FDTD 1D with CPML

skiloop(skiloop@126.com)

November 30, 2013

1 Maxwell Eqations

Equations in frequancy domain

$$j\omega\varepsilon E_x = -\frac{1}{s_z}\frac{\partial H_y}{\partial z} \tag{1}$$

$$j\omega\mu H_y = \frac{1}{s_z}\frac{\partial E_x}{\partial z} \tag{2}$$

where $s_z = 1$ in non-PML region and $s_x = \kappa_z + \frac{\sigma_z}{\alpha_z + j\omega\varepsilon_0}$ in PML region. Transform equations for PML to time domain we get

$$\frac{\partial(\varepsilon E_x)}{\partial t} + \sigma E_x = -\frac{1}{\kappa_z}\frac{\partial H_y}{\partial z} - \Psi_{Ezy} \tag{3}$$

$$\frac{\partial(\mu H_y)}{\partial t} + \sigma_m H_y = \frac{1}{\kappa_z}\frac{\partial E_x}{\partial z} + \Psi_{Hzx} \tag{4}$$

Here

$$\Psi_{Ezy} = \zeta_z(t) * \frac{\partial H_y}{\partial z} \tag{5}$$

and

$$\Psi_{Hzx} = \zeta_z(t) * \frac{\partial E_x}{\partial z} \tag{6}$$

where

$$\zeta_z(t) = -\frac{\sigma_z}{\varepsilon_0\kappa_z^2}\exp\left(-\left(\frac{\sigma_z}{\varepsilon_0\kappa_z} + \frac{\alpha_z}{\varepsilon_0}\right)t\right)u(t) \tag{7}$$

and $u(t)$ is the unit step function.

2 Discrete formula in PML region

$$\begin{aligned} &\varepsilon(k)\frac{E_x^{n+1}(k) - E_x^n(k)}{\Delta t} + \sigma(k)\frac{E_x^{n+1}(k) + E_x^n(k)}{2} = \\ &-\frac{1}{\kappa_z(k)}\frac{H_y^{n+1/2}(k+1/2) - H_y^{n+1/2}(k-1/2)}{\Delta z} - \Psi_{Ezy}^{n+1/2}(k) \end{aligned} \tag{8}$$

$$\begin{aligned} &E_x^{n+1}(k) = C_a(k)E_x^n(k) \\ &+ C_b(k)\left(H_y^{n+1/2}(k+1/2) - H_y^{n+1/2}(k-1/2)\right) + C_c(k)\Psi_{Ezy}^{n+1/2}(k) \end{aligned} \tag{9}$$

where

$$C_a(k) = \frac{1-b}{1+b} \tag{10}$$

$$C_c(k) = -\frac{\Delta t}{(1+b)\varepsilon(k)} \tag{11}$$

$$C_b(k) = \frac{C_c}{\kappa(k)\Delta z} \tag{12}$$

$$b = \frac{\sigma(k)\Delta t}{2\varepsilon(k)} \tag{13}$$

Magnetic

$$\begin{aligned} &\mu(k)\frac{H_y^{n+1/2}(k+1/2) - H_y^{n+1/2}(k+1/2)}{\Delta t} \\ &+ \sigma_m(k)\frac{H_x^{n+1/2}(k) + H_x^{n-1/2}(k+1/2)}{2} = \\ &\frac{1}{\kappa_z(k)}\frac{E_x^n(k+1) - E_x^n(k)}{\Delta z} + \Psi_{Hzx}^{n+1/2}(k+1/2) \end{aligned} \tag{14}$$

$$\begin{aligned} &H_y^{n+1/2}(k+1/2) = C_1(k+1/2)H_y^{n-1/2}(k+1/2) \\ &+ C_2(k+1/2)\left(E_x^n(k+1) - E_x^n(k)\right) + C_3(k+1/2)\Psi_{Hzx}^n(k+1/2) \end{aligned} \tag{15}$$

where

$$C_1(k) = \frac{1-b}{1+b} \tag{16}$$

$$C_3(k) = \frac{\Delta t}{(1+b)\mu(k)} \tag{17}$$

$$C_2(k) = \frac{C_3}{\kappa(k)\Delta z} \tag{18}$$

$$b = \frac{\sigma_m(k)\Delta t}{2\mu(k)} \tag{19}$$

3 Discrete formula in non-PML region

$$\begin{aligned} &\varepsilon(k)\frac{E_x^{n+1}(k) - E_x^n(k)}{\Delta t} + \sigma(k)\frac{E_x^{n+1}(k) + E_x^n(k)}{2} = \\ &-\frac{H_y^{n+1/2}(k+1/2) - H_y^{n+1/2}(k-1/2)}{\Delta z} \end{aligned} \tag{20}$$

$$\begin{aligned} &E_x^{n+1}(k) = C_a(k)E_x^n(k) \\ &+ C_b(k)\left(H_y^{n+1/2}(k+1/2) - H_y^{n+1/2}(k-1/2)\right) \end{aligned} \tag{21}$$

where

$$C_a(k) = \frac{1-b}{1+b} \tag{22}$$

$$C_b(k) = -\frac{\Delta t}{(1+b)\varepsilon(k)\Delta z} \tag{23}$$

$$b = \frac{\sigma(k)\Delta t}{2\varepsilon(k)} \tag{24}$$

Magnetic

$$\begin{aligned}
&\mu(k)\frac{H_y^{n+1/2}(k+1/2) - H_y^{n+1/2}(k+1/2)}{\Delta t} \\
&+ \sigma_m(k)\frac{H_x^{n+1/2}(k) + H_x^{n-1/2}(k+1/2)}{2} = \\
&\frac{E_x^n(k+1) - E_x^n(k)}{\Delta z}
\end{aligned} \tag{25}$$

$$\begin{aligned}
&H_y^{n+1/2}(k+1/2) = C_1(k+1/2)H_y^{n-1/2}(k+1/2) \\
&+ C_2(k+1/2)\left(E_x^n(k+1) - E_x^n(k)\right)
\end{aligned} \tag{26}$$

where

$$C_1(k) = \frac{1-b}{1+b} \tag{27}$$

$$C_2(k) = \frac{\Delta t}{(1+b)\mu(k)\Delta z} \tag{28}$$

$$b = \frac{\sigma_m(k)\Delta t}{2\mu(k)} \tag{29}$$

Note: the σ for updating E_x is different from σ_z that updating $\zeta_z(t)$.