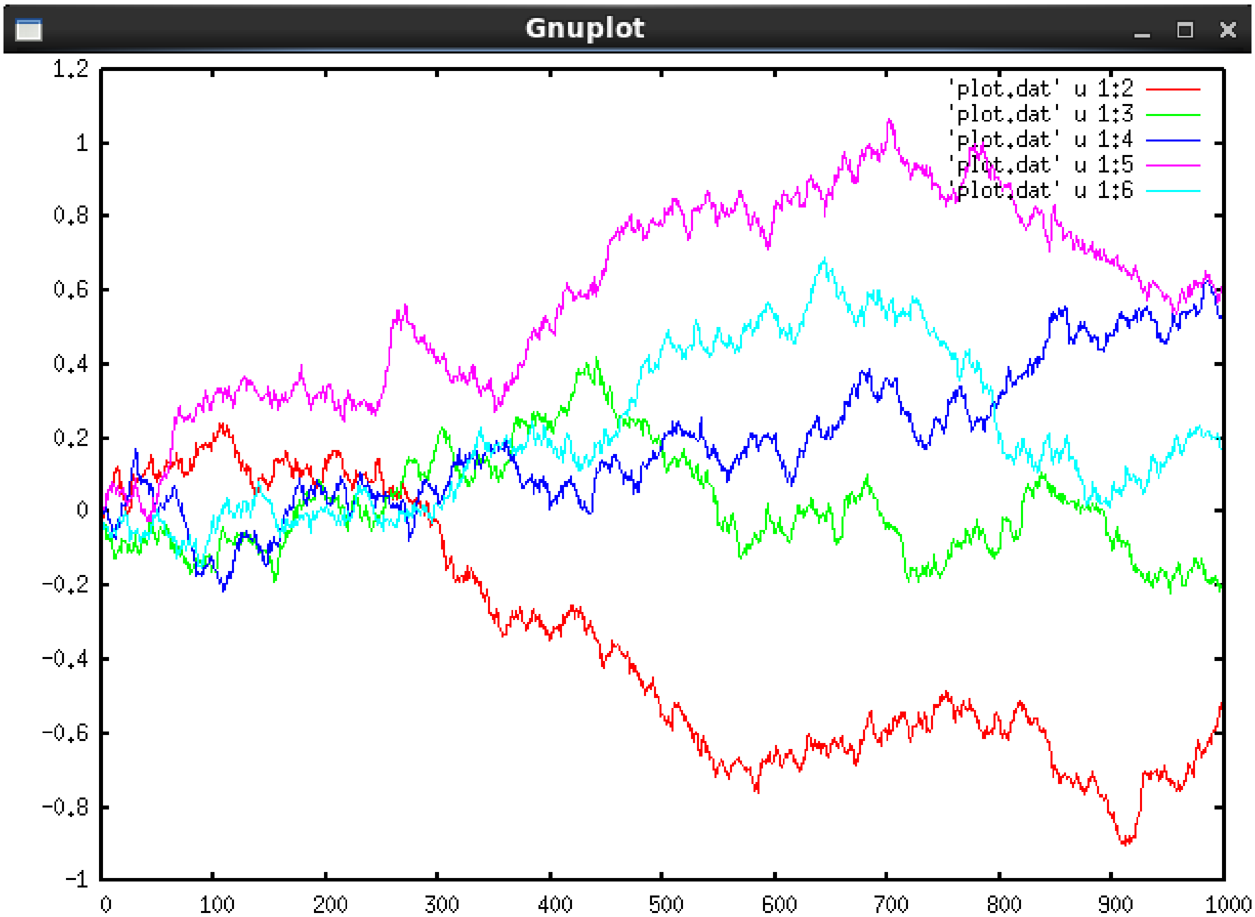
5637 Stochastic Differential Equations in Finance Report

Part 1: Implement random number generator to simulate brownian motion.

Before I wrote my own random number generator to simulate brownian motion, I used the gnu_ran_gaussian() from gsl library.

