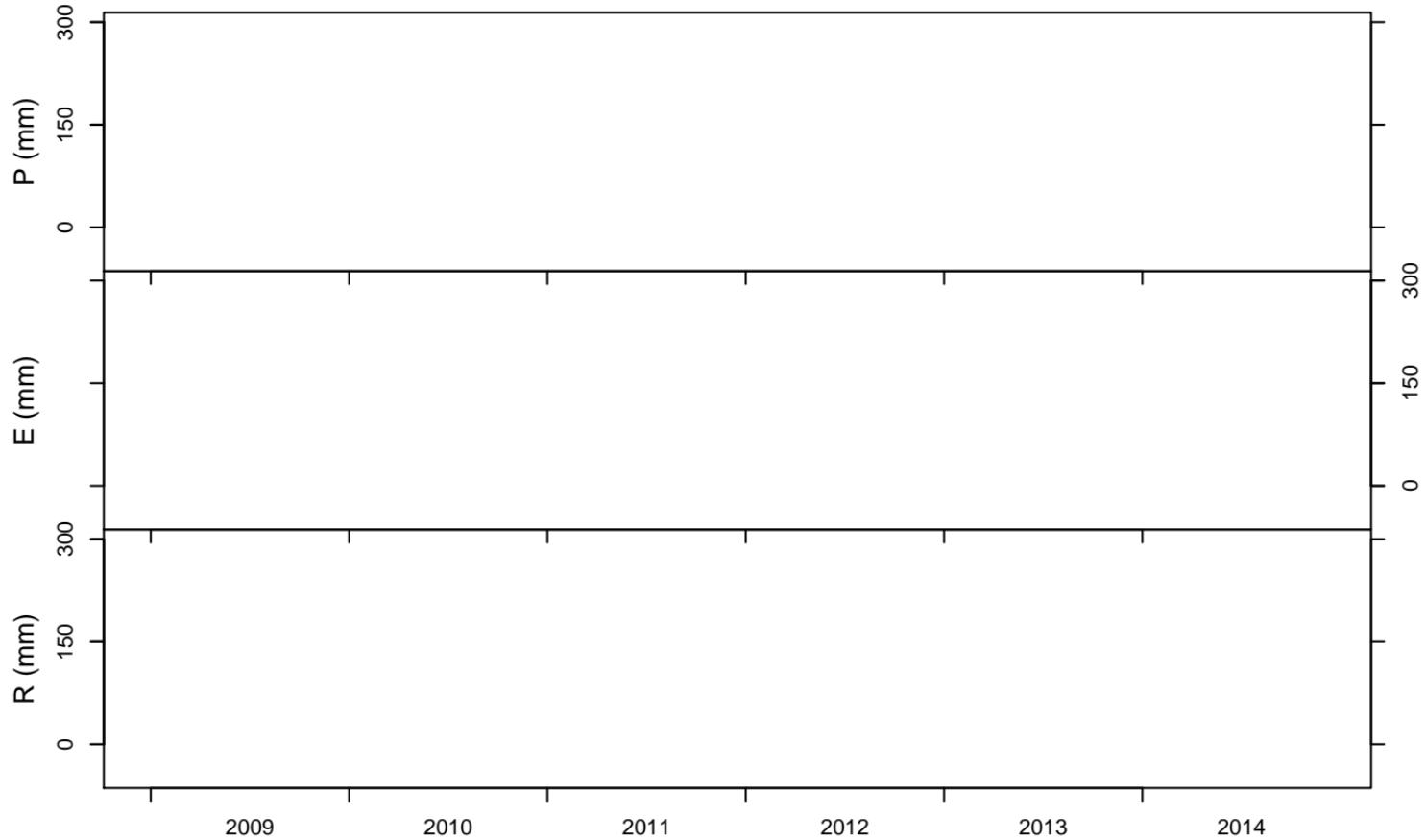
No releases published

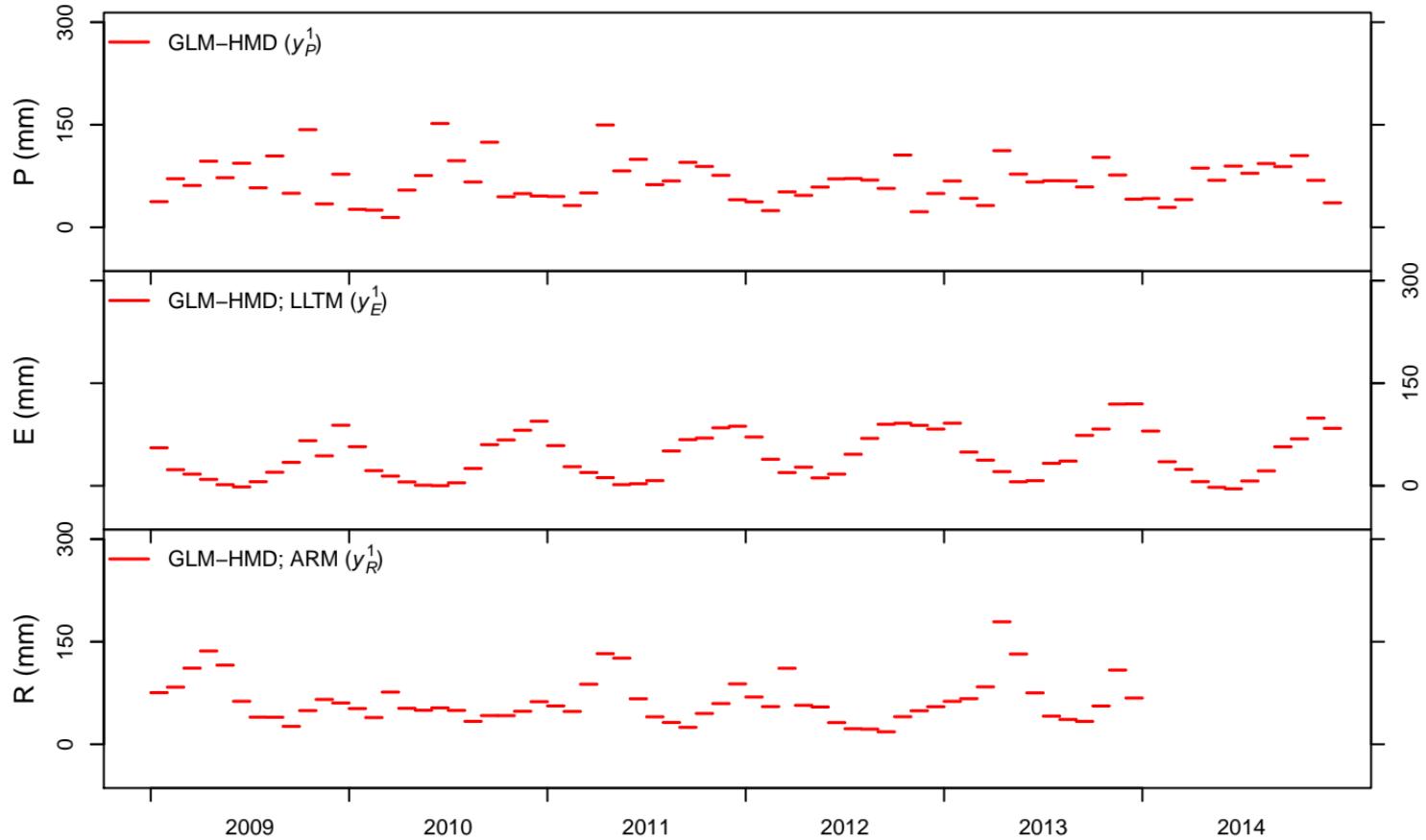
## Packages

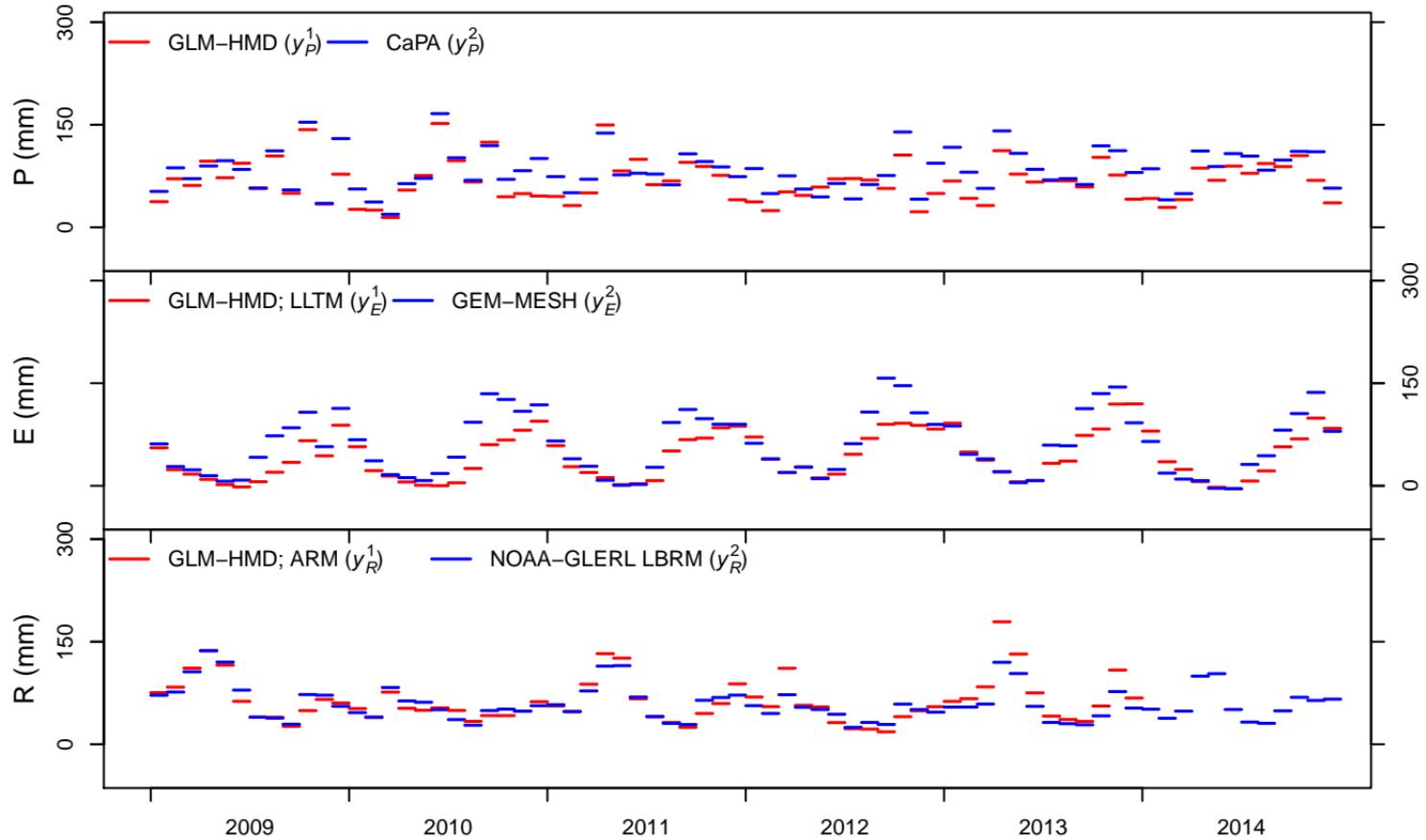
No packages published

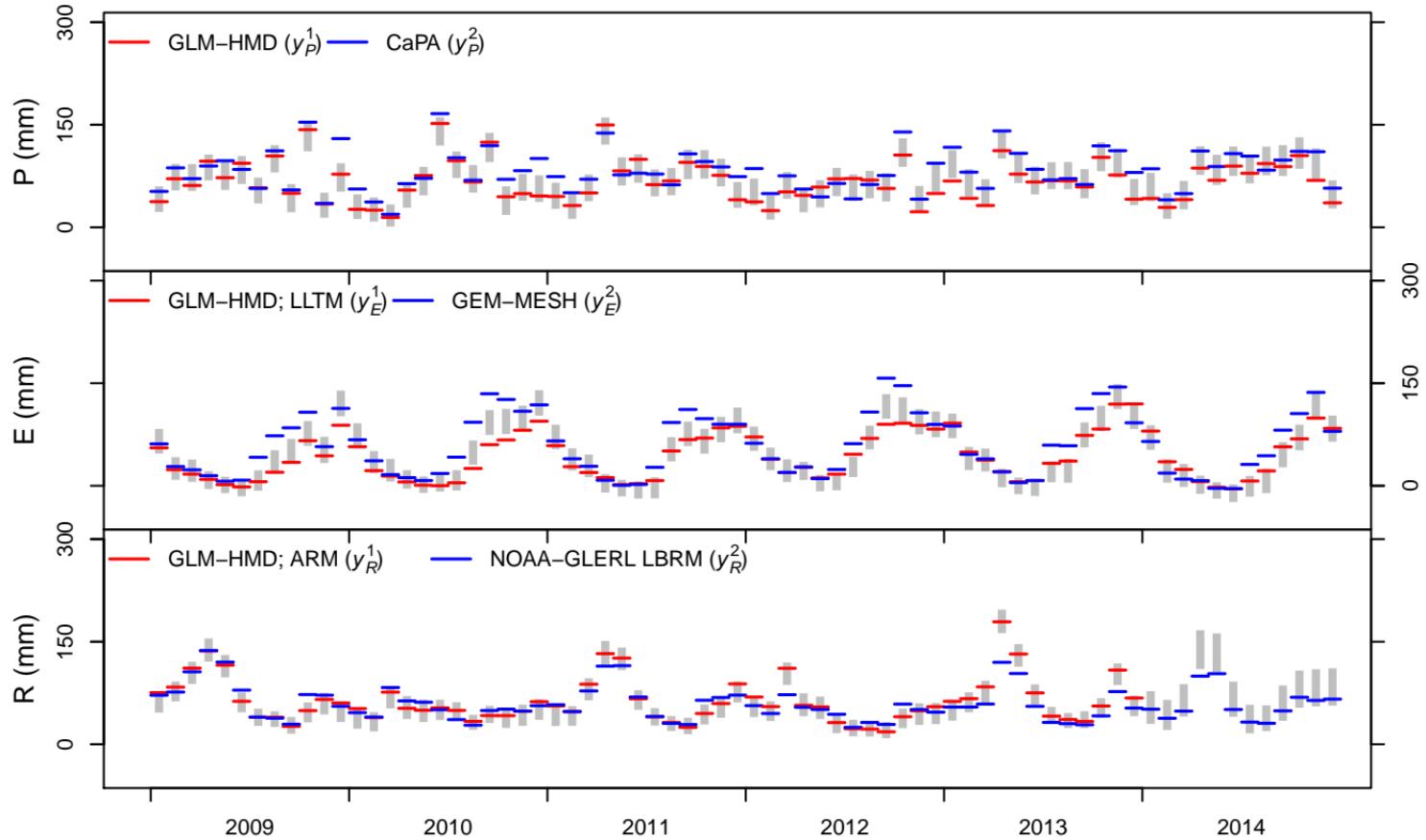
## Languages

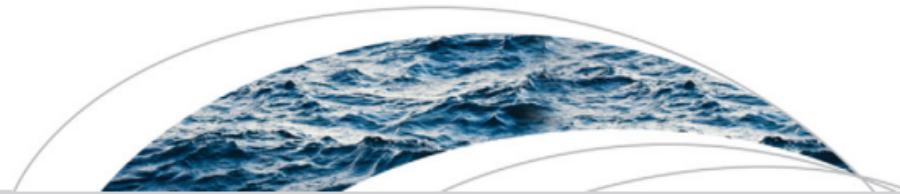
● HTML 90.3%   ● R 9.7%











## Water Resources Research

### RESEARCH ARTICLE

10.1002/2015WR018209

#### Key Points:

- Between January 2013 and December 2014, the two largest lakes on Earth rose at a record-setting rate
- We developed a Bayesian MCMC routine for inferring estimates of the water budget for this period
- The cold 2013–2014 winter contributed to reduced evaporation rates and rising water levels

### Hydrological drivers of record-setting water level rise on Earth's largest lake system

A. D. Gronewold<sup>1,2</sup>, J. Bruxer<sup>3</sup>, D. Durnford<sup>4</sup>, J. P. Smith<sup>5</sup>, A. H. Clites<sup>1</sup>, F. Seglenieks<sup>6</sup>, S. S. Qian<sup>7</sup>, T. S. Hunter<sup>1</sup>, and V. Fortin<sup>4</sup>

<sup>1</sup>National Oceanic and Atmospheric Administration, Great Lakes Environmental Research Laboratory, Ann Arbor, Michigan, USA, <sup>2</sup>Department of Civil and Environmental Engineering, University of Michigan, Ann Arbor, Michigan, USA, <sup>3</sup>Environment and Climate Change Canada, Great Lakes—St. Lawrence Regulation Office, Cornwall, Ontario, Canada, <sup>4</sup>Canadian Meteorological Centre, Dorval, Quebec, Canada, <sup>5</sup>Cooperative Institute for Limnology and Ecosystems Research, University of Michigan, Ann Arbor, Michigan, USA, <sup>6</sup>Environment and Climate Change Canada, Canada Centre for Inland Waters, Burlington, Ontario, Canada, <sup>7</sup>Department of Environmental Sciences, University of Toledo, Toledo, Ohio, USA

