

*Proof of Lemma 5:* Let  $\mathcal{Q}_n(q_0, \Delta) = \{q: \text{for some } \sigma \in L(\Delta, \mathbb{S}) \text{ such that } |\sigma| = n, q = \delta(q_0, \sigma)\}$ . Then, we prove that  $\forall n \in \mathbb{N}, \mathcal{Q}_n(q_0, \Delta_L) \subseteq \mathcal{Q}_n(q_0, \Delta_J)$  by mathematical induction on the length of the event string  $|\sigma|$  as follows.

If  $|\sigma| = n = 0$ , it is clear that  $\mathcal{Q}_0(q_0, \Delta_L) = \mathcal{Q}_0(q_0, \Delta_J) = \{q_0\}$ . Thus,  $\mathcal{Q}_n(q_0, \Delta_L) \subseteq \mathcal{Q}_n(q_0, \Delta_J)$  when  $n = 0$ .

Suppose that  $\mathcal{Q}_n(q_0, \Delta_L) \subseteq \mathcal{Q}_n(q_0, \Delta_J)$  when  $|\sigma| = n > 0$ . Now, we prove  $\mathcal{Q}_{n+1}(q_0, \Delta_L) \subseteq \mathcal{Q}_{n+1}(q_0, \Delta_J)$ . Suppose that  $\exists q \in \mathcal{Q}_n(q_0, \Delta_L) \cap \mathcal{Q}_n(q_0, \Delta_J)$  and  $\pi \in \xi(q)$  that  $q_1 = \delta(q, \pi) \in \mathcal{Q}_{n+1}(q_0, \Delta_L)$  but  $q_1 \notin \mathcal{Q}_{n+1}(q_0, \Delta_J)$ . In other words,  $q_1$  satisfies inequalities in RO,  $\mathcal{D}$ , and NHC<sub>2</sub> but violates  $\zeta$ . Let  $M = \mathcal{N}(\chi(q_1))$  and  $M' = \mathcal{N}(\chi(q))$ . We consider two cases of  $\pi \in \sum_c \cup \sum_u$ .

Case 1)  $\pi = \alpha_{jk} \in \sum_c$ . Since  $q_1$  satisfies NHC<sub>2</sub>,  $M(\varphi(r_m)) + M(\varphi(r_n)) < C(r_m) + C(r_n)$ ,  $\forall r_m, r_n \in R_F$ . Since  $q_1$  satisfies  $\mathcal{D}$ ,  $M(\varphi(r_m)) \leq C(r_m)$ , i.e.,  $M(r_m) \geq 0$ ,  $\forall r_m \in R_F$ . Suppose that  $N_f$  does not contain  $\xi$ -resources. Since  $q$  satisfies  $\zeta$  and  $q_1$  violates  $\zeta$ ,  $t_{jk}$  (corresponding to  $\alpha_{jk}$ ) is not resource-enabled at  $M'$  in  $N_f(t_{jk}$  is process-enabled since  $\alpha_{jk} \in \xi(q)$ ) or there is a saturated MPRT-circuit  $\theta$  at  $M$  in  $N_f$ . If  $t_{jk}$  is not resource-enabled at  $M'$ , then  $M(r_m) < 0$ , where  $r_m = R(p_{jk}, N_f) \in R_F$ . It is a contradiction to the assumption that  $q_1$  satisfies  $\mathcal{D}$ . Thus, there is a saturated MPRT-circuit  $\theta$  at  $M$  in  $N_f$ . That means  $M(\varphi(r_m)) = C(r_m)$ ,  $\forall r_m \in \mathcal{N}[\theta]$ . Since  $\mathcal{N}[\theta] \subseteq R_F$  and  $|\mathcal{N}[\theta]| \geq 2$ , it is a contradiction to the assumption that  $q_1$  satisfies NHC<sub>2</sub>.

Suppose that  $(N_f, M_{f0}), (N_f(r_1), M_{fA1}), \dots$ , and  $(N_f(r_1, \dots, r_{z-1}), M_{fA(z-1)})$  contain  $\xi$ -resources but  $(N_f(r_1, \dots, r_z), M_{fAz})$  does not. Let  $M_1 = Y(M')$ ,  $M_2 = Y(M_1)$ ,  $\dots$ , and  $M_z = Y(M_{z-1})$  denote the markings of  $N_f(r_1)$ ,  $N_f(r_1, r_2)$ ,  $\dots$ , and  $N_f(r_1, \dots, r_z)$  corresponding to  $M'$ , respectively. Since  $q$  satisfies  $\zeta$  and  $q_1$  violates  $\zeta$ ,  $t_{jk}$  is not resource-enabled ( $t_{jk}$  is process-enabled since  $\alpha_{jk} \in \xi(q)$ ) at  $M_v$  in  $N_f(r_1, \dots, r_v)$ , where  $v \in \{1, 2, \dots, z\}$  or there is a saturated MPRT-circuit  $\theta$  at  $M_{1z}$  in  $N_f(r_1, \dots, r_z)$ , where  $M_z[t_{jk}] > M_{1z}$ .