The following picture is a sample of 5 brownian motions plotted by gnuplot with data generated from my cpp code:



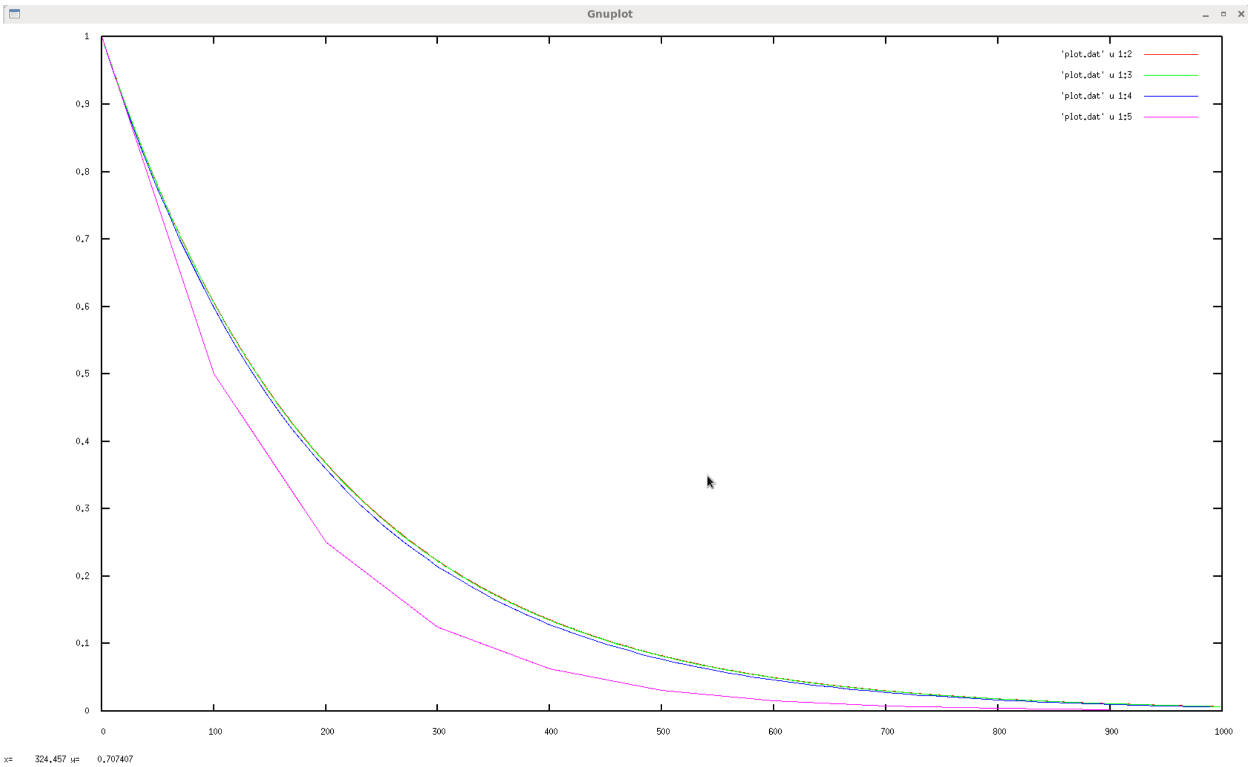
Part 2: Write software to solve the differential equation.

The equation given is:
$dx = -5x\, dt$, $x(0) = 1$,

Solving this equation step by step I got $x = e^{(-5t)}$ and in c/c++ language should be written as x = exp(-5*t).

Solving this equation by euler method, I got Xn+1 = Xn + h*(f(tn, Xn)).