$$\Delta H_{j,w}$$

$$\Delta H_{j,w} = H_{j+w} - H_j$$

$$\Delta H_{j,w} = H_{j+w} - H_j = \sum_{i=j}^{j+w-1} (P_i - E_i + R_i + I_i - Q_i + D_i + \epsilon_i)$$

$y_{\Delta H, j, w}$

$$y_{\Delta H,j,w} = y_{H,j+w} - y_{H,j}$$

$$y_{\Delta H,j,w} = y_{H,j+w} - y_{H,j} \sim \mathcal{N}$$

$$y_{\Delta H,j,w} = y_{H,j+w} - y_{H,j} \sim N(\Delta H_{j,w},$$

$$y_{\Delta H,j,w} = y_{H,j+w} - y_{H,j} \sim N(\Delta H_{j,w}, \tau_{\Delta H,w})$$

$$y_{t,\theta}^n \sim N(\theta_t + \eta_{\theta,c_t}^n, \tau_{t,\theta}^n)$$

$$\pi(E_t) = \mathcal{N}(\mu_{E,c_t}, \tau_{E,c_t}/2)$$

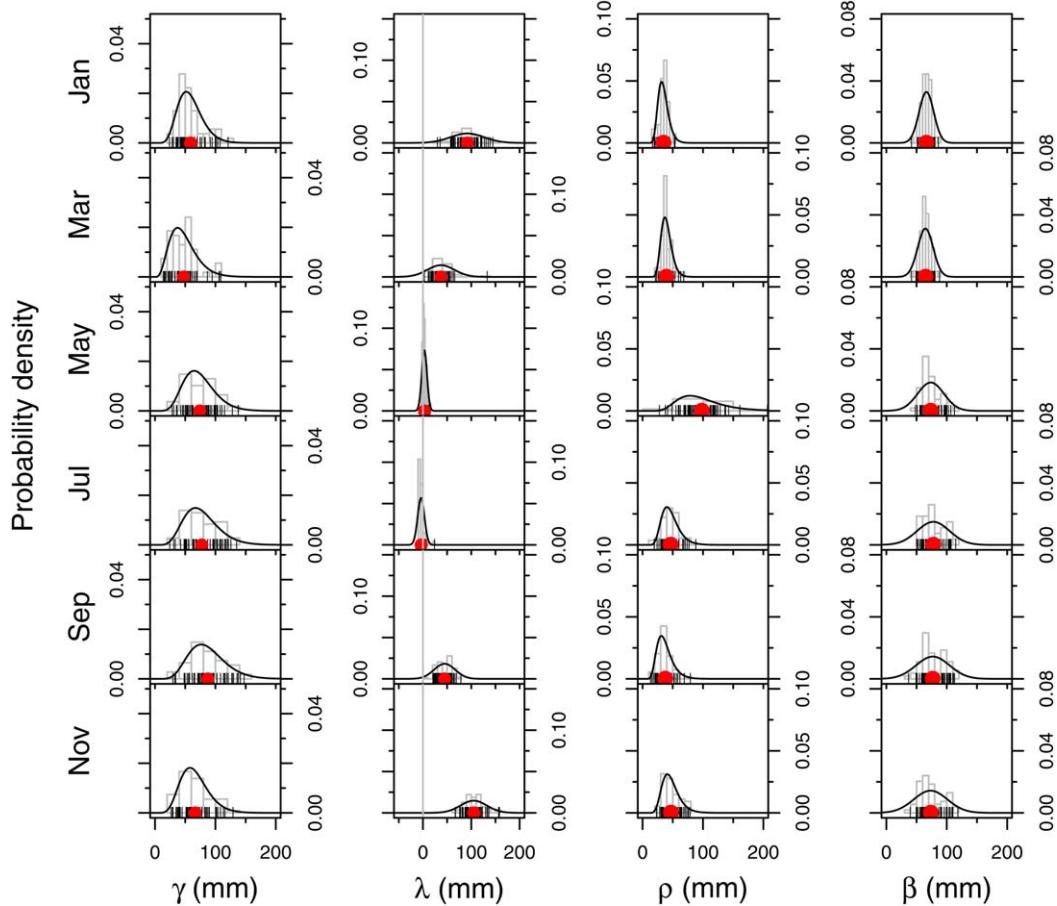
$$\pi(Q'_t) = \mathcal{N}(\mu_{Q',c_t}, \tau_{Q,c_t})$$

$$\pi(D'_t) = \mathcal{N}(\mu_{D',c_t}, \tau_{D',c_t})$$

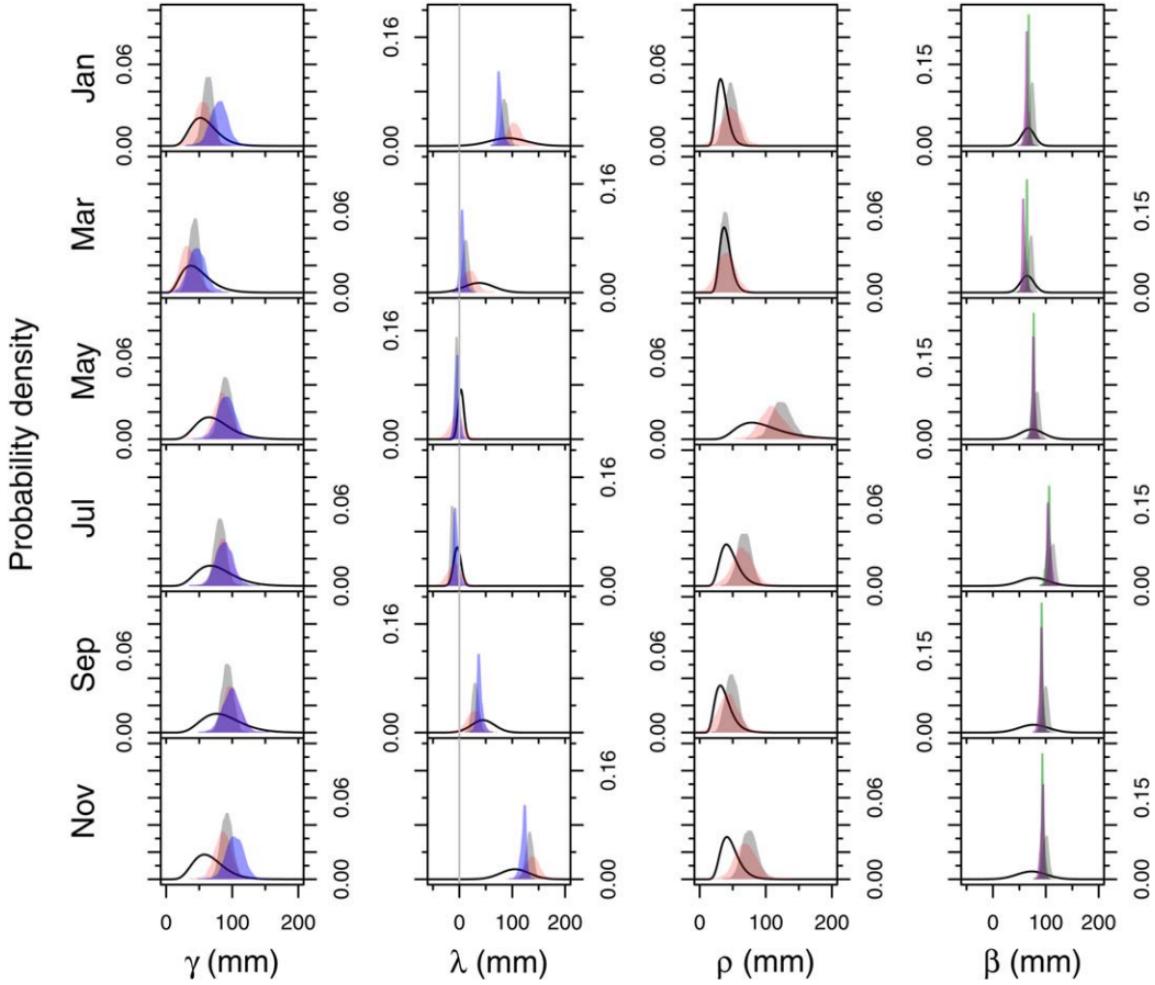
$$\pi(R_t) = \text{LN}(\mu_{ln(R), c_t}, \tau_{ln(R), c_t})$$

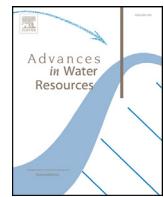
$$\epsilon_t = \epsilon_{ct}$$

$$\pi(\epsilon_{ct}) = N(0, 0.01)$$



**Figure 2.** Development of prior probability distributions for Lake Superior monthly water budget components. Plot includes every other month (starting in January) for clarity and simplicity. Prior probability distributions for all months for both Lakes Superior and Michigan-Huron are included in supporting information Figures S1 and S2. Vertical tick marks along x axis and histograms in each plot are based on values from the historical record from 1950 to 2004 in the GLM-HMD [Hunter et al., 2015]. Red dots represent the historical mean, and thin black curves represent the “fitted” prior probability distributions.





