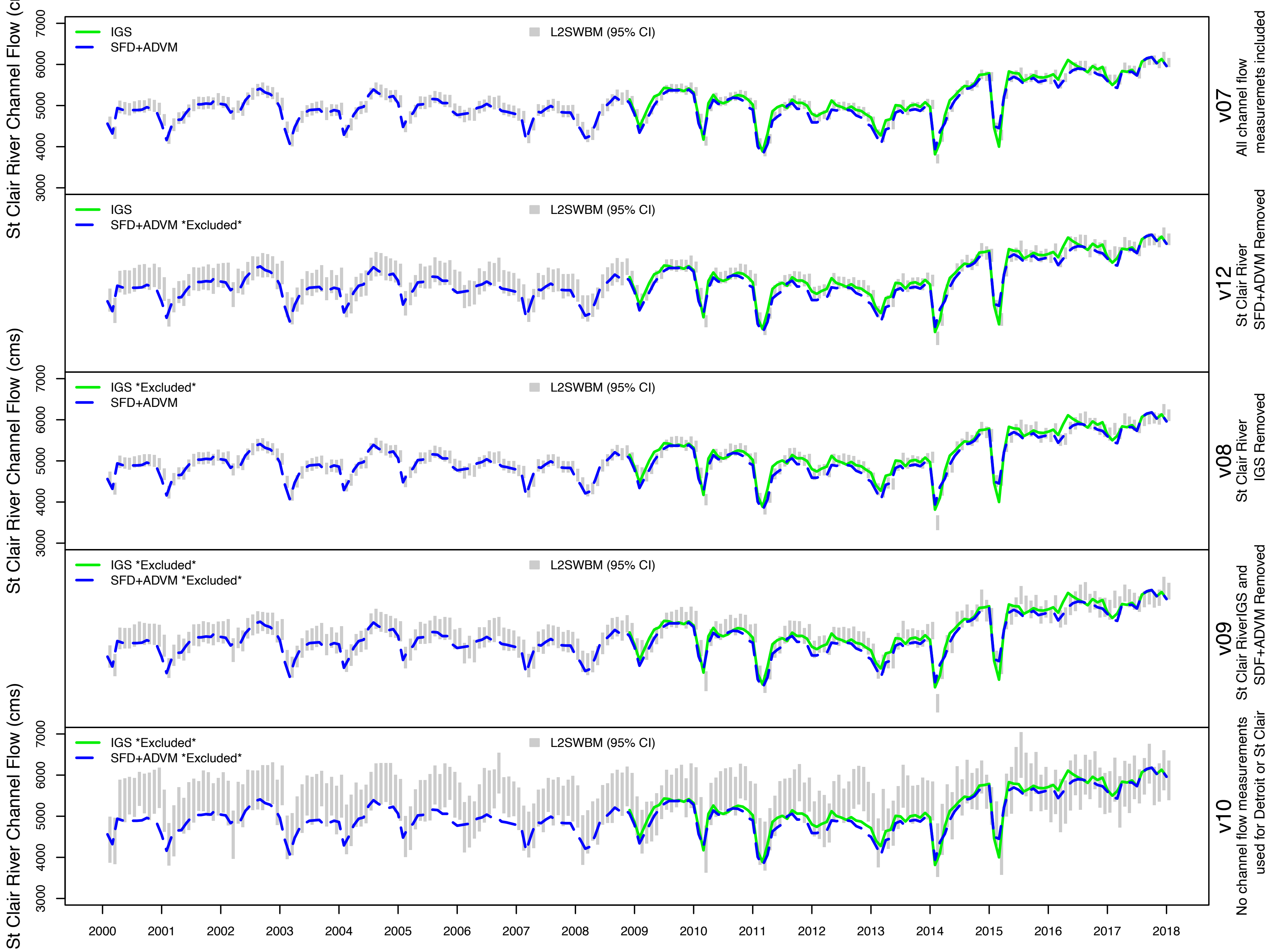
Dataset Assimilation Tests (historical Q datasets)

All experiments run with 200,000 iterations; 2 Chains
Prior: 1982 – 1999 ; Posterior: 2000 – 2017

Run ID	St. Clair River IGS	St. Clair River ADVM	Detroit River IGS	Detroit River ADVM	Description of Notable Run Conditions
V07					All channel flow observations incorporated
V08					
V09					No observations incorporated for St Clair River
V10					No channel flow observations incorporated
V11					
V12					
V13					
V14					No observations incorporated for Detroit River