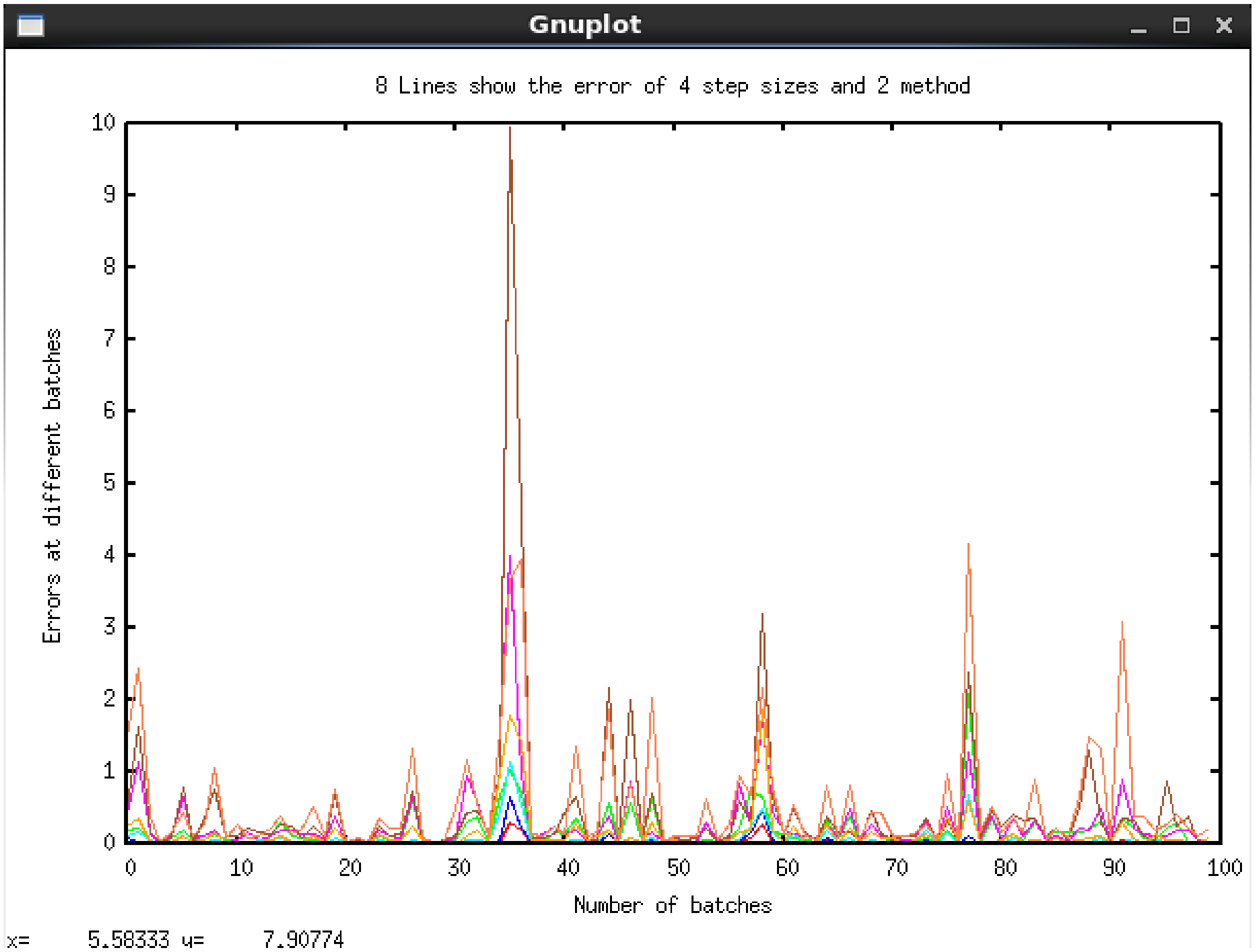
I implemented euler method with 3 different step sizes in [0,1]: 0.1 is the purple line in the below picture, 0.01 is the blue line and 0.001 is the green line. At the same time the analytic solution is also plotted as the red line with step size 0.001. We can see in the graph red line and green line coincide with each other.



Part 3: Solution of one dimensional SDEs.

(a):
A very basic SDE problem should be solved in this part: dX = aXdt + bXdW. I set a=2, b=2 and X (0)=1.

