## Recap of Central Results

- **Linear Mixed Models:** Explicitly model the mean as well as sources of correlation.
  - Combines random effects and serial structures
  - Clustered and nested data
  - Mean model and a covariance model
- Estimation via ML and REML:
  - Assume multivariate normality
  - ullet  $\widehat{oldsymbol{eta}}$  and  $\widehat{oldsymbol{ heta}}$
  - REML: Correction for degrees of freedom in  $oldsymbol{eta}$

## Recap of Central Results (2)

- Estimation (Prediction) of Random effects
  - **b**<sub>i</sub> for understanding individual trajectories
  - Mixed Model Equations/Empirical Bayes/Penalized Likelihood
  - Shrinkage
- Inference in Mixed Models
  - Wald and LRT
  - Testing on boundary of parameter space
- Model fitting and diagnostics
  - Mean, covariance (random effects and serial correlation)

## Key References

- Mixed Models: Laird & Ware (1982, Biometrics)
- REML Estimation: Harville (1974, Biometrika)
- Random Effect Estimation: Harville (1977, JASA) and Robinson (1991, Stat Science)
- Testing VCs: Self & Liang (1987, JASA); Stram and Lee (1994, Biometrics)
- Serial Correlation: Diggle (1988, Biometrics)
- Additional Topics:
  - Influence Diagnostics: Lesaffre & Verbeke (1998, Biometrics)
  - Alternative Random Eff Dist: Butler & Louis (1992, Stat in Med)
  - Non/Semi-parametric Regression: Lin & Zhang (1999, JRSSB)

## Where we are in 571...

- 1. Introduction to Correlated Data
- 2. Linear Mixed Models (HW1)
- 3. Review of GLMs and Quasi-likelihood
- 4. Generalized Estimating Equations (HW2)
- 5. Generalized Linear Mixed Models (HW3)
- 6. Missing Data
- 7. Classical and Modern Multivariate Analysis
- 8. Group Papers Due