Dataset incorporated into L2SWBM   Dataset *not* incorporated into L2SWBM

St Clair River Channel Flow Experiments



Fully constrained by St Clair & Detroit River observations

Unconstrained by St Clair River observations

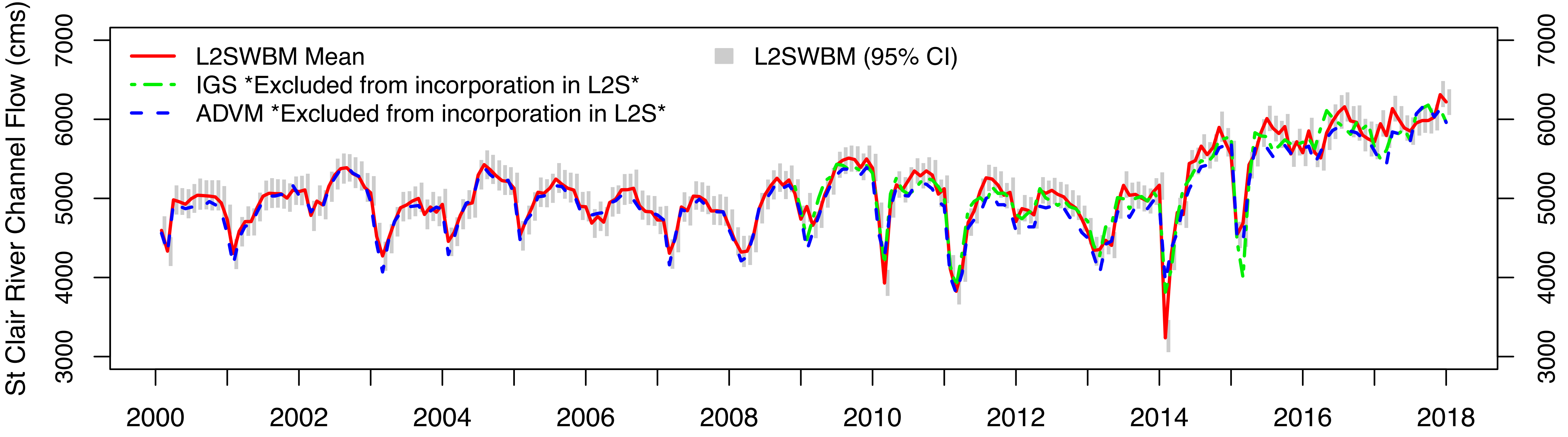
Unconstrained by St Clair & Detroit River observations

Lake-to-Lake Routing Status

Priors for this run:
c : mean = 250 ; SD = 50
d : mean = 1.6 ; SD = 2

Posteriors for this run:
c : mean = 249.7 ; 95% CI = 152.2 – 349.3
d : mean = 1.41 ; 95% CI = 0.77 – 2.05

Iterations: 500,000
Chains: 3



Project Objectives

Funding from the US Army Corps of Engineers
Funding Opportunity No.: W81EWF-20-SOI-0028
CFDA No.: 12.630

"...Focus on quantifying variability and uncertainty over time in the Detroit River and St. Clair River discharge estimates."

(source: funding proposal)

1. <u>Sensitivity testing</u>: Understand impact of assimilating different combinations of **historical flow measurements** on estimated Q.
2. <u>Directly Encode Lake-to-Lake Routing Equations</u>: Generate new, **independent estimates of Q** constrained <u>only</u> by the water balance.
3. <u>Estimate coefficients for Lake-to-Lake routing equations:</u> Faithful to the Great Lakes water balance.
4. <u>Assess and compare new machine learning datasets</u>: **Assimilate new USACE estimates of flow** & compare to estimates from water balance.

APPENDIX A - Data

This Appendix includes a summary (Table A1) of data sources for populating water balance component "observations" (y) and for calculating prior probability distribution hyperparameters.

