### Recap of Central Results

- Longitudinal modeling of non-Gaussian outcomes is hard!
- Subject specific model  $\rightarrow$  Random effects model  $\rightarrow$  GLMMs
  - Conditional independence assumption
  - Note: LMM permits additional residual correlation
- GLMM vs. GEE
  - Underlying effects of covariates are the same (biology), but how we model them is different
    - Interpretations are model specific
    - Equal only in special cases
  - Subject specific vs. marginal interpretation
  - Challenging to interpret some parameters

# Recap of Central Results (2)

- Inference and estimation
  - MLE from marginal model
  - Theoretical result follow from ML theory: consistency and asymptotic normality of  $\widehat{m{\beta}}$
  - Main challenge: computation
- Approaches to integration
  - Numerical integration via Adaptive Guassian Quadrature (or others)
  - Approximation: Laplace, Solomon-Cox, PQL and CPQL procedures
  - EM algorithm
  - Note that all of these can be problematic in different situations and slow!

## Comments on Integration/Solving in R

- Tools in R:
  - integrate() and cubature package for integration
  - nlm() and optim() for optimization
  - numDeriv() for getting Hessians
- Ime4 bundles a lot of this stuff for you
- SAS is (again) probably a little bit better
- None of the R packages are perfect and need to look at the output carefully
- GEE is a bit more stable (numerically)

#### **GLMM** Issues

- Assume: Mean model is correct
- Assume: Conditional independence: given  $b_i$ ,  $Y_{ij}$ 's are independent
- Careful: causal statements on what happens if we change X by one unit

### Advantages of GLMMs

- If we are interested in conditional inference
- If we want subject specific trajectories
- Major: dealing with missing data

# **Key References**

- Breslow & Day (1989)
- Breslow & Clayton (1993)
- Zeger, Liang, Albert (1988)
- Neuhaus, Kalbfleish, Hauck (1991)
- Zeger & Liang (1992)
- Breslow & Lin (1995)
- Lin & Breslow (1996)
- Lin (1997)

### Where we are in 571...

- 1. Introduction to Correlated Data
- 2. Linear Mixed Models (HW2)
- 3. Review of GLMs and Quasi-likelihood
- 4. Generalized Estimating Equations (HW3)
- 5. Generalized Linear Mixed Models (HW4)
- 6. Missing Data
- 7. Classical and Modern Multivariate Analysis (HW5??)
- 8. Group Papers Due (HW5/6)