Let  $M_{11} = Y(M)$ ,  $M_{12} = Y(M_{11})$ ,  $\dots$ , and  $M_{1z} = Y(M_{1(z-1)})$  denote the markings of  $N_f(r_1)$ ,  $N_f(r_1, r_2)$ ,  $\dots$ , and  $N_f(r_1, \dots, r_z)$  corresponding to  $M$ , respectively. If  $t_{jk}$  is not resource-enabled at  $M_v$ , then there is a resource  $r_w$  ( $w \neq 1, \dots, v$ ) in  $N_f(r_1, \dots, r_v)$  that  $M_{1v}(r_w) < 0$ . Then, there is at least one part in  $p \in H(r_w, N_f(r_1, \dots, r_v))$  at  $M_{1v}$  and  $R(p, N_f) = r_m \in \{r_1, \dots, r_v\}$  (if there is no such part, then  $M_{1v}(r_w) = M(r_w) \geq 0$ ). Since  $r_1, \dots, r_v$  are  $\xi$ -resources,  $C(r_n) = 1$ ,  $\forall r_n \in \{r_1, \dots, r_v\}$ . Thus, only one such part participates in this violation (if two or more such parts participate in this violation then  $q_1$  violates NHC<sub>2</sub>). Since  $M_{1v}(r_w) < 0$ ,  $M(r_w) = M_{1v}(r_w) + 1 \leq 0$ . Since  $M(r_w) \geq 0$ ,  $M(r_w) = 0$ , i.e.,  $M(\varphi(r_w)) = C(r_w)$ . Since there is another resource  $r_m \in \{r_1, \dots, r_v\}$  and  $r_m \neq r_w$  that  $M(\varphi(r_m)) = C(r_m) = 1$ , it is a contradiction to the assumption that  $q_1$  satisfies NHC<sub>2</sub>.

Thus there is a saturated MPRT-circuit  $\theta$  at  $M_{1z}$  in  $N_f(r_1, \dots, r_z)$ . Then,  $M_{1z}(r_m) = 0$ ,  $\forall r_m \in \mathcal{N}[\theta]$ . Suppose that there is not a part in  $p \in H(r_m, N_f(r_1, \dots, r_z))$  at  $M_{1z}$ , where  $r_m \in \mathcal{N}[\theta]$ , and  $R(p, N_f) = r_t \in \{r_1, \dots, r_z\}$ . Then  $\forall r_m \in \mathcal{N}[\theta]$ ,  $M(r_m) = M_{1z}(r_m) = 0$ , i.e.,  $M(\varphi(r_m)) = C(r_m)$ . This is a contradiction to the assumption that  $q_1$  satisfies NHC<sub>2</sub>. Since  $r_1, \dots, r_z$  are  $\xi$ -resources,  $C(r_t) = 1$ ,  $\forall r_t \in \{r_1, \dots, r_z\}$ . Thus, only one such part participates in this saturation (if two or more such parts participate in this saturation then  $q_1$  violates NHC<sub>2</sub>). Suppose that this part is in  $p \in H(r_w, N_f(r_1, \dots, r_z))$  at  $M_{1z}$ , where  $r_w \in \mathcal{N}[\theta]$ , and  $R(p, N_f) = r_s \in \{r_1, \dots, r_z\}$ . Then,  $\forall r_m \in \mathcal{N}[\theta] \setminus \{r_w\}$ ,  $M(r_m) = M_{1z}(r_m) = 0$ , i.e.,  $M(\varphi(r_m)) = C(r_m)$ , and  $M(\varphi(r_s)) = C(r_s) = 1$ . Since  $|\mathcal{N}[\theta]| \geq 2$  and  $r_s \notin \mathcal{N}[\theta]$ , it is a contradiction to the assumption that  $q_1$  satisfies NHC<sub>2</sub>.

Case 2)  $\pi \in \sum_u$ . If  $\pi = \beta_{jk}, \kappa$  or  $\eta$ ,  $M' = M$ . Since  $M'$  is accepted by  $\zeta$ ,  $M$  is accepted by  $\zeta$ , too. It is a contradiction to the assumption that  $q_1$  violates  $\zeta$ .

Thus,  $\mathcal{Q}_{n+1}(q_0, \Delta_L) \subseteq \mathcal{Q}_{n+1}(q_0, \Delta_J)$ . So,  $\forall n \in \mathbb{N}, \mathcal{Q}_n(q_0, \Delta_L) \subseteq \mathcal{Q}_n(q_0, \Delta_J)$ . Since  $\mathcal{Q}_R(q_0, \Delta) = \{q: \text{for some } \sigma \in L(\Delta, \mathbb{S}), q = \delta(q_0, \sigma)\} = \cup_{n \in \mathbb{N}} \mathcal{Q}_n(q_0, \Delta)$ ,  $\mathcal{Q}_R(q_0, \Delta_L) \subseteq \mathcal{Q}_R(q_0, \Delta_J)$ . ■