Variable or parameter	Data source and reference(s)	Years used
$y_{\Delta H}$	CCGLBHHD (Gronewold et al., 2018)	1980 - 2015
y_P^1	GLM-HMD (Hunter et al., 2015)	1980 - 2015
y_P^2	CaPA (Lespinas et al., 2015)	2006 - 2015
μ_P, $\mu_{ln(P)}$	GLM-HMD (Hunter et al., 2015)	1950 - 1979
y_E^1	GLM-HMD; LLTM (Hunter et al., 2015)	1980 - 2015
y_E^2	GEM-MESH (Deacu et al., 2012)	2004 - 2014
μ_E	GLM-HMD; LLTM (Hunter et al., 2015)	1950 - 1979
y_R^1	GLM-HMD; ARM (Hunter et al., 2015)	1980 - 2015
y_R^2	GEM-MESH (Superior and Michigan-Huron)	2004 (June) - 2009
y_R^2	GEM-MESH (Erie and Ontario)	2004 (June) - 2013
$\mu_{ln(R)}$	GLM-HMD; ARM (Hunter et al., 2015)	1950 - 1979
$y_{NBS'}^1$	GLM-HMD (Hunter et al., 2015)	1980 - 2015
$y_{NBS'}^2$	GEM-MESH (Deacu et al., 2012)	2004 (June) - 2012
$y_{NBS'}^3$	CCGLBHHD Residual (Gronewold et al., 2018)	1980 - 2015
$\mu_{NBS'}$	CCGLBHHD Residual (Gronewold et al., 2018)	1950 - 1979
$y_{Q'}^1$	CCGLBHHD (Gronewold et al., 2018)	1980 - 2015
$y_{Q'}^2$	IGS (for St. Marys, St. Clair, and Detroit Rivers only)	2008 (Nov) - 2014
$\mu_{Q'}$	CCGLBHHD (Gronewold et al., 2018)	1950 - 1979
$y_{D'}$	CCGLBHHD (Gronewold et al., 2018)	1980 - 2015
$\mu_{D'}$	CCGLBHHD (Gronewold et al., 2018)	1950 - 1979

Table A1: Summary of data sets used in our study. Unless indicated otherwise, date ranges include the entire calendar year. Variable definitions are included in table B1.

Lake, Channel Flow or Diversion	λ
Superior Outflow (St. Marys River)	0.02
Superior Diversion (Ogoki, Long-Lac)	0.04
Michigan-Huron Outflow (St. Clair River)	0.03
Michigan-Huron Diversion (Chicago)	0.04
St. Clair Outflow (Detroit River)	0.03
Erie Outflow (Niagara)	0.02
Erie Diversion (Welland Canal)	0.04
Ontario Outflow (St. Lawrence River)	0.02

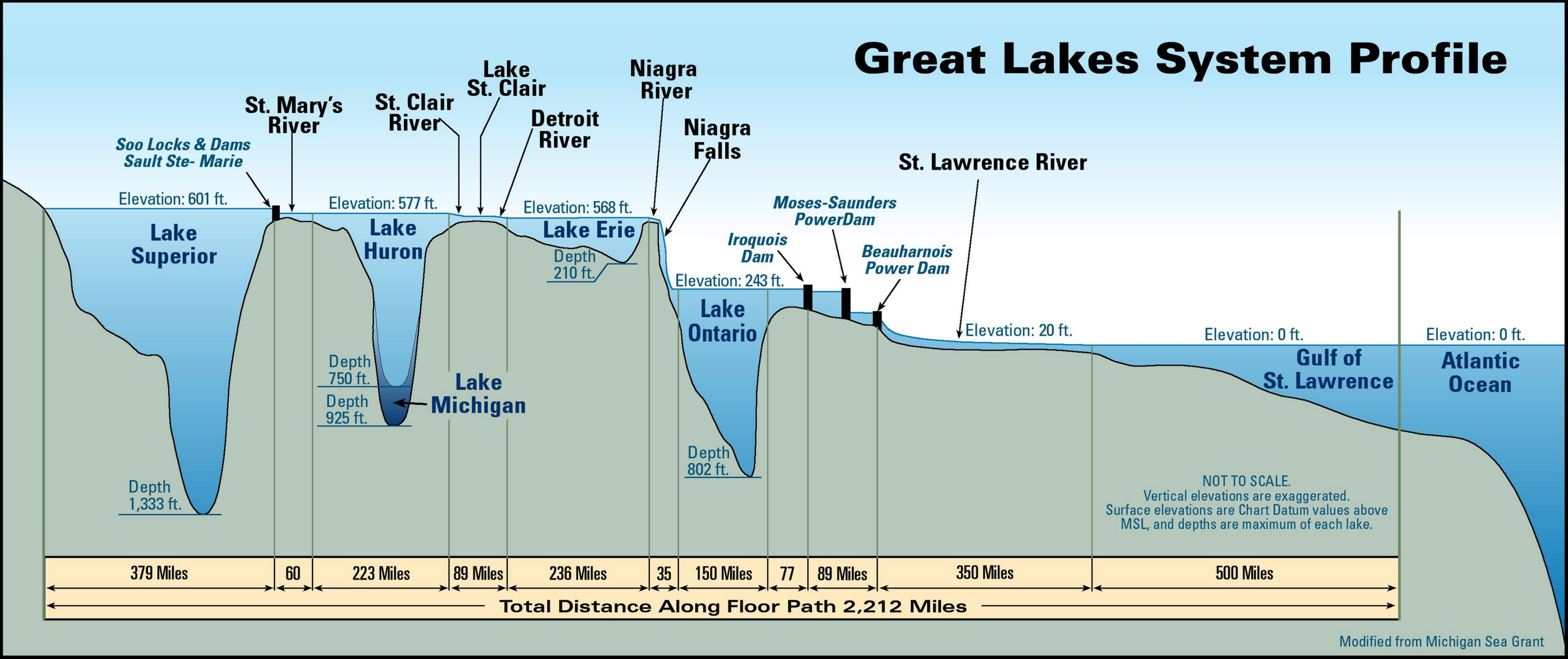
Table 2: Coefficients of variation for empirical estimation of prior standard deviation on the bias in historical data sources for the Great Lakes.

Symbol	Description
Indices and related variables	
$c(t)$	Calendar month $c \in [1, 12]$ of time step t
i	Index for months within a water balance window of length w
j	Index $j \in [1, T-w+1]$ for the first month of a rolling window
l	Index for an individual lake; $l \in [SUP, MHU]$
n	Index of data sources for a particular water balance component; $n \in [1, N]$
N	Total number of data sources for a water balance component (in this study, typically 2)
t	Index for month number in the sequence $[1, T]$
T	Total number of months in study. Here, $T = 120$ (January 2005 through December 2014)
w	Length of rolling window (in months) for water balance inference
"True" (unobserved) monthly average water balance components (all in mm over lake surface)	
$\Delta H_{l,j,w}$	Change in water level for lake l from beginning of month j to beginning of month $j+w$
$D_{l,t}$	Monthly diversion from lake l in month t
$E_{l,t}$	Evaporation from lake l in month t
$I_{l,t}$	Connecting channel inflow for lake l in month t
$P_{l,t}$	Precipitation over lake l in month t
$Q_{l,t}$	Connecting channel outflow for lake l in month t
$R_{l,t}$	Basin runoff into lake l in month t
θ	A parameter from Θ
Θ	The parameter set P, E, R, Q, D
Data and model-based estimates (subscript l removed from each for clarity)	
$y_{H,t}$	Water level measurement at beginning of month t
$y_{\Delta H,j,w}$	Observed water level difference from beginning of month j to beginning of month $j+w$
$y^n_{\theta,t}$	n^{th} estimate of θ_t
Prior probability distribution parameters (subscripts l and $c(t)$ removed from each for clarity)	
μ_E, μ_Q, μ_D	Historical empirical mean of E, Q, and D from the GLM-HMD
$\mu_{ln(R)}$	Historical empirical log-mean of R from the GLM-HMD
τ_E, τ_Q, τ_D	Historical empirical precision of E, Q, and D from the GLM-HMD
$\tau_{ln(R)}$	Historical empirical precision of natural logarithm of R from GLM-HMD
ψ^1, ψ^2	Shape and rate parameters for $\pi(P)$
Hyperparameters (subscripts l removed for clarity)	
$\epsilon_t = \epsilon_{c(t)}$	Water balance model process error for calendar month $c(t)$
$\eta^n_{\theta,c(t)}$	Seasonal bias of $y^n_{\theta,t}$ by calendar month $c(t)$
$\tau_{\Delta H,w}$	Precision of $y_{\Delta H,j,w}$ for all j
τ^n_θ	Precision of $y^n_{\theta,t}$ (for all t)

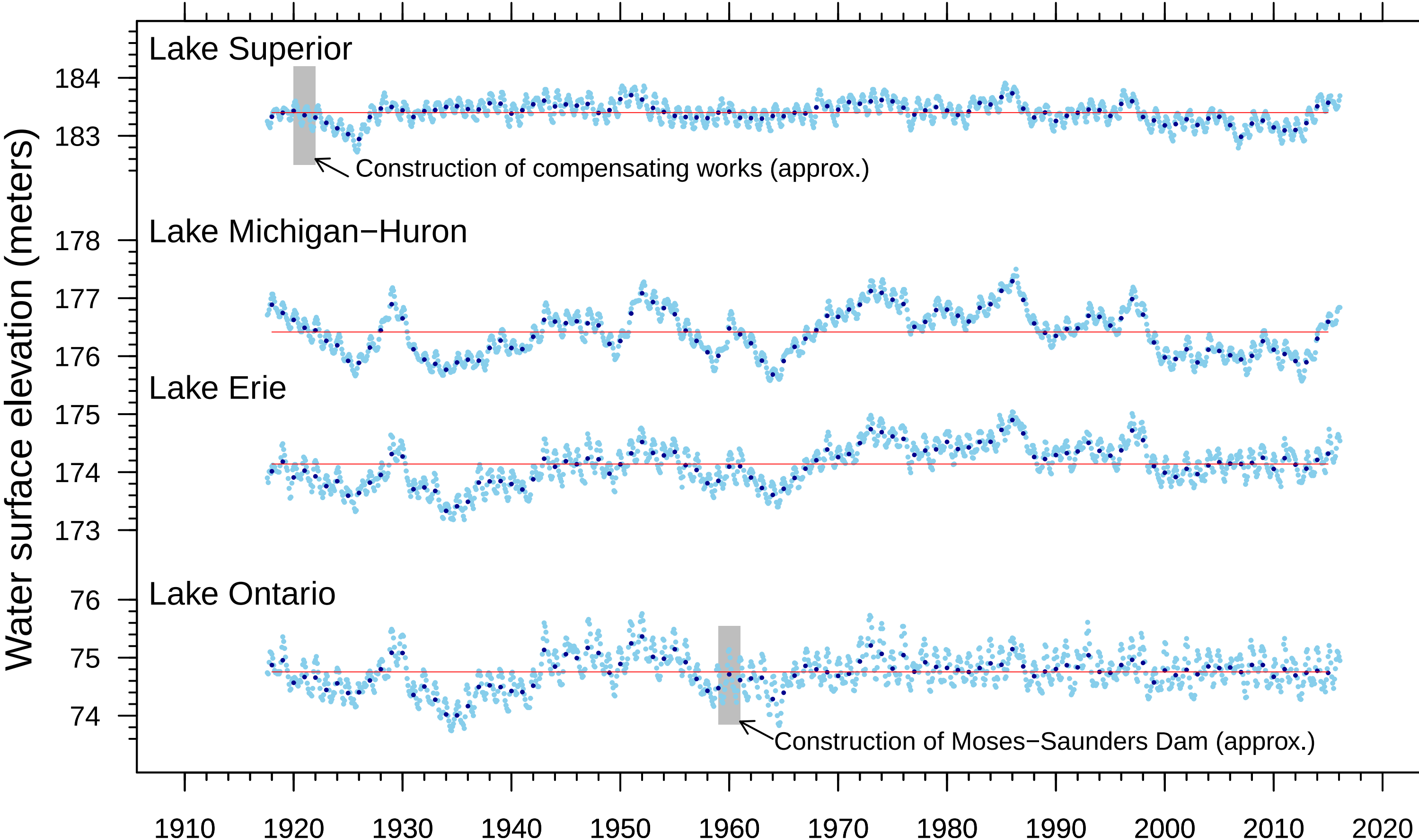
Table B1: Summary of notation used in our study.


0
125
250
500 Kilometers

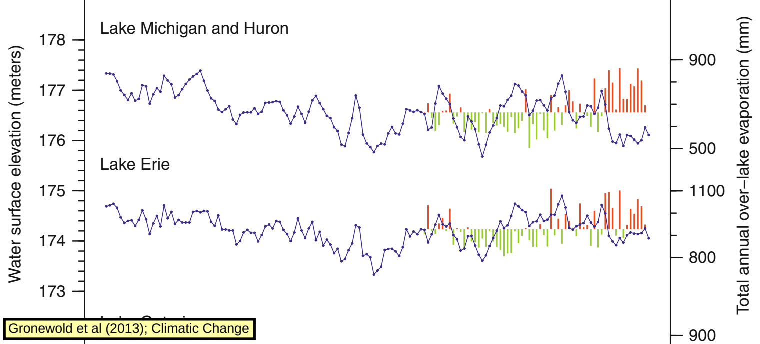

Ogoki Diversion
Nipigon River
Long Lac Diversion
Statute Miles
50 0 50 100 150 200
Kilometers
100 0 100 200 300
MINNESOTA
Duluth
LAKE SUPERIOR
ONTARIO
MICHIGAN
St. Marys River
Sault Ste. Marie
Ottawa River
St. Lawrence River
QUEBEC
WISCONSIN
Straits of Mackinac
LAKE HURON
GEORGIAN BAY
Cornwall
Massena
Green Bay
LAKE MICHIGAN
MICHIGAN
Toronto
LAKE ONTARIO
Welland Canal
Buffalo
NEW YORK
Milwaukee
St. Clair River
Niagara River
Detroit
Detroit River
Chicago
LAKE ERIE
ILLINOIS
Chicago Sanitary and Ship Canal
Cleveland
PENNSYLVANIA
INDIANA
OHIO



Great Lakes System Profile
Soo Locks & Dams
Sault Ste- Marie
St. Mary's River
St. Clair River
Lake St. Clair
Detroit River
Niagra River
Niagra Falls
St. Lawrence River
Moses-Saunders PowerDam
Iroquois Dam
Beauharnois Power Dam
Elevation: 601 ft.
Elevation: 577 ft.
Elevation: 568 ft.
Elevation: 243 ft.
Elevation: 20 ft.
Elevation: 0 ft.
Elevation: 0 ft.
Lake Superior
Lake Huron
Lake Erie
Lake Ontario
Lake Michigan
Gulf of St. Lawrence
Atlantic Ocean
Depth 210 ft.
Depth 750 ft.
Depth 925 ft.
Depth 802 ft.
Depth 1,333 ft.
NOT TO SCALE.
Vertical elevations are exaggerated.
Surface elevations are Chart Datum values above MSL, and depths are maximum of each lake.
379 Miles
60
223 Miles
89 Miles
236 Miles
35
150 Miles
77
89 Miles
350 Miles
500 Miles
Total Distance Along Floor Path 2,212 Miles
Modified from Michigan Sea Grant

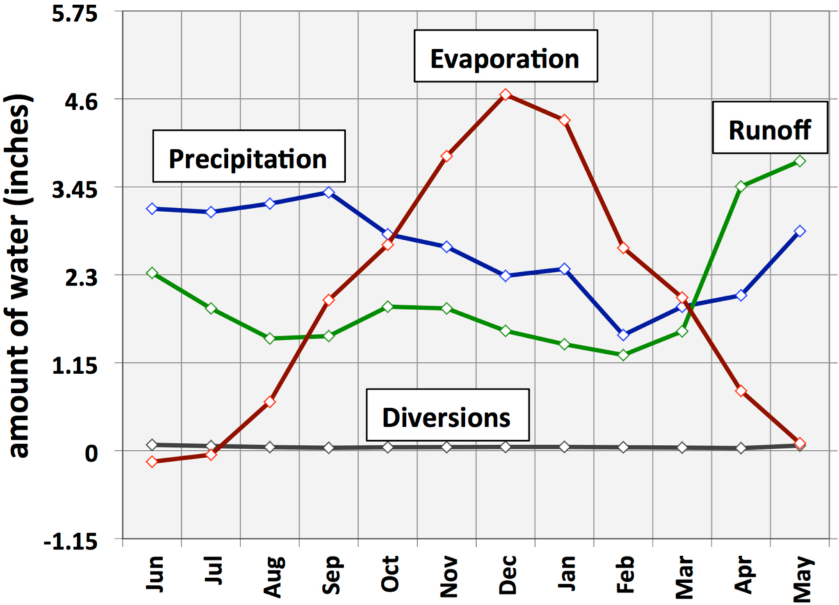

Lake Superior
Construction of compensating works (approx.)
Lake Michigan−Huron
Lake Erie
Lake Ontario
Construction of Moses−Saunders Dam (approx.)
Water surface elevation (meters)
184
183
178
177
176
175
174
173
76
75
74
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
2010
2020


Lake Michigan and Huron
Lake Erie
Water surface elevation (meters)
Total annual basin–wide precipitation (mm)
178
177
176
175
174
173
1100
600
1100
600
Gronewold et al (2013); Climatic Change