## Reconciling the water balance of large lake systems

Andrew D. Gronewold <sup>a,b,\*</sup>, Joeseph P. Smith <sup>c</sup>, Laura K. Read <sup>d</sup>, James L. Crooks <sup>e,f</sup>



<sup>a</sup> School for Environment and Sustainability, University of Michigan, Ann Arbor, MI 48109, USA

<sup>b</sup> Department of Civil and Environmental Engineering, University of Michigan, Ann Arbor, MI 48109, USA

<sup>c</sup> Cooperative Institute for Great Lakes Research, University of Michigan, Ann Arbor, MI 48109, USA

<sup>d</sup> National Center for Atmospheric Research, Boulder, CO 80307, USA

<sup>e</sup> Division of Biostatistics and Bioinformatics, National Jewish Health, Denver, CO 80206, USA

<sup>f</sup> Department of Epidemiology, Colorado School of Public Health, Aurora, CO 80045, USA

### ARTICLE INFO

#### Keywords:

Hydrologic cycle  
Large lakes  
Statistical modeling  
Bayesian inference  
Water balance

### ABSTRACT

Water balance models are commonly employed to improve understanding of drivers behind changes in the hydrologic cycle across multiple space and time scales. Generally, these models are physically-based, a feature that can lead to unreconciled biases and uncertainties when a model is not encoded to be faithful to changes in water storage over time. Statistical methods represent one approach to addressing this problem. We find, however, that there are very few historical hydrological modeling studies in which bias correction and uncertainty quantification methods are routinely applied to ensure fidelity to the water balance. Importantly, we know of none (aside from preliminary applications of the model we advance in this study) applied specifically to large lake systems. We fill this gap by developing and applying a Bayesian statistical analysis framework for inferring water balance components specifically in large lake systems. The model behind this framework, which we refer to as the L2SWBM (large lake statistical water balance model), includes a conventional water balance model encoded to iteratively close the water balance over multiple consecutive time periods. Throughout these iterations, the L2SWBM can assimilate multiple preliminary estimates of each water balance component (from either historical model simulations or interpolated *in situ* monitoring data, for example), and it can accommodate those estimates even if they span different time periods. The L2SWBM can also be executed if data for a particular water balance component are unavailable, a feature that underscores its potential utility in data scarce regions. Here, we demonstrate the utility of our new framework through a customized application to the Laurentian Great Lakes, the largest system of lakes on Earth. Through this application, we find that the L2SWBM is able to infer new water balance component estimates that, to our knowledge, are the first ever to close the water balance over a multi-decadal historical period for this massive lake system. More specifically, we find that posterior predictive intervals for changes in lake storage are consistent with observed changes in lake storage across this period over simulation time intervals of both 6 and 12 months. In addition to introducing a framework for developing definitive long-term hydrologic records for large lake systems, our study provides important insights into the origins of biases in both legacy and state-of-the-art hydrological models, as well as regional and global hydrological data sets.

### 1. Introduction

Hydrological models that simulate and forecast the water balance across a variety of space and time scales are needed to facilitate water resources management planning and, ultimately, to ensure human and environmental health (Vörösmarty et al., 2000; Pekel et al., 2016). This need is particularly pronounced in regions where rapid population growth coincides with changes in the spatiotemporal distribution of fresh water, and where the sustainability of future water supplies is uncertain (Schewe et al., 2014). To address this need, hydrological models need to clearly differentiate components of the hydrologic cycle that

are often confounded (including, for example, evapotranspiration and irrigation water demand) and to quantify changes in those components over time (Nijssen et al., 2001; Kebede et al., 2006; Raes et al., 2006; Li et al., 2007; Gronewold and Stow, 2014).

Global, continental, and basin-scale water balance modeling research typically focuses on improving representation of terrestrial and atmospheric physical processes collectively governing precipitation, evapotranspiration, and streamflow (Crow et al., 2008; Kim and Stricker, 1996; Milly and Dunne, 2017; Senay et al., 2011; Vörösmarty et al., 1998). This body of research, while providing foundational hydrologic data for much of the planet's land surface, rarely explicitly

\* Corresponding author: School for Environment and Sustainability, University of Michigan, Ann Arbor, MI 48109, USA.

E-mail address: [drewgron@umich.edu](mailto:drewgron@umich.edu) (A.D. Gronewold).

# SCIENTIFIC DATA



OPEN

DATA DESCRIPTOR

## Seventy-year long record of monthly water balance estimates for Earth's largest lake system

Hong X. Do<sup>1,2</sup>✉, Joeseph P. Smith<sup>1</sup>, Lauren M. Fry<sup>4,5</sup> & Andrew D. Gronewold<sup>1</sup>

We develop new estimates of monthly water balance components from 1950 to 2019 for the Laurentian Great Lakes, the largest surface freshwater system on Earth. For each of the Great Lakes, lake storage changes and water balance components were estimated using the Large Lakes Statistical Water Balance Model (L2SWBM). Multiple independent data sources, contributed by a binational community of research scientists and practitioners, were assimilated into the L2SWBM to infer feasible values of water balance components through a Bayesian framework. A conventional water balance model was used to constrain the new estimates, ensuring that the water balance can be reconciled over multiple time periods. The new estimates are useful for investigating changes in water availability, or benchmarking new hydrological models and data products developed for the Laurentian Great Lakes Region. The source code and inputs of the L2SWBM model are also made available, and can be adapted to include new data sources for the Great Lakes, or to address water balance problems on other large lake systems.

# Outline

## 1 Introduction

## 2 Day 1 (Monday) - L2SWBM Technical Updates

- L2SWBM background and technical overview
- Recent and ongoing research projects
- Version control and code repositories
- Reality check (under the hood)

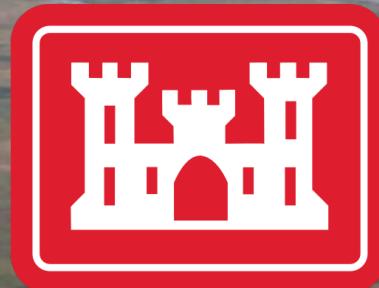
## 3 Day 2 (Tuesday) - Copulas and Federal Perspectives

- Using Copulas
- Federal Agency Partners

## 4 Day 3 (Wednesday) - Beyond the (Laurentian) Great Lakes

- African Great Lakes
- Mono Lake
- NSF large lakes proposal (and other ideas)

# Project Updates: Investigating Uncertainty Associated with the Great Lakes Water Balance



Fall 2021 Coordinating Committee Hydrology Subcommittee Meeting

Thursday, October 21<sup>st</sup>, 2021

Sarah Katz, Jennani Jayaram, Drew Gronewold

# L2SWBM Background

- Large Lakes Statistical Water Balance Model
  - Originally developed at NOAA-GLERL
  - Continued development by the SEAS Hydro Team at the University of Michigan
- Assimilates data on water budget components to provide new estimates of water budget components that are faithful to the Great Lakes water balance.



Drew Gronewold



Hong Do



Dani Cohen



Sarah Katz



Jennani Jayaram



Ryan  
Armbrustmacher



$$\Delta H_{j,w} = H_{j+w} - H_j = \sum_{i=j}^{j+w-1} (P_i - E_i + R_i + I_i - Q_i + D_i + \epsilon_i)$$

$$\Delta H_{j,w} = H_{j+w} - H_j = \sum_{i=j}^{j+w-1} (P_i - E_i + R_i + I_i - \boxed{Q_i} + D_i + \epsilon_i)$$

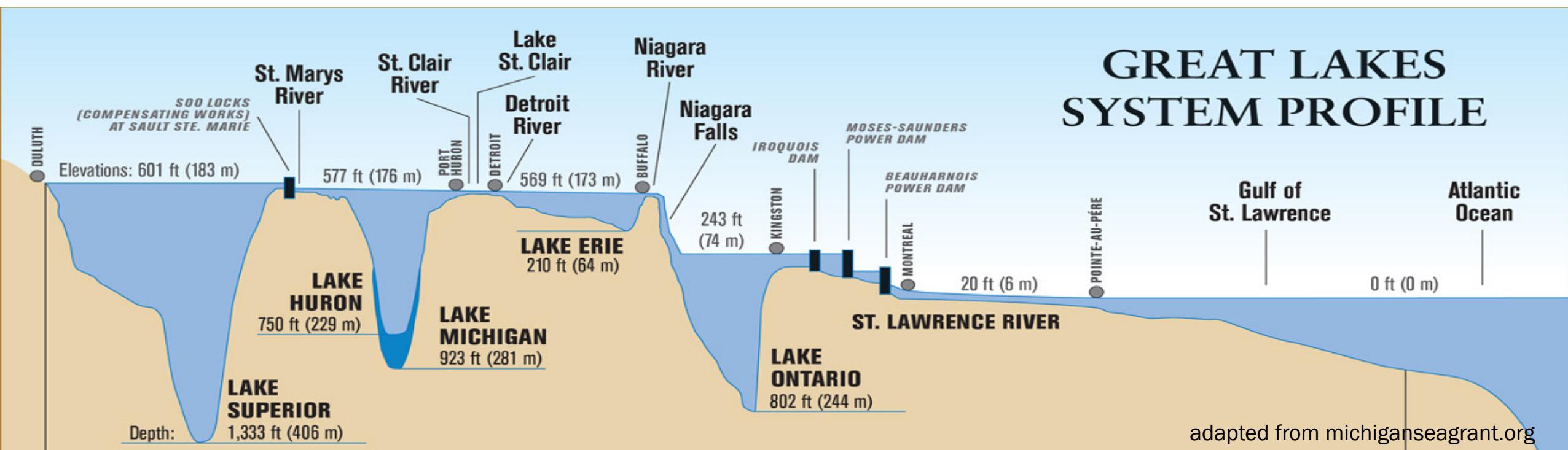
# Encoding Lake-to-Lake routing equations

$$Q_{STC} = c \left( \frac{(z_1 + z_2)}{2} - y_m \right)^d$$

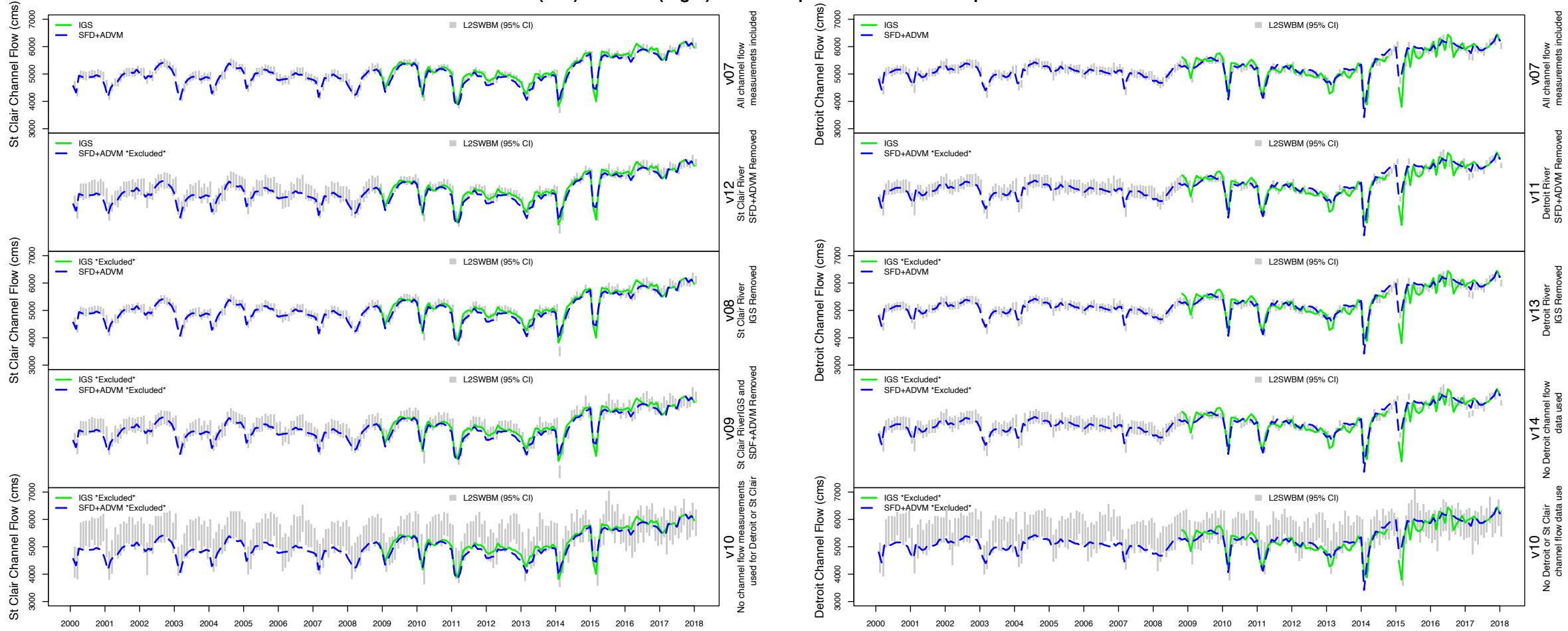
$z_1$  = MH BOM water levels  
 $z_2$  = STC BOM water levels  
 $Q_{STC}$  = St Clair River channel flow  
 $c$  = coefficient term  
 $d$  = exponent term  
 $y_m$  = 167

\* This formulation used for initial tests, additional runs forthcoming with fall component

\* L2SWBM can be used to iteratively solve for coefficient values that are faithful to the water balance



### St Clair (Left) & Detroit (Right) River Compiled Channel Flow Experiments



\* Large uncertainties in L2SWBM when channel flow is unconstrained by observations (4<sup>th</sup> and 5<sup>th</sup> rows)

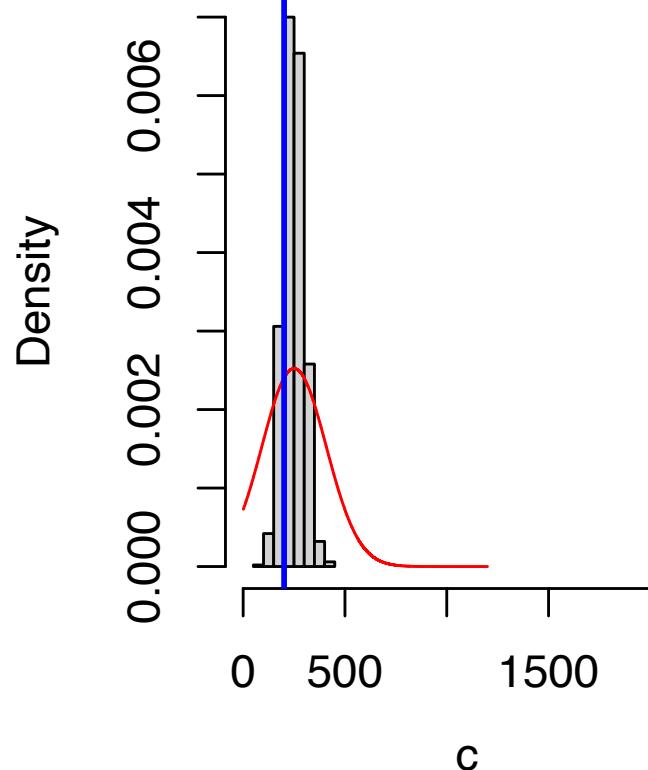
\* Incorporating channel flow datasets improves L2SWBM performance and reduces model uncertainties (top row)

# Lake-to-Lake Routing Status

Priors for this run:

$c$  : mean = 250 ; SD = 50

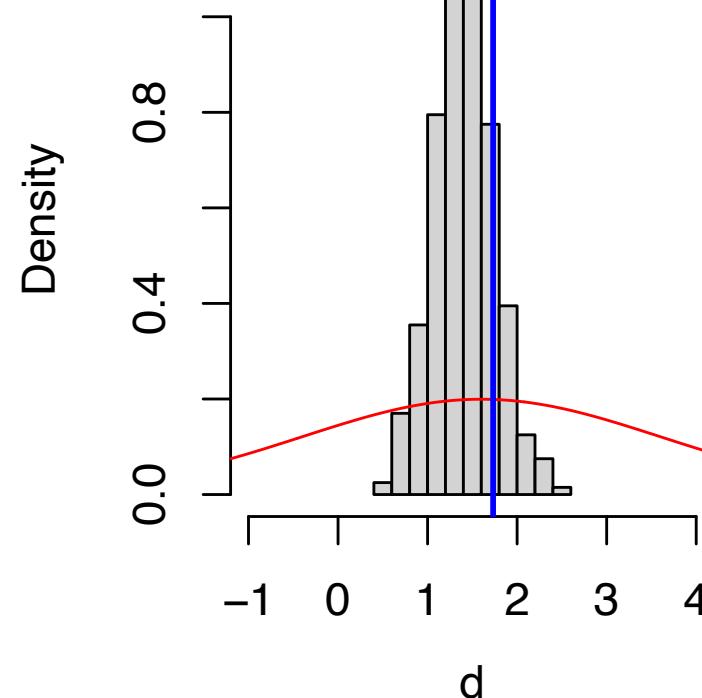
$d$  : mean = 1.6 ; SD = 2



Posteriors for this run:

$c$  : mean = 249.7 ; 95% CI = 152.2 – 349.3

$d$  : mean = 1.41 ; 95% CI = 0.77 – 2.05



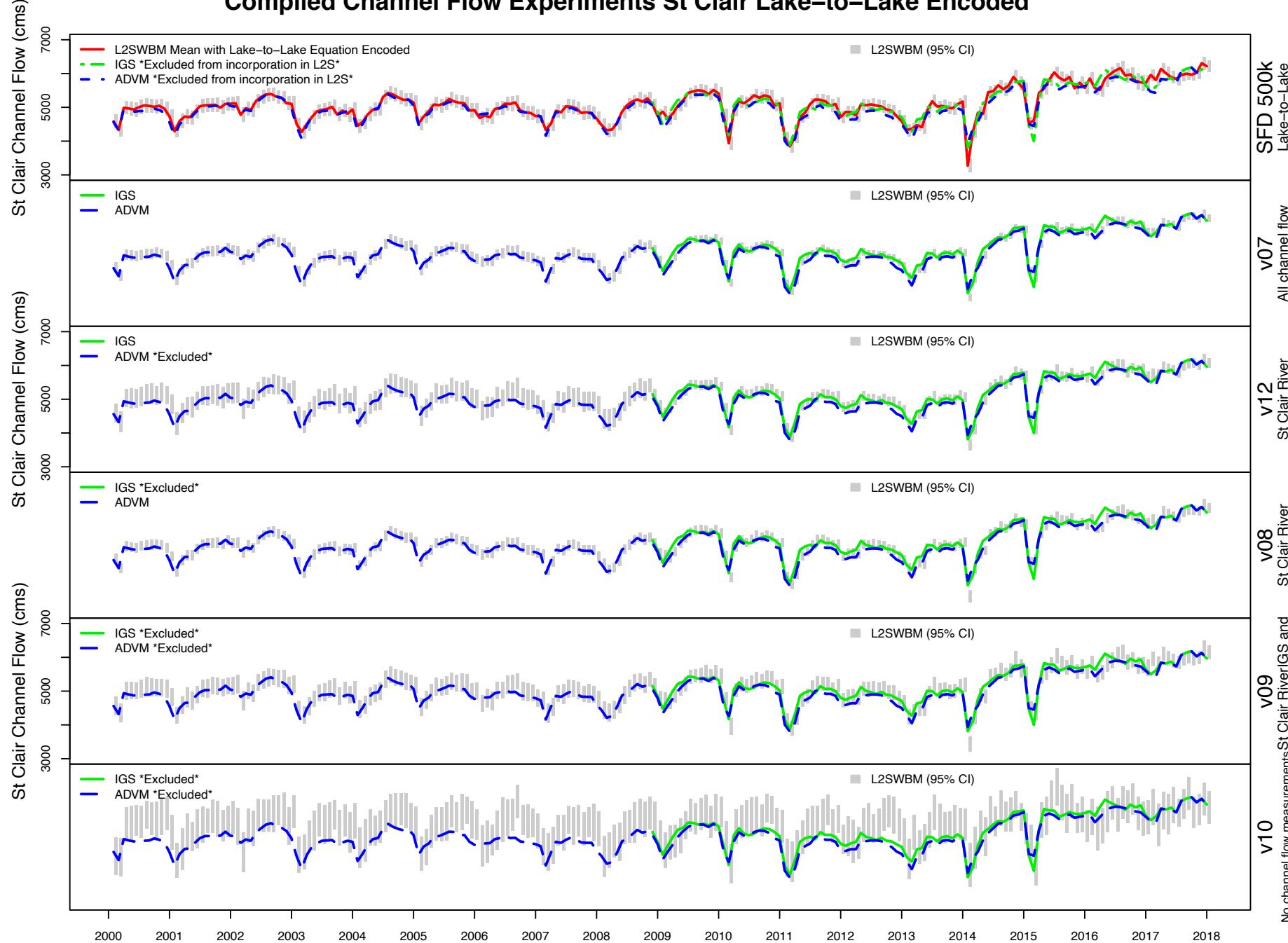
Iterations: 500,000

Chains: 3

- Assigned prior
- USACE-provided value
- Posterior distribution



## Compiled Channel Flow Experiments St Clair Lake-to-Lake Encoded



New result constrained by  
MH & STC BOM and Lake-  
to-Lake equation

Fully constrained by St  
Clair & Detroit River  
observations

Unconstrained by St Clair  
River observations

Unconstrained by St Clair  
& Detroit River  
observations

# Lake-to-Lake Routing Status

Priors for this run:

$c$  : mean = 250 ; SD = 50

$d$  : mean = 1.6 ; SD = 2

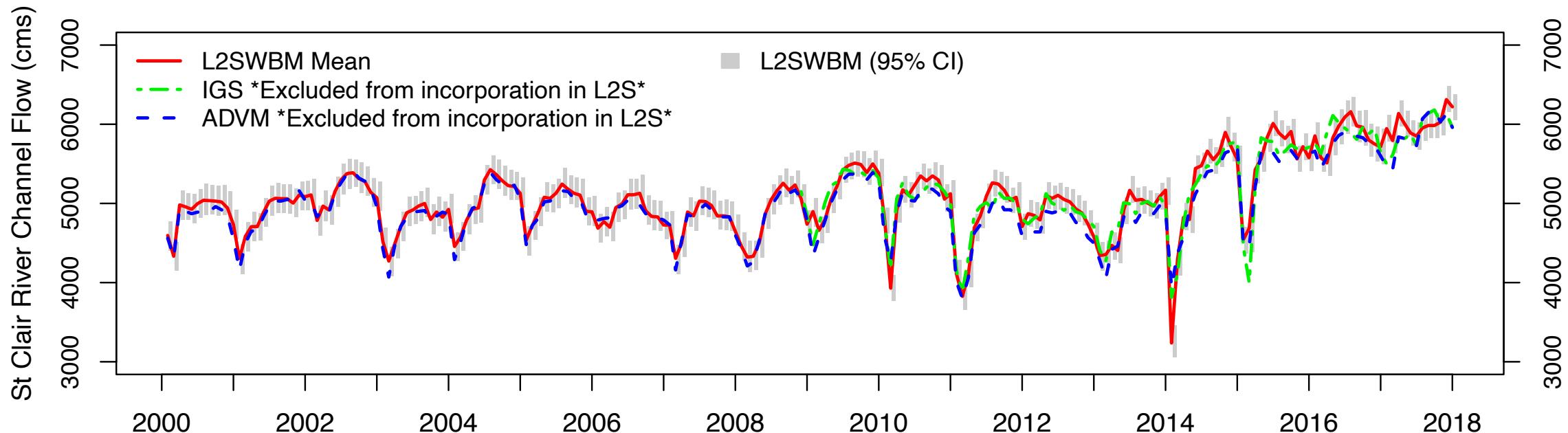
Posteriors for this run:

$c$  : mean = 249.7 ; 95% CI = 152.2 – 349.3

$d$  : mean = 1.41 ; 95% CI = 0.77 – 2.05

Iterations: 500,000

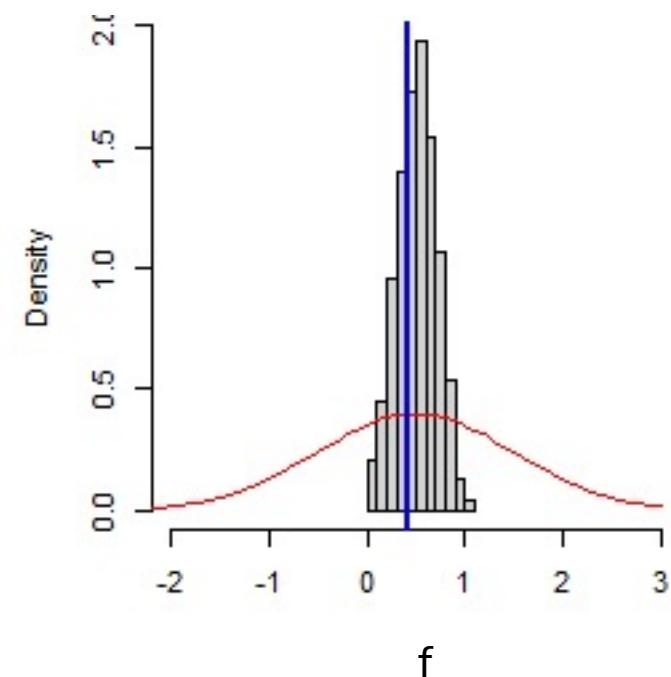
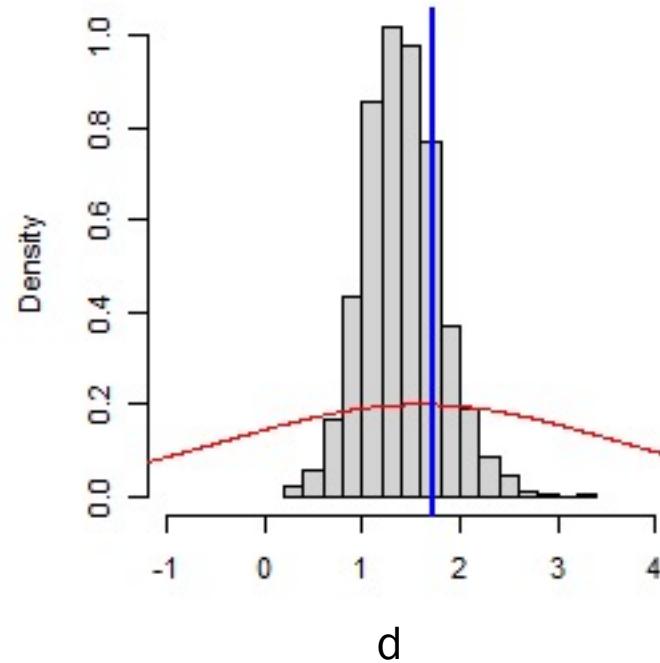
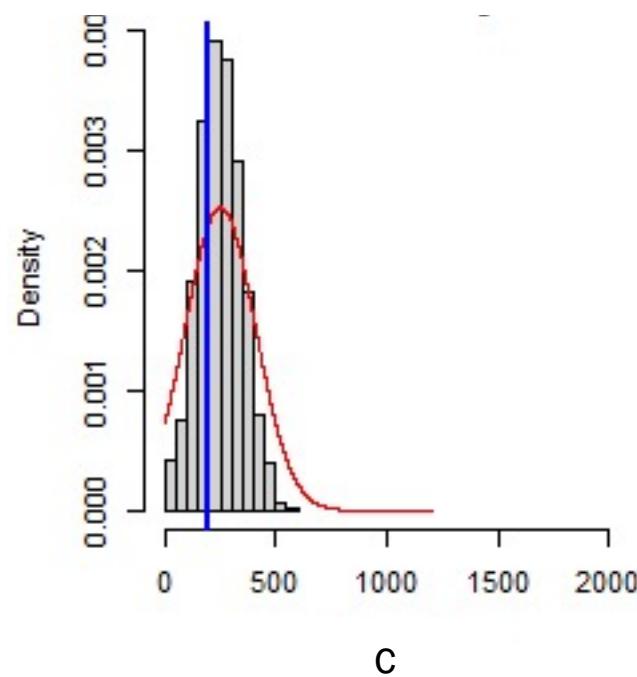
Chains: 3



# Lake-to-Lake Routing Status

\*\* Using a formulation of Lake-to-Lake equation incorporating fall  
→ Note: Small number of iterations, more testing needed!!