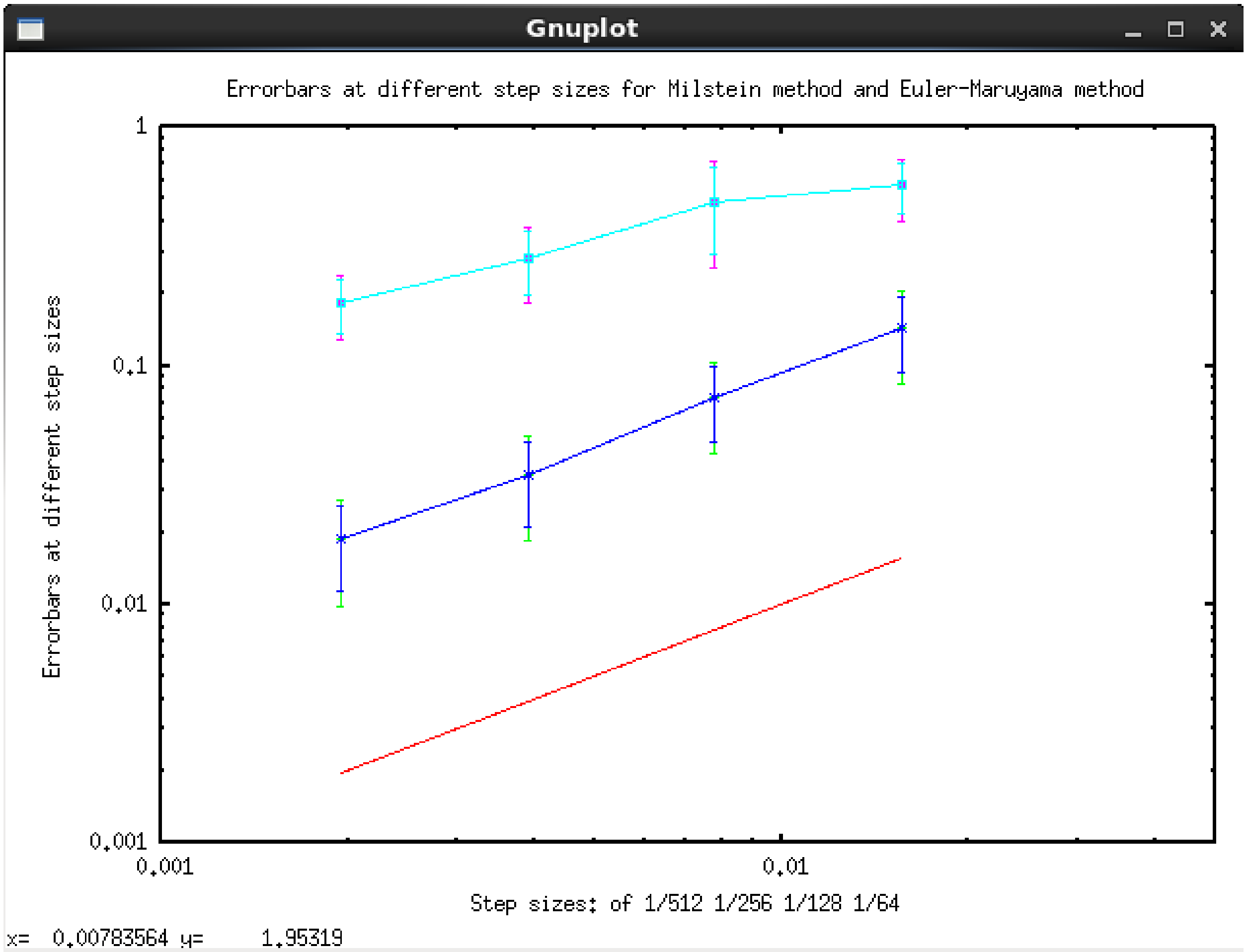
In the following graph, X axis is the number of batches and Y axis is the value of error of the batc h. 8 lines in the graph stand for 4 step sizes (1/512, 1/256, 1/128, 1/64) and 2 methods (Milstein method and Euler-Maruyama method).



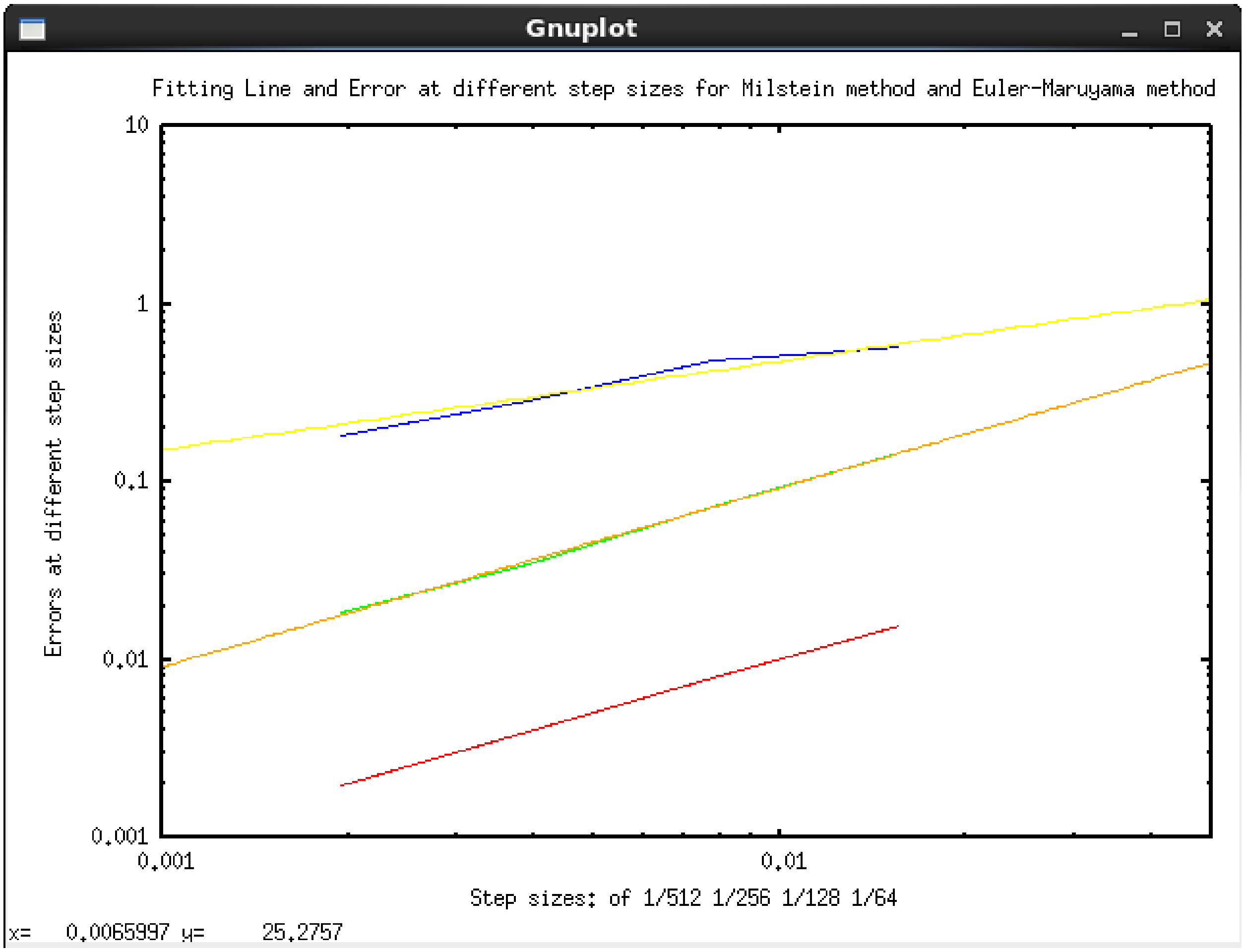
Red line: dt = 1/512 for Milstein method.
Green line: dt = 1/512 for Euler-Maruyama method.
Dark blue line: dt = 1/256 for Milstein method.
Purple line: dt = 1/256 for Euler-Maruyama method.
Light blue line: dt = 1/128 for Milstein method.
Brown line: dt = 1/128 for Euler-Maruyama method.
Yellow line: dt = 1/64 for Milstein method.
Light Brown line: dt = 1/64 for Euler-Maruyama method.