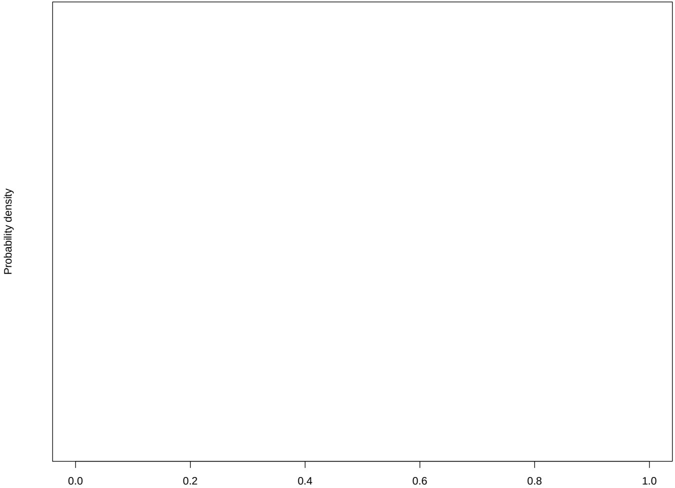


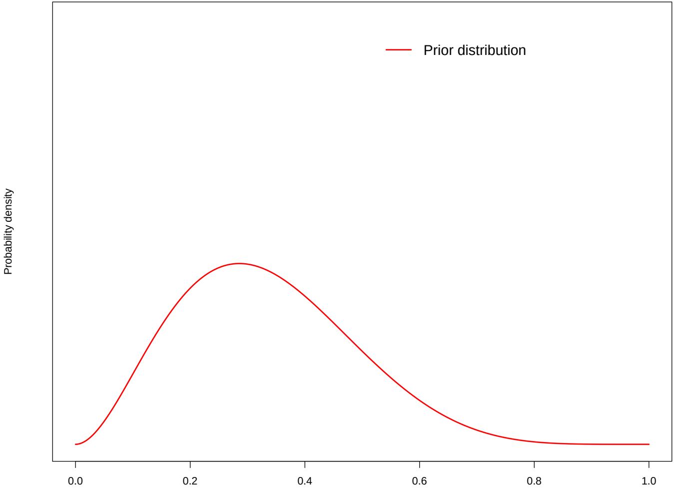
Lake Michigan and Huron
Lake Erie
Water surface elevation (meters)
Total annual over-lake evaporation (mm)
178
177
176
175
174
173
900
500
1100
800
900
Gronewold et al (2013); Climatic Change

Lake Superior water balance



From: Lenters et al (2013); Assessing the impacts of climate variability and change on Great Lakes evaporation


Probability density
0.0
0.2
0.4
0.6
0.8
1.0



Prior distribution
Probability density
0.0
0.2
0.4
0.6
0.8
1.0

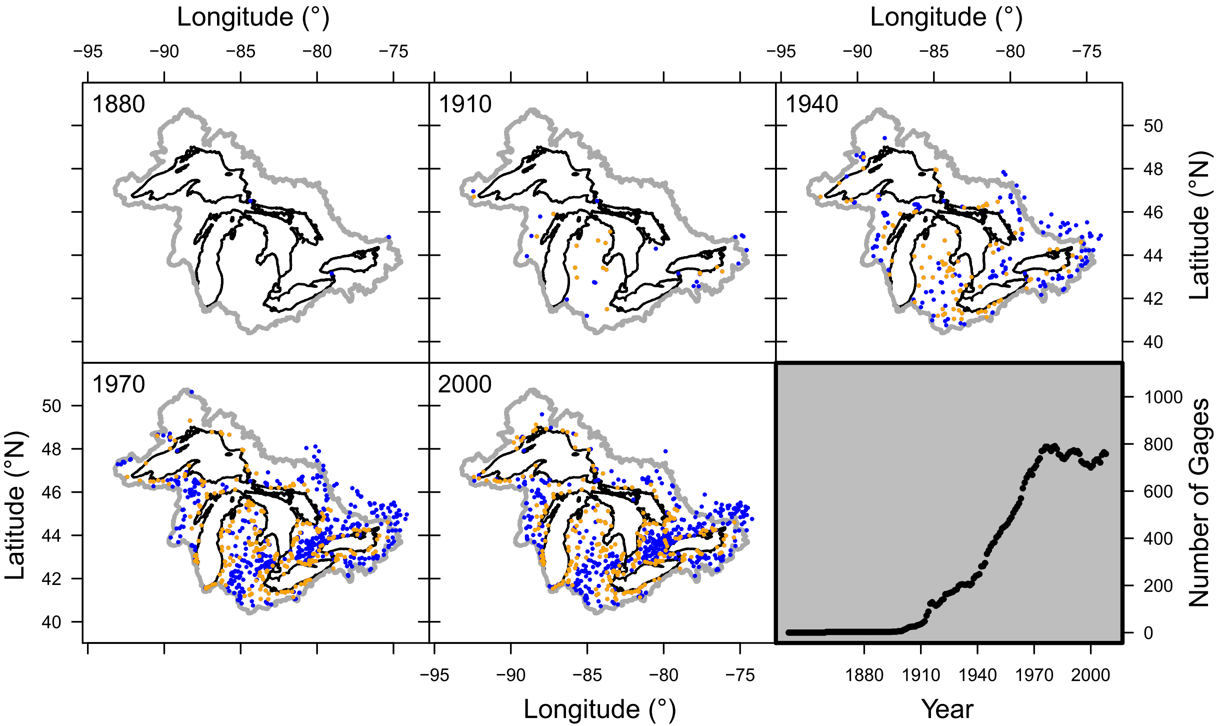
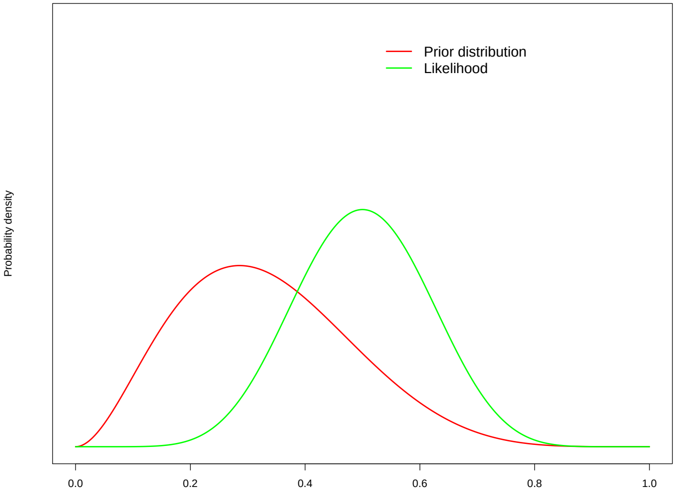
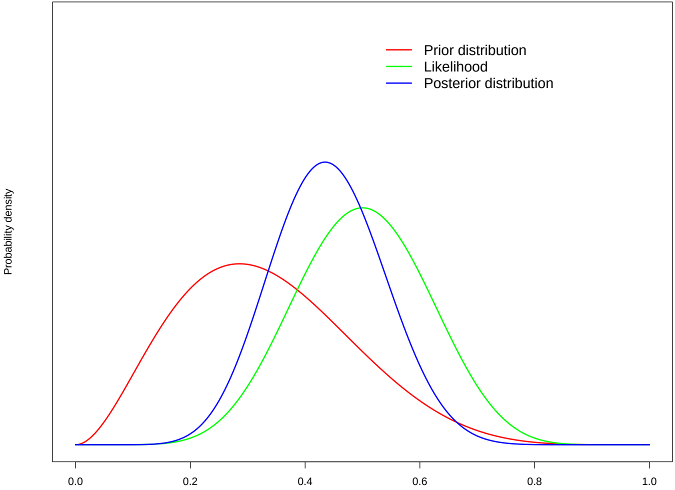
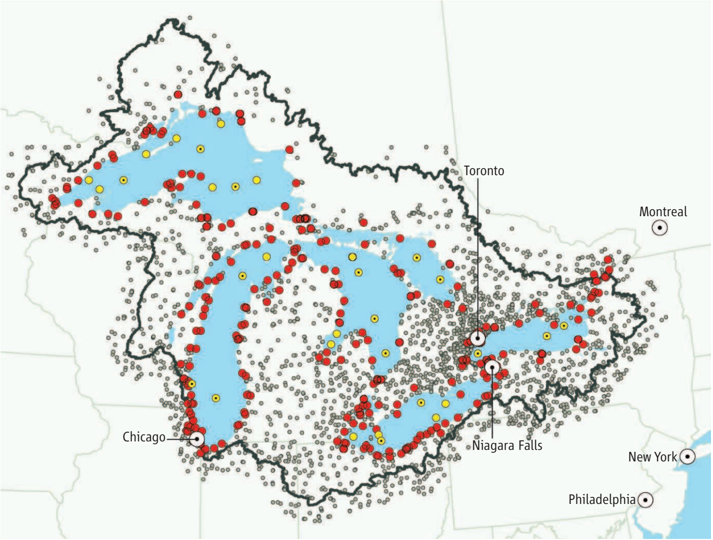


Fig. 6. Spatial distribution of USGS and WSC streamflow gages across the Great Lakes basin (boundary represented by gray line) reporting daily measurements in 1880, 1910, 1940, 1970, and 2000. Yellow dots represent the subset of stations that meet GLERL-ARM selection criteria (blue dots represent stations that do not meet the criteria). Bottom right-hand panel indicates total number of gages reporting daily values from 1840 to present.


Prior distribution
Likelihood
Probability density
0.0
0.2
0.4
0.6
0.8
1.0


Prior distribution
Likelihood
Posterior distribution
Probability density
0.0
0.2
0.4
0.6
0.8
1.0



From: Gronewold & Stow (2014), *Science*