- Assigned prior
- USACE-provided value
- Posterior distribution





SCIENCE PRODUCTS NEWS CONNECT ABOUT

Latest Earthquakes |

JOHN WESLEY POWELL CENTER FOR ANALYSIS AND SYNTHESIS | SCIENCE

# Improving representation of groundwater in foundational Great Lakes hydrologic and hydrodynamic models and data sets

By [John Wesley Powell Center for Analysis and Synthesis](#) September 30, 2020

ACTIVE

[Overview](#) [Connect](#)

Groundwater plays a critical role in the water balance, however the groundwater component of the hydrologic cycle is frequently overlooked at basin scales because it is difficult to observe and quantify. We address this problem through a novel framework that combines existing hydrological models and data sets with groundwater flux estimates across Earth's largest system of lakes; the Laurentian Great Lakes. Aside from serving as a template for combining surface and ground water data and models, the Laurentian Great Lakes recently transitioned from a period characterized by water scarcity (water levels on the lakes were persistently below average from 1998 through 2013) to extreme water abundance (all-time high water levels were set in 2017 and 2019). Throughout this transition, we know of no comprehensive data record or modeling system that explicitly linked changes in observed land surface hydrology with critical subsurface groundwater processes. In addition to utilizing our novel framework for combining existing surface and ground water data sets and model simulations, we will also demonstrate the impact of ground water representation in existing lake physics models that serve as the basis for understanding lake evaporation, circulation patterns, and pollutant fate and transport dynamics.

## **Principal Investigators:**

Andrew D. Gronewold (*University of Michigan*)  
Sandra M. Eberts (*USGS - Water Mission Area*)  
Juliane Mai (*University of Waterloo*)  
Howard W. Reeves (*USGS - Upper Midwest Water Science Center*)

The Great Lakes from space.

## **Contacts**

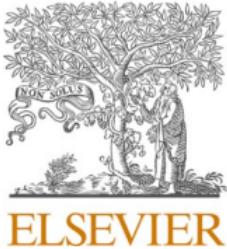
### **John Wesley Powell Center for Earth System Analysis and Synthesis**

Fort Collins Science Center (FORT)  
2150 Centre Ave Building C  
Fort Collins, CO 80526  
United States

Email: [powellcenter@usgs.gov](mailto:powellcenter@usgs.gov)  
Phone: [970-226-9103](tel:970-226-9103)

### **Howard W Reeves**

Research Hydrologist  
**Upper Midwest Water Science Center**  
Email: [hwreeves@usgs.gov](mailto:hwreeves@usgs.gov)  
Phone: [517-887-8914](tel:517-887-8914)



Contents lists available at [ScienceDirect](#)

# Journal of Hydrology

journal homepage: [www.elsevier.com/locate/jhydrol](http://www.elsevier.com/locate/jhydrol)



Research papers

## Investigating groundwater-lake interactions in the Laurentian Great Lakes with a fully-integrated surface water-groundwater model



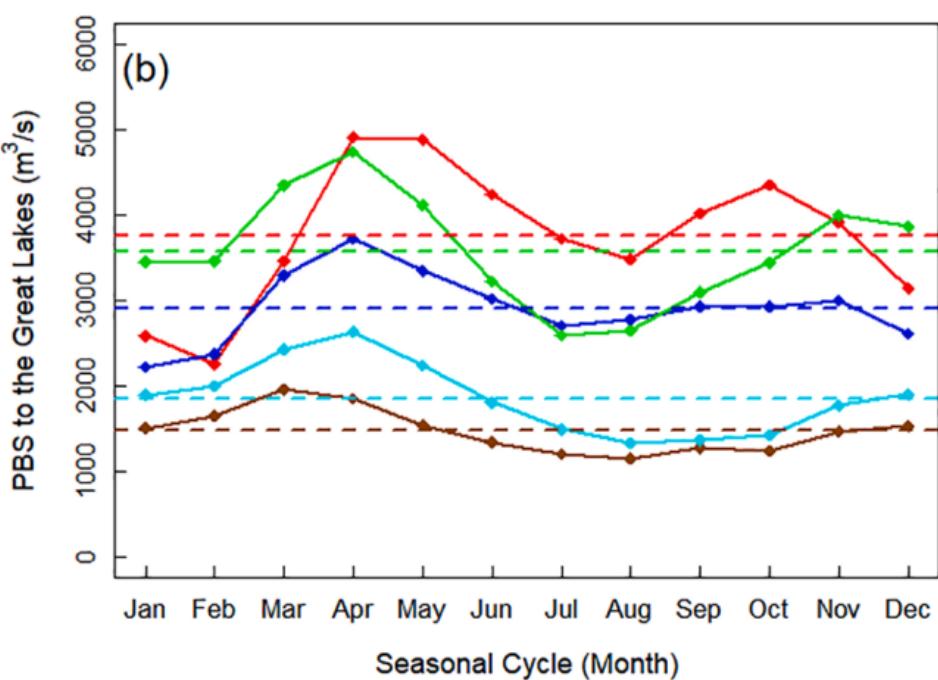
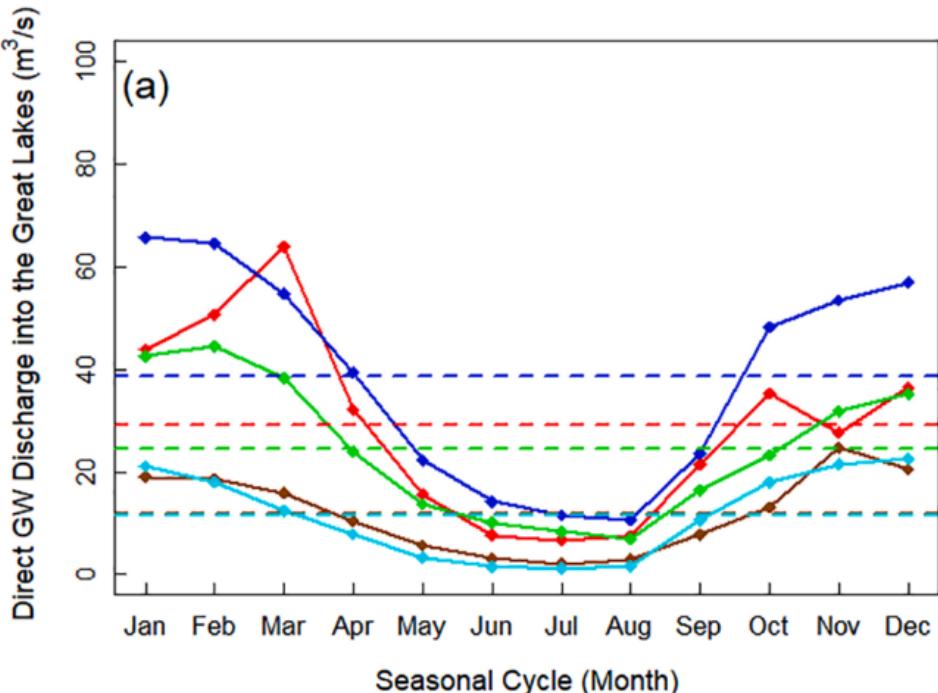
Shu Xu <sup>a</sup>, S.K. Frey <sup>a,b,\*</sup>, A.R. Erler <sup>a,d</sup>, O. Khader <sup>a</sup>, S.J. Berg <sup>a,b</sup>, H.T. Hwang <sup>a,b</sup>, M.V. Callaghan <sup>a</sup>, J.H. Davison <sup>c</sup>, E.A. Sudicky <sup>a,b</sup>

<sup>a</sup> Aquanty, 564 Weber St. N., Waterloo, ON N2L 5C6, Canada

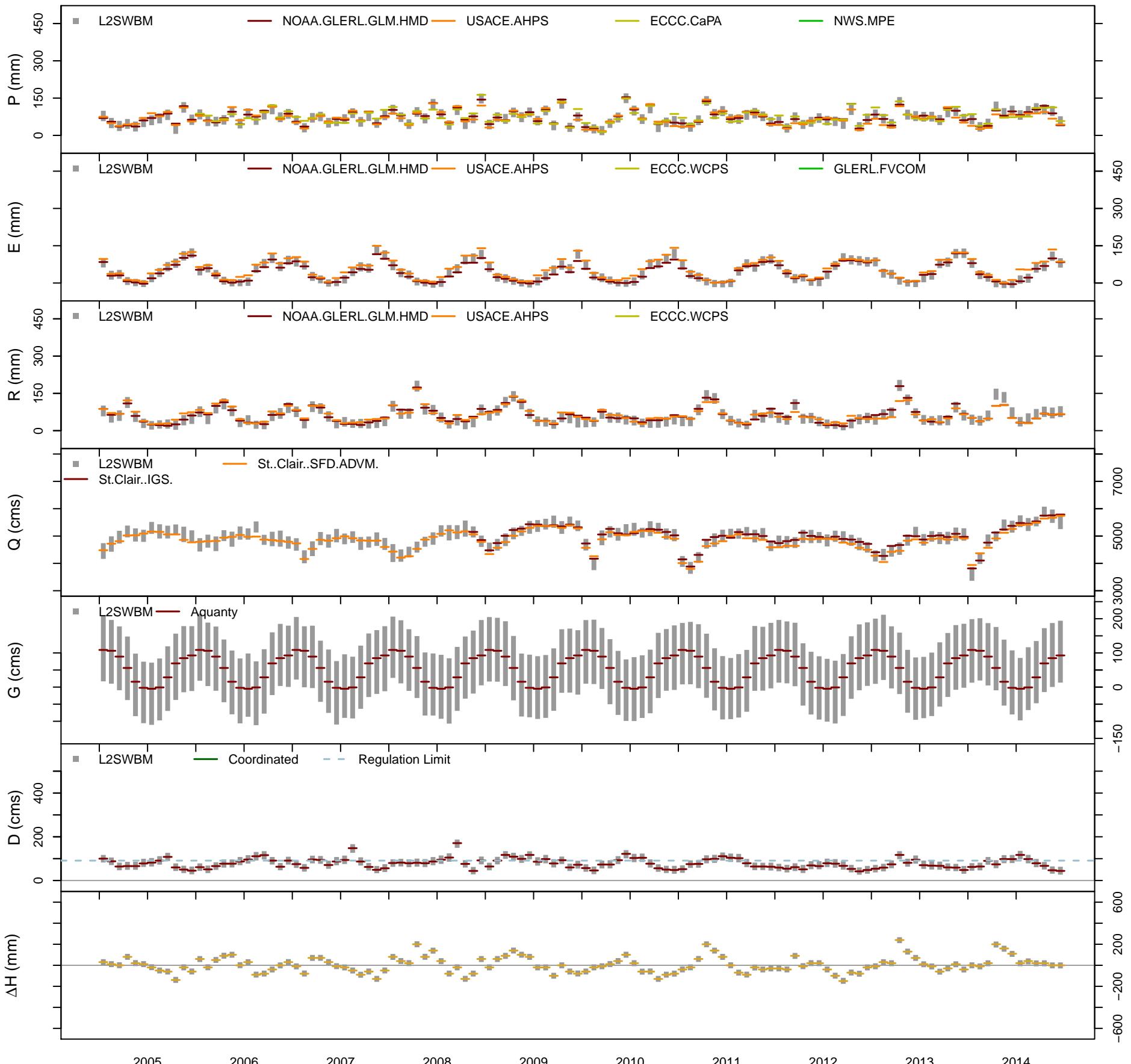
<sup>b</sup> University of Waterloo, Department of Earth and Environmental Sciences, 200 University Ave. W., Waterloo, ON N2L 3G1, Canada

<sup>c</sup> The Catholic University of America, Department of Civil and Environmental Engineering, 620 Michigan Ave., N.E., Washington, DC 20064, United States

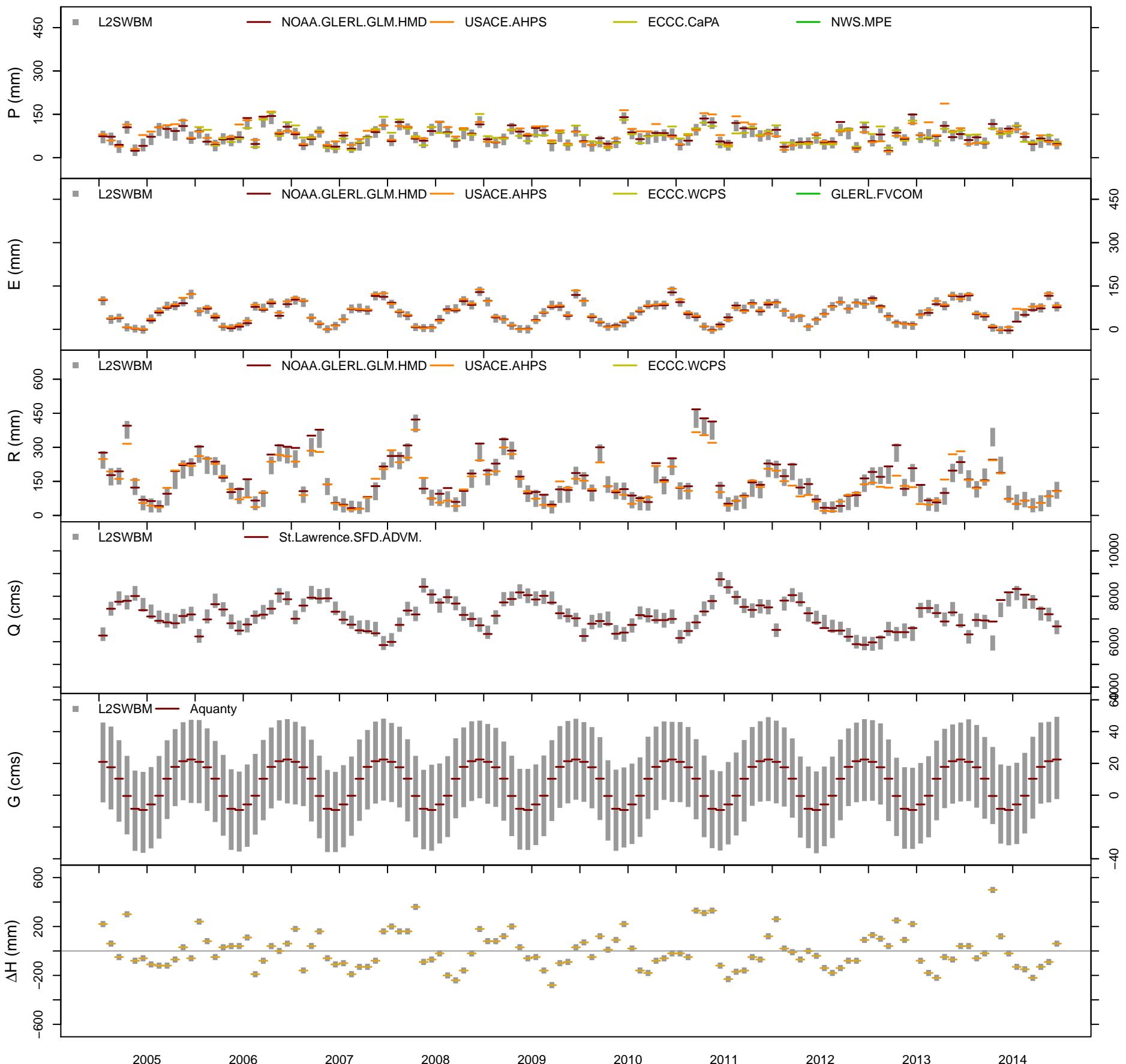
<sup>d</sup> University of Waterloo, Department of Geography and Environmental Management, 200 University Ave. W., Waterloo, ON N2L 3G1, Canada



### Michigan–Huron – posterior inferences



### Ontario – posterior inferences



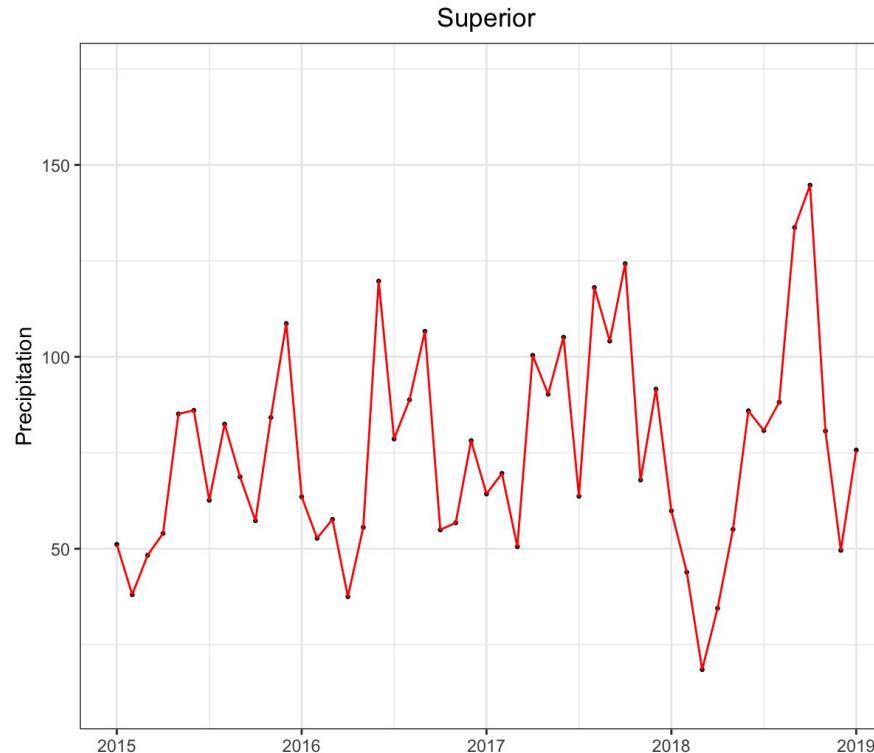
Lake Superior	Average magnitude	Average monthly estimate uncertainty	
Component of water balance	ft <sup>3</sup> /s	Low	High
Connecting-channel inflow	N/A	N/A	N/A
Connecting-channel outflow	79,118	5%	15%
Change in storage	N/A	0.12 in (3 mm)	0.47 in (12 mm)
Over-lake precipitation	73,047	15%	45%
Runoff	55,619	15%	35%
Evaporation	54,233	10%	35%
Long Lac diversion	1,412	10%	30%
Ogoki diversion	4,035	10%	30%
Ground water	Unknown	100% of 850 ft <sup>3</sup> /s	100% of 3,500 ft <sup>3</sup> /s
<b>Component NBS</b>	70,398	11%	31%
<b>Residual NBS</b>	73,671	4%	12%

Table extracted from: Neff, Brian P. and J. R. Nicholas, 2005, Uncertainty in the Great Lakes Water Balance, Date Posted: November 23, 2005: U.S. Geological Survey Scientific Investigations Report 2004-5100, 42 p. <https://pubs.water.usgs.gov/sir2004-5100/>

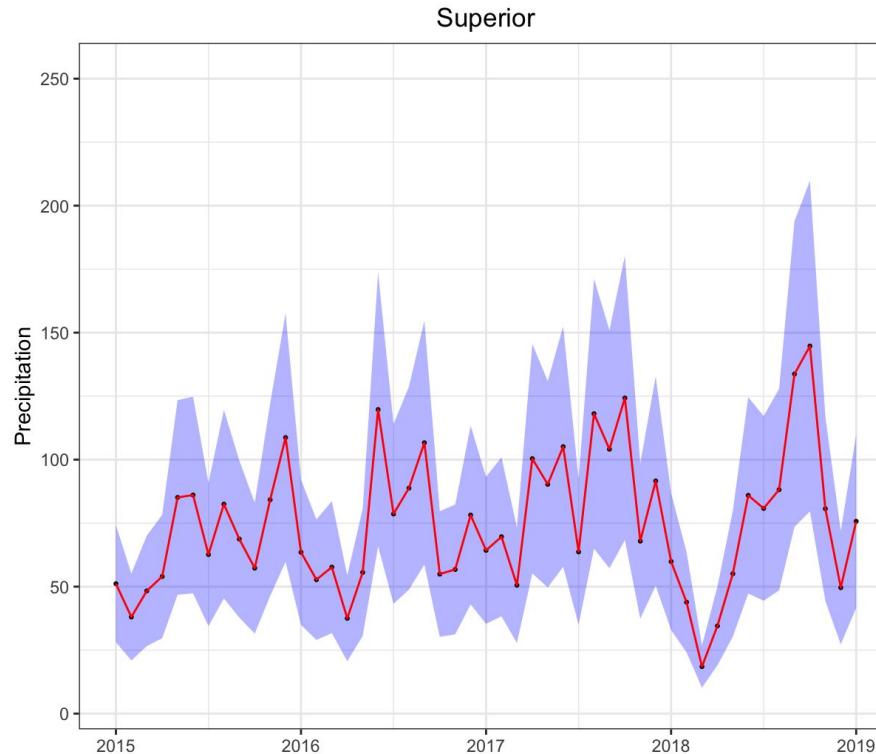
Lake Superior	Average magnitude	Average monthly estimate uncertainty	
Component of water balance	ft <sup>3</sup> /s	Low	High
Connecting-channel inflow	N/A	N/A	N/A
Connecting-channel outflow	79,118	5%	15%
Change in storage	N/A	0.12 in (3 mm)	0.47 in (12 mm)
Over-lake precipitation	73,047	15%	45%
Runoff	55,619	15%	35%
Evaporation	54,233	10%	35%
Long Lac diversion	1,412	10%	30%
Ogoki diversion	4,035	10%	30%
Ground water	Unknown	100% of 850 ft <sup>3</sup> /s	100% of 3,500 ft <sup>3</sup> /s
<b>Component NBS</b>	70,398	11%	31%
<b>Residual NBS</b>	73,671	4%	12%

Table extracted from: Neff, Brian P. and J. R. Nicholas, 2005, Uncertainty in the Great Lakes Water Balance, Date Posted: November 23, 2005: U.S. Geological Survey Scientific Investigations Report 2004-5100, 42 p. <https://pubs.water.usgs.gov/sir2004-5100/>

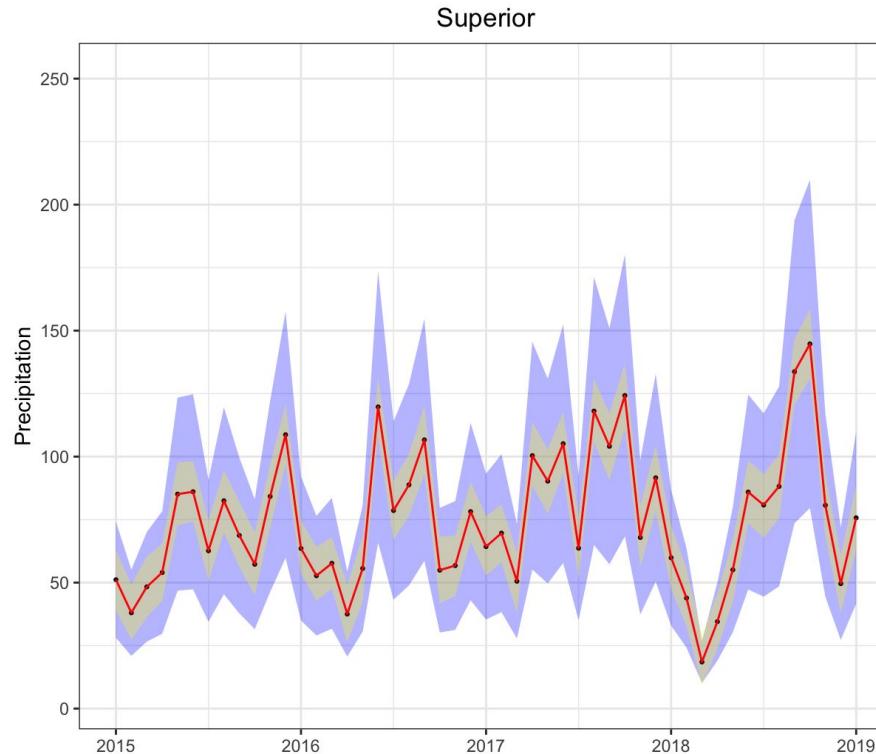
# L2SWBM output: Lake Superior precipitation (2015-2019)

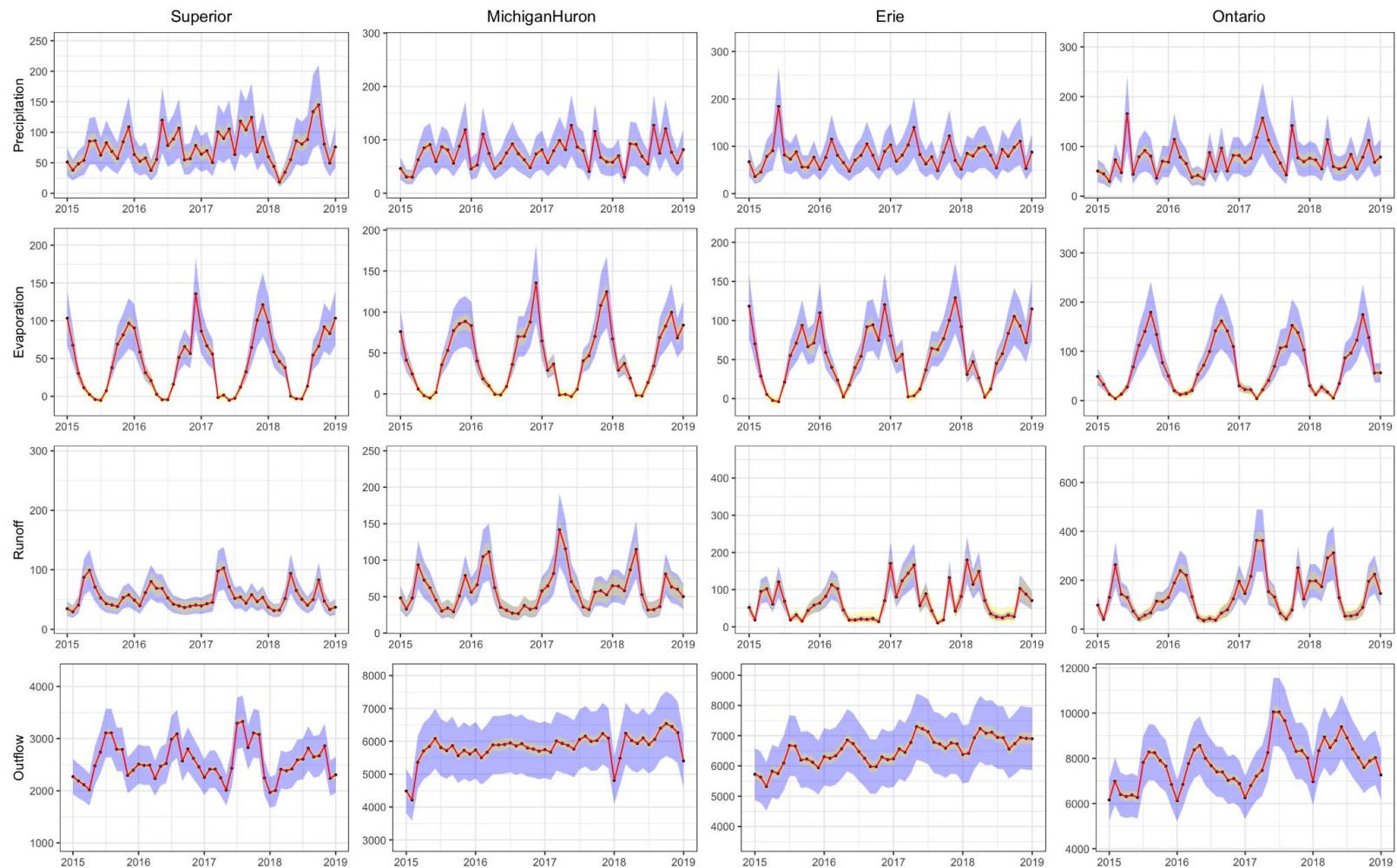


# L2SWBM output: Lake Superior precipitation (2015-2019)

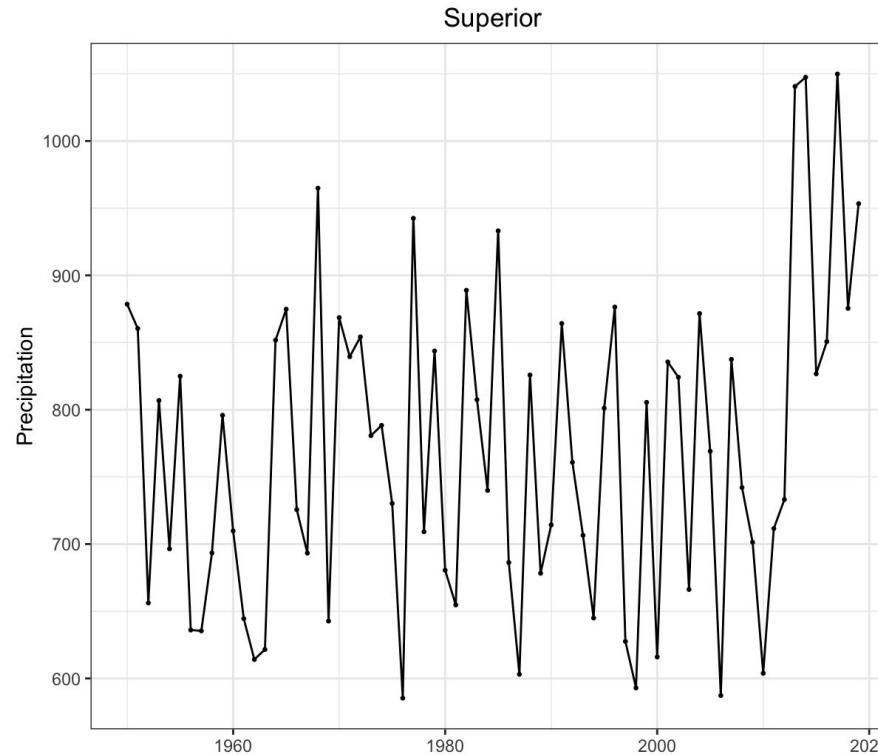


# L2SWBM output: Lake Superior precipitation (2015-2019)



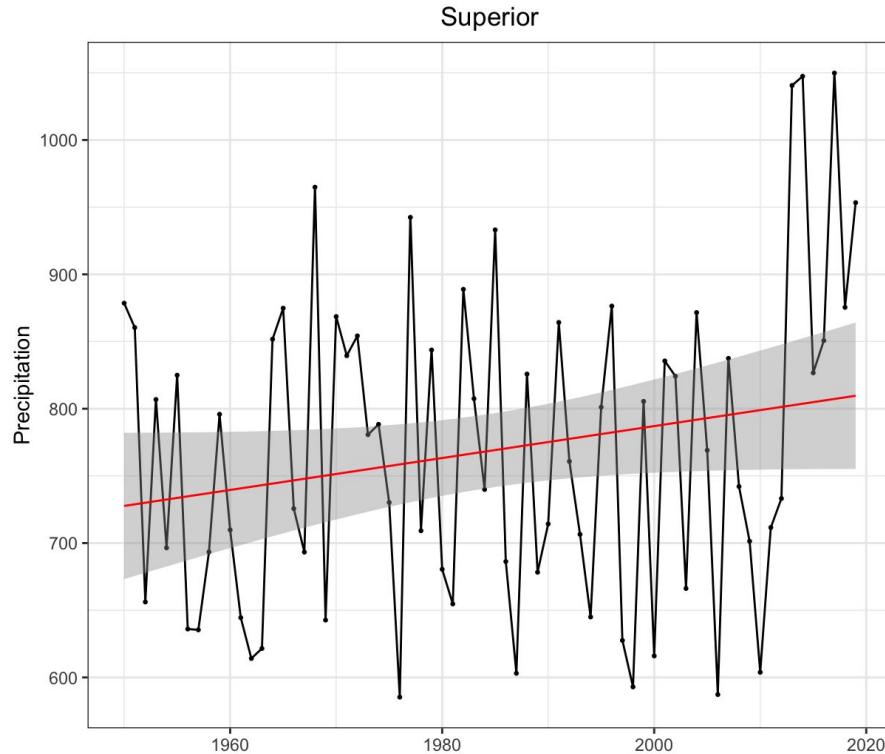


# Lake Superior annual precipitation (1950-2019)



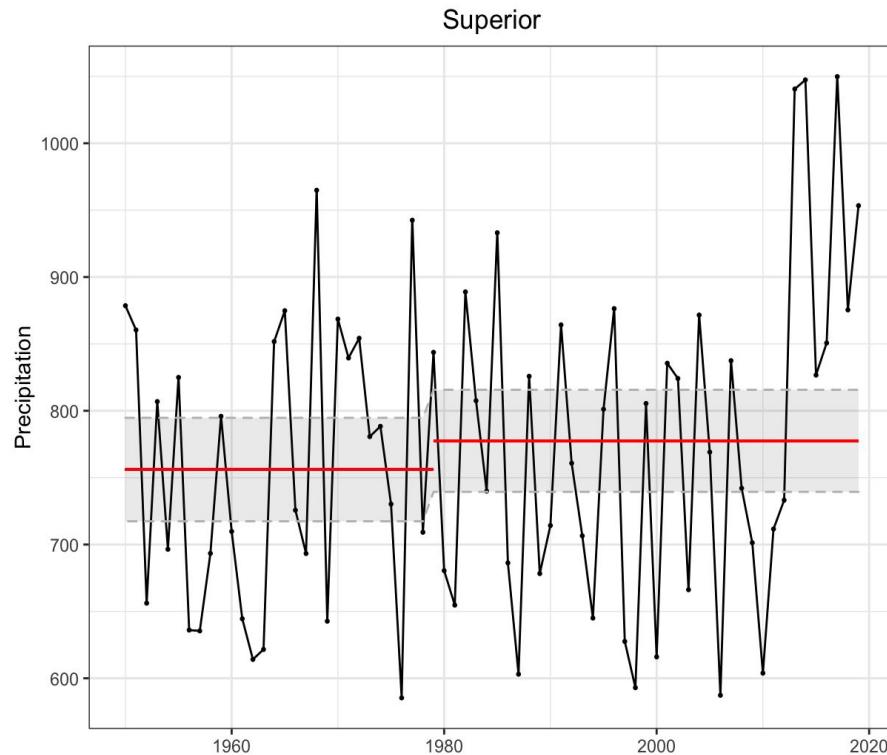
# Method 1: Linear regression

- A simple and quick process of finding a line that best fits the data
- Determines the underlying trends, and can be used to forecast future water balance



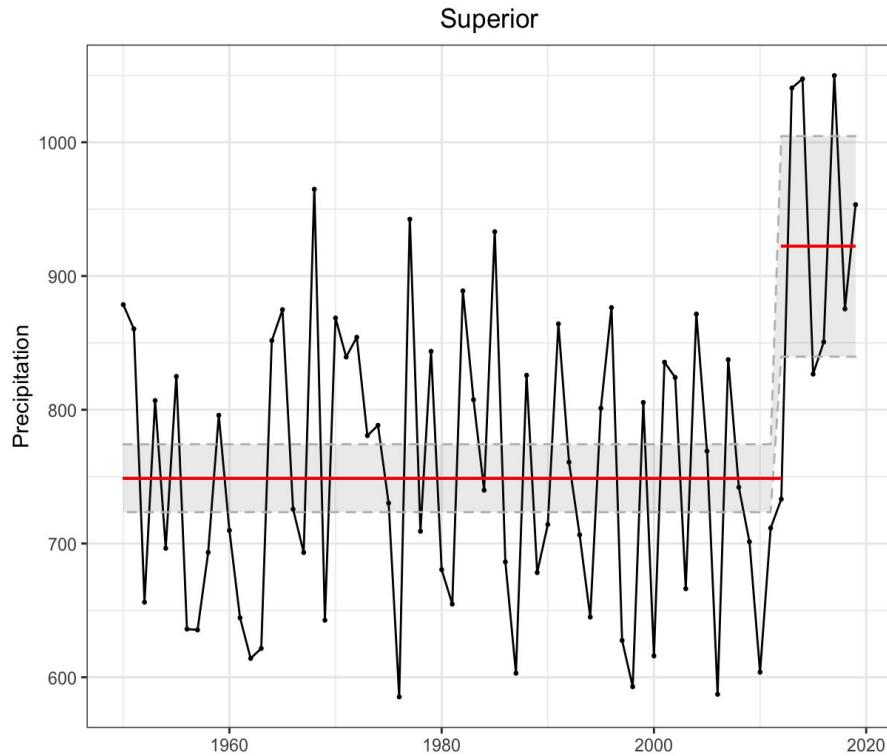
# Method 2a: Fixed reference period (1950-1979 vs 1980-present)

- Filters out interannual variation or anomalies
- Shows longer climatic trends



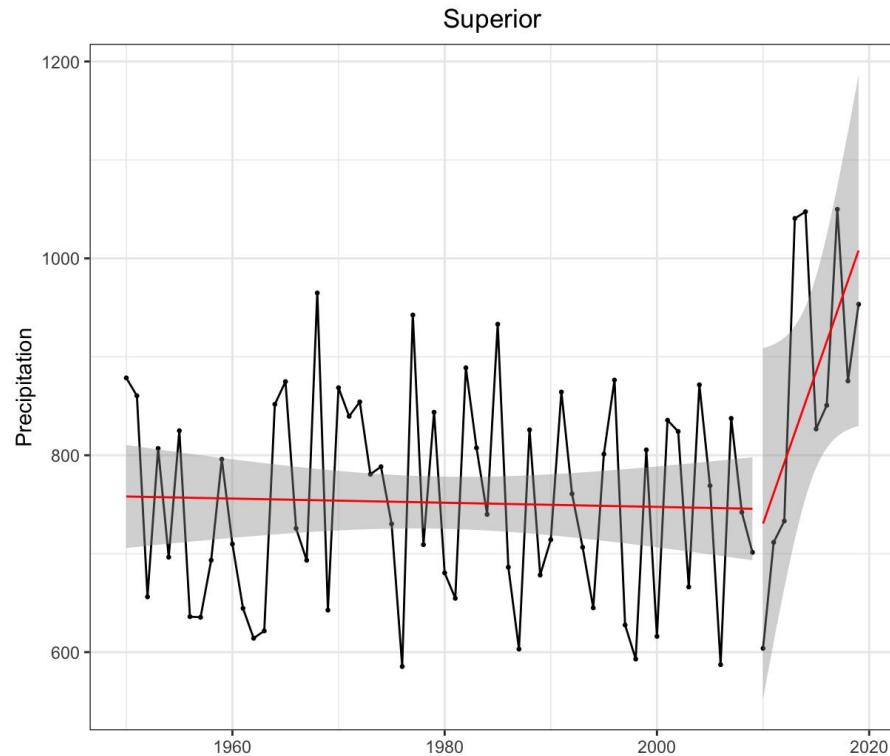
# Method 3: Change point detection

- Automatically detects where a statistically significant shift in the data occurs
- Divides a time series into two segments with distinct statistical characteristics



# Method 4: “Hockey stick” segmented regression

- Automatically selects the “best” breakpoint where there is a significant trend change over time
- Widely used in estimating intervention effects on interrupted time series studies



# Outline

## 1 Introduction

## 2 Day 1 (Monday) - L2SWBM Technical Updates

- L2SWBM background and technical overview
- Recent and ongoing research projects
- Version control and code repositories
- Reality check (under the hood)

## 3 Day 2 (Tuesday) - Copulas and Federal Perspectives

- Using Copulas
- Federal Agency Partners

## 4 Day 3 (Wednesday) - Beyond the (Laurentian) Great Lakes

- African Great Lakes
- Mono Lake
- NSF large lakes proposal (and other ideas)

# Outline

## 1 Introduction

## 2 Day 1 (Monday) - L2SWBM Technical Updates

- L2SWBM background and technical overview
- Recent and ongoing research projects
- Version control and code repositories
- Reality check (under the hood)

## 3 Day 2 (Tuesday) - Copulas and Federal Perspectives

- Using Copulas
- Federal Agency Partners

## 4 Day 3 (Wednesday) - Beyond the (Laurentian) Great Lakes

- African Great Lakes
- Mono Lake
- NSF large lakes proposal (and other ideas)

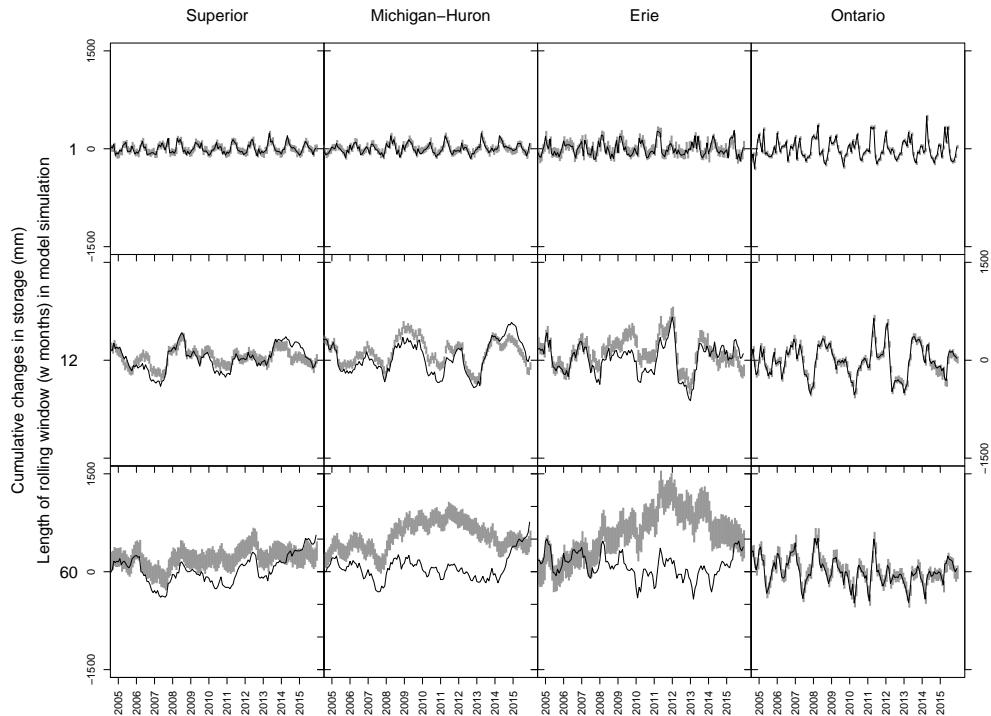


Figure 3: Water balance closure assessment for the version of our model configured with a 1-month inference window. Simulated changes in storage from the model are presented as grey bands (95% posterior predictive intervals) over cumulative one month (top row), 12 month (middle row), and 60 month periods. Observed cumulative changes in storage over corresponding time horizons are represented by a black line.

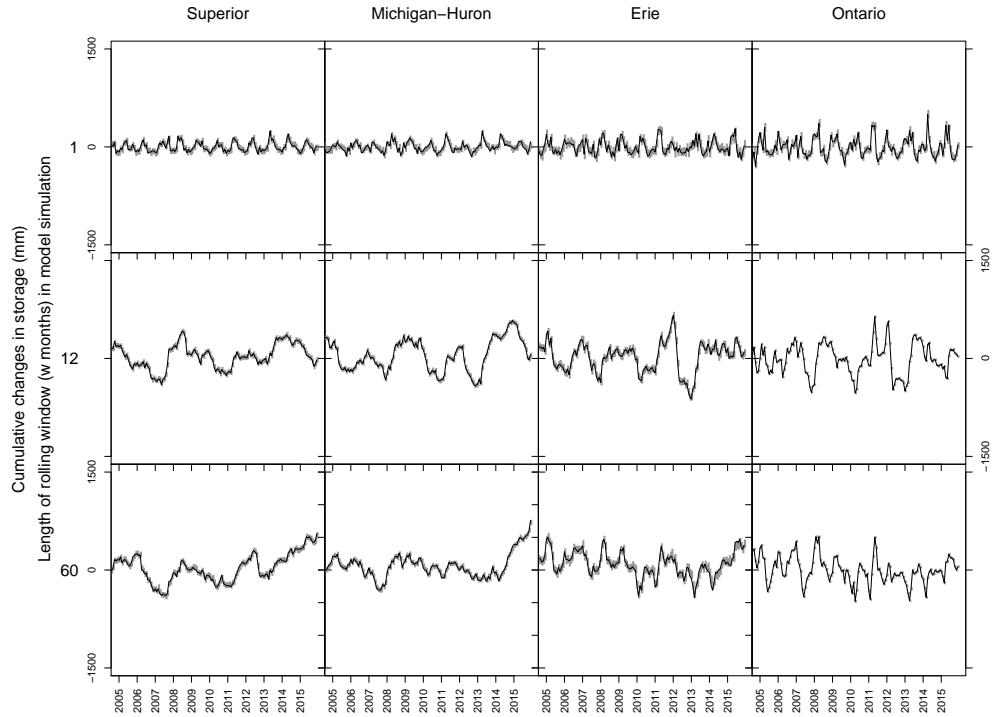


Figure 4: Water balance closure assessment for the version of our model configured with a 12-month inference window. Simulated changes in storage from the model are presented as grey bands (95% posterior predictive intervals) over cumulative one month (top row), 12 month (middle row), and 60 month periods. Observed cumulative changes in storage over corresponding time horizons are represented by a black line. Vertical axis scale in each panel is the same as in figure 3 to facilitate comparison.

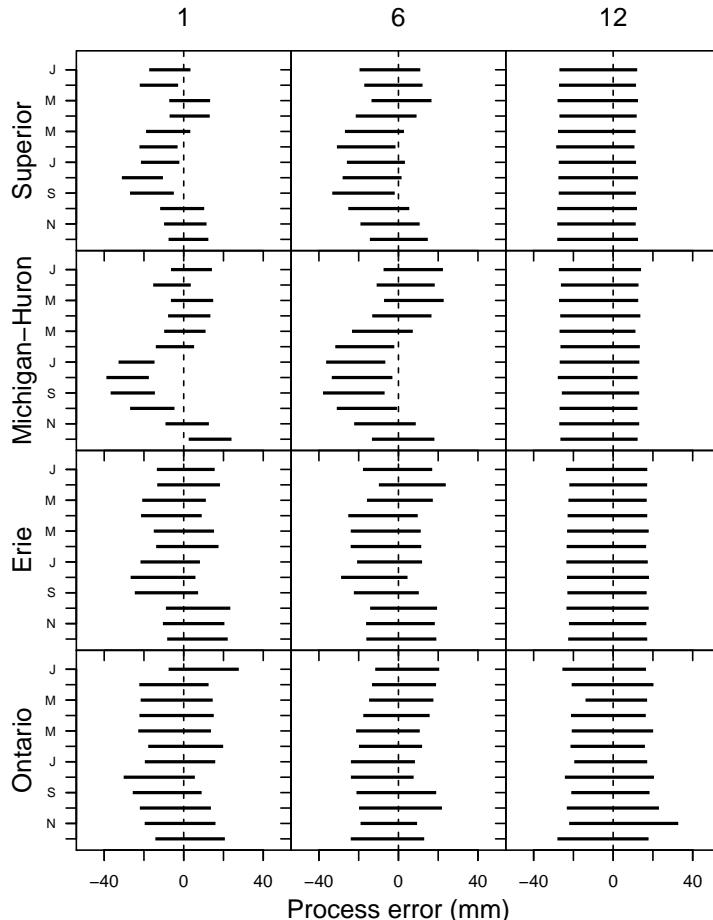


Figure 2: 95% credible intervals for model process errors  $\epsilon_{ct}$  from model configurations with a 1-month inference window (left), 6-month inference window (center), and a 12-month inference window (right).

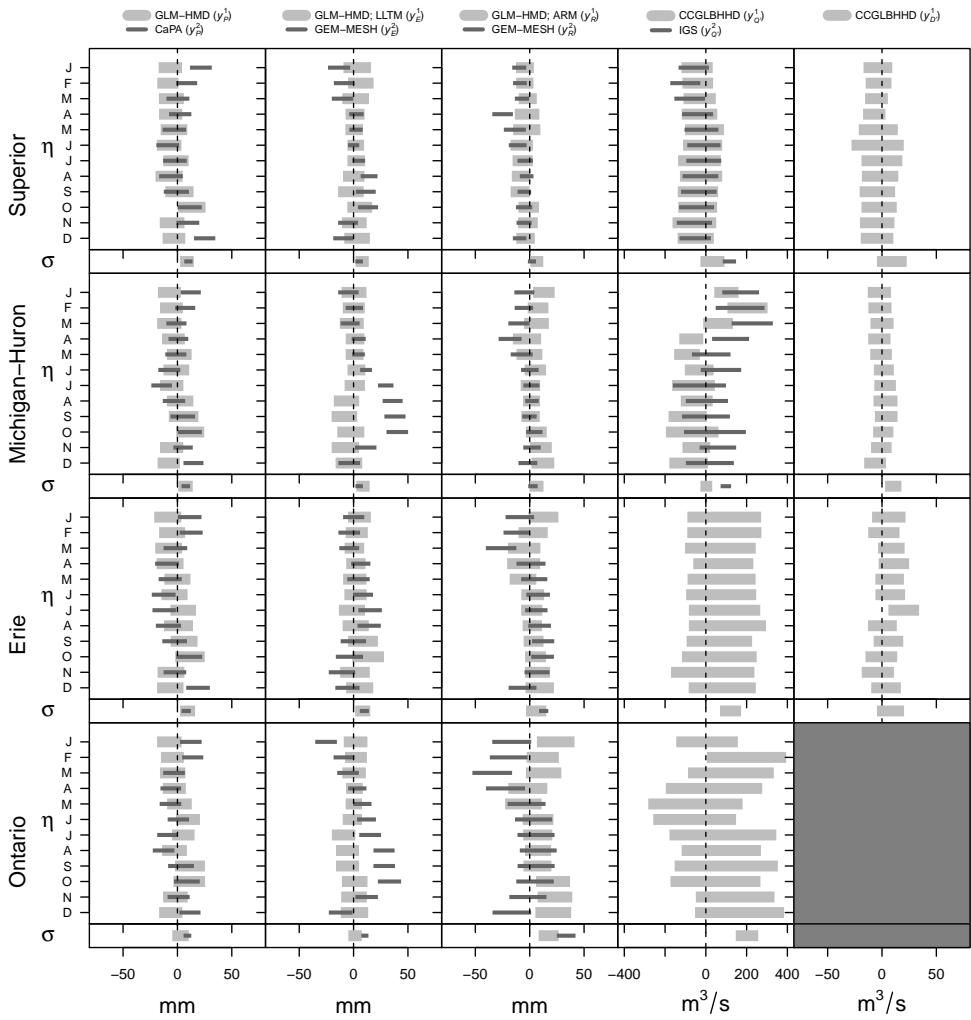
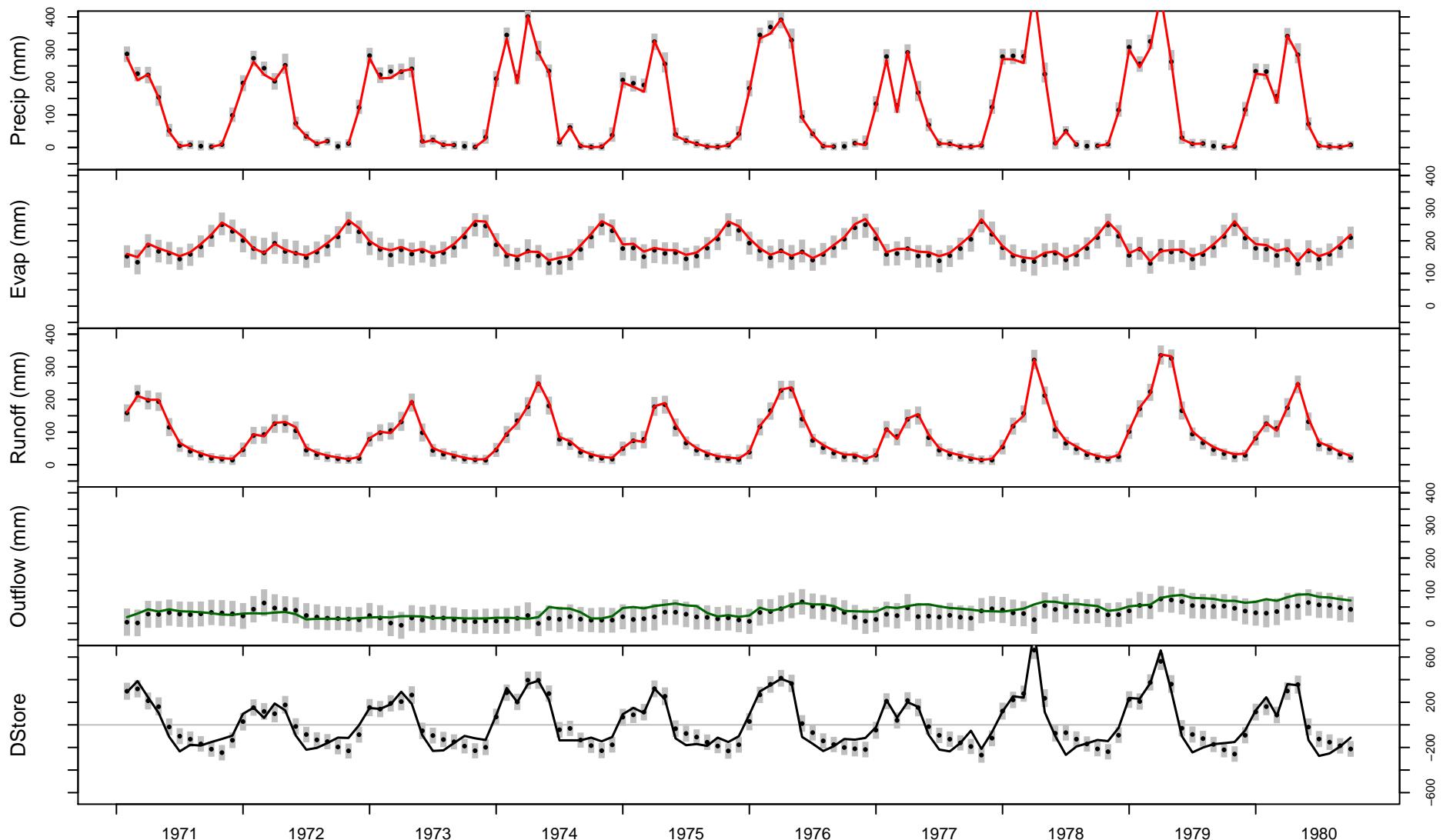


Figure 6: Inferred bias ( $\eta$ ) and error ( $\sigma$ ) in historical water balance component data based a configuration of our model with a 12-month inference window. Columns, from left to right, correspond to monthly data for over-lake precipitation, over-lake evaporation, lateral tributary runoff, outflow, and interbasin diversions. Horizontal bars represent 95% credible intervals. For details on data sources, see table A1.

### Malawi Time Series



# Outline

## 1 Introduction

## 2 Day 1 (Monday) - L2SWBM Technical Updates

- L2SWBM background and technical overview
- Recent and ongoing research projects
- Version control and code repositories
- Reality check (under the hood)

## 3 Day 2 (Tuesday) - Copulas and Federal Perspectives

- Using Copulas
- Federal Agency Partners

## 4 Day 3 (Wednesday) - Beyond the (Laurentian) Great Lakes

- African Great Lakes
- Mono Lake
- NSF large lakes proposal (and other ideas)