We can see from the above graph that Milstein method lead to smaller error than Euler-Maruyama method as well as smaller step size lead to smaller error. Smallest error come from dt=1/512 Milstein method and largest error come from dt=1/64 Euler-Maruyama method.

In the graph below, X axis is the step sizes of the numerical solution of the SDE and Y axis is the error of each step size and respect errorbar. The upper light blue line shows the error of Euler-Maruyama method. The middle dark blue line shows the error of Milstein method. Red line at bottom is the standard comparing line defined by four points (1/512, 1/512), (1/256, 1/256), (1/128, 1/128), (1/64, 1/64).



In the graph below, X axis is the step sizes of the numerical solution of the SDE and Y axis is the error of each step size and respect errorbar. The upper blue line is the error line of Euler-Maruyama method and the yellowing line is its fitting line with parttern f(x)=a*x^b. The middle green line is the error line of Milstein method and the orange line is its fitting line with parttern f(x)=a*x=b. The red line at bottom is same as above graph.



Fitting of the two error lines are done by gnuplot which also generated the graph. Fitting of blue li
ne started from guessing a=3, b=-1 and after 47 iterations converged.

```
After 47 iterations the fit converged.
final sum of squares of residuals : 2.76504e-06
rel. change during last iteration : -1.85496e-08

degrees of freedom    (FIT_NDF)                        : 2
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.00117581
variance of residuals (reduced chisquare) = WSSR/ndf   : 1.38252e-06

Final set of parameters            Asymptotic Standard Error
=======================            ==========================

a1              = 9.57883          +/- 0.6988       (7.296%)
b1              = 1.00732          +/- 0.01663      (1.651%)


correlation matrix of the fit parameters:

               a1     b1
a1              1.000
b1              0.995  1.000
```

Fitting of the two error lines are done by gnuplot which also generated the graph. Fitting of blue li ne started from guessing a=5, b=-1 and after 14 iterations converged.

```
After 14 iterations the fit converged.
final sum of squares of residuals : 0.00611084
rel. change during last iteration : -1.66782e-07

degrees of freedom    (FIT_NDF)                        : 2
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.0552758
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.00305542

Final set of parameters            Asymptotic Standard Error
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a2              = 4.73215          +/- 2.342        (49.5%)
b2              = 0.497962         +/- 0.1051       (21.11%)


correlation matrix of the fit parameters:

               a2     b2
a2              1.000
b2              0.991  1.000
```

(b):

Part 4: Two dimensional SDE.

Part 5: Heston stochastic volatility model.

Part 6: Solve.

Part 7: