

**Supplementary File for "A Petri Net-based Deadlock Avoidance Policy for Flexible Manufacturing Systems with Assembly Operations and Multiple Resource Acquisition"**

To test the efficiency in terms of permissiveness of MBA, a large example is given, as shown in Fig. 5. The system contains six types of robots, i.e.,  $r_1$ - $r_6$  and eight types of machines, i.e.,  $m_1$ - $m_8$ . The capacities of  $m_1$ ,  $m_7$ ,  $m_8$ ,  $r_1$ , and  $r_6$  are all 1, those of  $m_2$ ,  $m_3$ ,  $m_4$ ,  $m_5$ ,  $m_6$ ,  $r_2$ ,  $r_3$ , and  $r_4$  are all 2, and that of  $r_5$  is 4. There are five types of raw parts, which can be manufactured and assembled into three types of products. The part type set is  $J = \{J_i \mid i \in Z_5\}$ . A raw  $J_1$  part is taken from  $I_1$  by  $r_1$ . After being processed by  $m_1$  and  $m_3$  simultaneously, it is moved to  $m_2$  by  $r_2$ . After being processed by  $m_2$ , a product is completed and is moved to  $O_1$  by  $r_1$ . A raw  $J_2$  part is taken from  $I_2$  by  $r_1$ . After being processed by  $m_2$  and  $m_4$  simultaneously, it is moved to  $m_1$  by  $r_2$ . A raw  $J_3$  part is taken from  $I_3$  by  $r_5$ . After being processed by  $m_4$  and  $m_5$  simultaneously, it is moved to  $m_6$  by  $r_4$ . After a  $J_2$  part being processed by  $m_1$  and a  $J_3$  part being processed by  $m_6$ , they are moved to  $m_3$  by  $r_4$  and  $r_3$ , respectively. After being assembled in  $m_3$ , a product is completed and is moved to  $O_2$  by  $r_3$ . A raw  $J_4$  part is taken from  $I_4$  by  $r_6$ . After being processed by  $m_7$ , it is moved either to  $m_6$  by  $r_3$  or to  $m_8$  by  $r_6$ . A raw  $J_5$  part is taken from  $I_5$  by  $r_6$ . Then, it is processed by  $m_8$ . After a  $J_4$  part being processed by  $m_6$  or  $m_8$  and two  $J_5$  parts being processed by  $m_8$ , they are moved to  $m_5$  and  $m_7$  by  $r_2$  and  $r_5$ , respectively. After being assembled by  $m_5$  and  $m_7$  simultaneously, a product is completed and is moved to  $O_3$  by  $r_6$ . In Fig. 5, the labeled notation, namely,  $t_j$  ( $j \in Z_{33}$ ) beside each arc and leading to a resource indicates that the resource is required in order to trigger the event represented by  $t_j$ . The number of required resources is represented by the weight of the outgoing arc. The occurrence of an event stands for moving one or more parts from the current resource to a required one. The number of moved parts is represented by the integer before  $t_j$ . For example,  $2t_{25}$  in Fig. 5 represents that the occurrence of  $t_{25}$  moves 2 parts from  $r_5$ . For simplicity, the integer 1 is omitted.

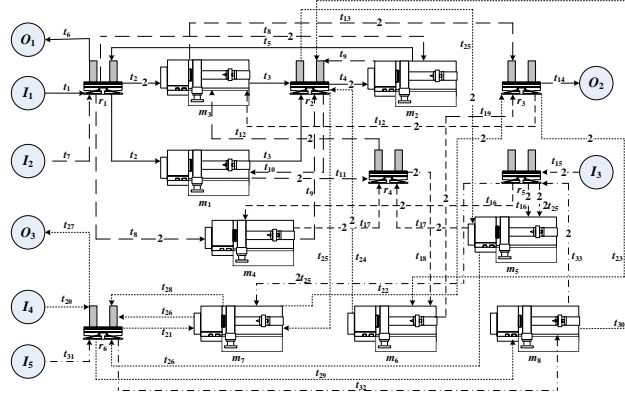


Fig. 5. The block diagram of an AMS.

The system's PN model is denoted as  $(N_3, M_{30})$  as shown in Fig. 6. As it allows flexible routes,  $H_{15}$  cannot be applied to it. Since  $\exists W(p_{37}, t_{25}) > 1$ ,  $(N_3, M_{30})$  is not an AEMG. Thus,  $H_{13}$  cannot be applied to it. The first two conditions in  $W_{08}$  can be changed to twelve constraints as follows:

- 1)  $M(p_2) + M(p_3) + M(p_5) + M(p_6) + M(p_9) + M(p_{10}) + M(p_{12}) + M(p_{14}) \leq 1$ ;
- 2)  $M(p_2) + M(p_3) + M(p_4) + M(p_5) + M(p_6) + M(p_9) + M(p_{10}) + M(p_{11}) + M(p_{12}) + M(p_{13}) + M(p_{14}) + M(p_{15}) + M(p_{19}) + M(p_{20}) + M(p_{22}) + M(p_{26}) + M(p_{28}) \leq 2$ ;
- 3)  $M(p_2) + M(p_3) + M(p_5) + M(p_6) + M(p_9) + M(p_{10}) + M(p_{12}) + M(p_{13}) + M(p_{14}) + M(p_{15}) + M(p_{19}) +$

$$\begin{aligned}
& M(p_{20}) + M(p_{22}) + M(p_{26}) \leq 1; \\
& 4) M(p_{38}) + M(p_{39}) + M(p_{40}) + M(p_{46}) + M(p_{47}) \geq 7; \\
& 5) M(p_3) + M(p_{10}) + M(p_{13}) + M(p_{14}) + M(p_{15}) + M(p_{19}) + M(p_{20}) + M(p_{21}) + M(p_{22}) + M(p_{26}) + M(p_{27}) \\
& + M(p_{29}) \leq 1; \\
& 6) M(p_3) + M(p_{10}) + M(p_{13}) + M(p_{14}) + M(p_{15}) + M(p_{18}) + M(p_{19}) + M(p_{20}) + M(p_{21}) + M(p_{22}) + M(p_{26}) \\
& + M(p_{27}) + M(p_{29}) + M(p_{37}) \leq 2; \\
& 7) M(p_{42}) + M(p_{45}) + M(p_{51}) \geq 3; \\
& 8) M(p_{18}) + M(p_{19}) + M(p_{24}) + M(p_{25}) + M(p_{29}) + M(p_{30}) + M(p_{32}) + M(p_{33}) + M(p_{35}) + M(p_{36}) + \\
& M(p_{37}) \leq 2; \\
& 9) M(p_{15}) + M(p_{19}) + M(p_{21}) + M(p_{22}) + M(p_{24}) + M(p_{25}) + M(p_{26}) + M(p_{27}) + M(p_{29}) + M(p_{30}) + \\
& M(p_{32}) + M(p_{35}) \leq 1; \\
& 10) M(p_4) + M(p_{11}) + M(p_{15}) + M(p_{19}) + M(p_{21}) + M(p_{22}) + M(p_{24}) + M(p_{25}) + M(p_{26}) + M(p_{27}) + \\
& M(p_{28}) + M(p_{29}) + M(p_{30}) + M(p_{32}) + M(p_{35}) \leq 2; \\
& 11) M(p_{19}) + M(p_{24}) + M(p_{25}) + M(p_{29}) + M(p_{30}) + M(p_{32}) + M(p_{33}) + M(p_{35}) + M(p_{36}) \leq 1; \\
& 12) M(p_4) + M(p_{11}) + M(p_{19}) + M(p_{24}) + M(p_{25}) + M(p_{28}) + M(p_{29}) + M(p_{30}) + M(p_{32}) + M(p_{33}) + \\
& M(p_{35}) + M(p_{36}) \leq 2.
\end{aligned}$$

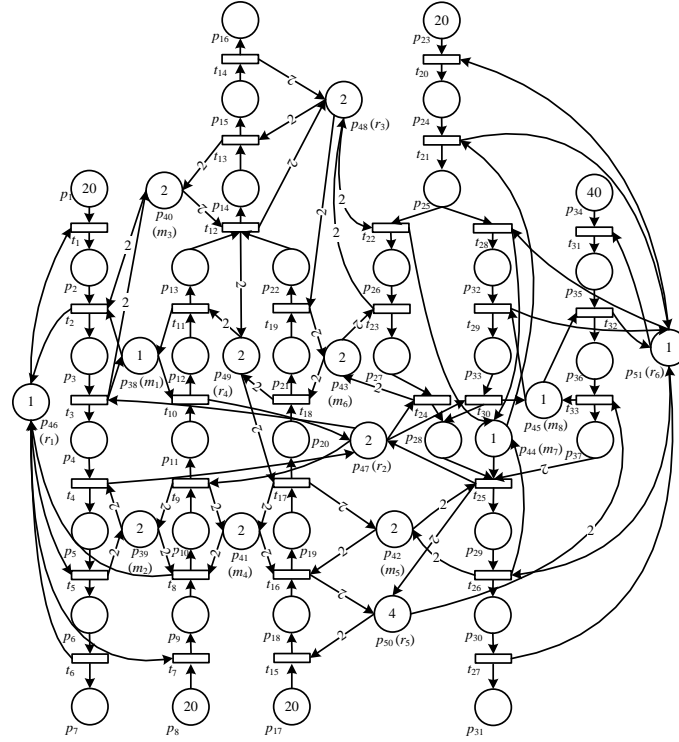


Fig. 6. Petri net model of the AMS in Fig. 5.

For  $(N_3, M_{30})$ , the number of reachable markings under these twelve constraints is 114, which means that the number of reachable markings under  $W_{08}$  is no more than 114. The number of reachable markings under MBA is 2999. Obviously, MBA admits much more markings than  $W_{08}$ . Since  $W_{08}$  admits much more markings than [24], MBA owns the highest permissiveness among these policies.

Now, we use AMS in Figs. 3, 4, and 6 to test the runtime of MBA. Note that  $H_{13}$  and  $H_{15}$  are both offline control policies that are established in advance. Comparing MBA with them in terms of runtime is meaningless. Thus, we only compare the runtime of MBA with that of  $W_{08}$ . All algorithms are

implemented in C++. They are compiled by MSBuild 4.0 and run on a 3.4 GHz desktop computer with 16G RAM. Its operating system is Windows 7 Professional. Simulation results are shown in Table I. From it, we know that the average runtime of  $W_{08}$  is much shorter than that of MBA. Although MBA is slower than  $W_{08}$ , it can detect the safety of markings of tested AMSs in 4.409  $\mu$ s averagely. Besides, from Table I, we find that  $AR \approx 0.00013 \times AN^2 \times |T|^2 \mu$ s. It means that MBA can detect the safety of markings of a system with  $|T| = 600$  and  $AN = 140$  in 0.917 s averagely. Thus, MBA is capable of handling large scale AMSs.

TABLE I  
SIMULATION RESULTS OF MBA AND  $W_{08}$ .

	$(N_1, M_{10})$ $ T  = 24, AN = 5.33$		$(N_2, M_{20})$ $ T  = 16, AN = 6.62$		$(N_3, M_{30})$ $ T  = 33, AN = 5.68$	
methods	$W_{08}$	MBA	$W_{08}$	MBA	$W_{08}$	MBA
NTM	5134	5134	19401	19401	20049	20049
TR ( $\mu$ s)	64	10692	316	28209	353	88396
AR ( $\mu$ s)	0.012	2.083	0.016	1.454	0.018	4.409

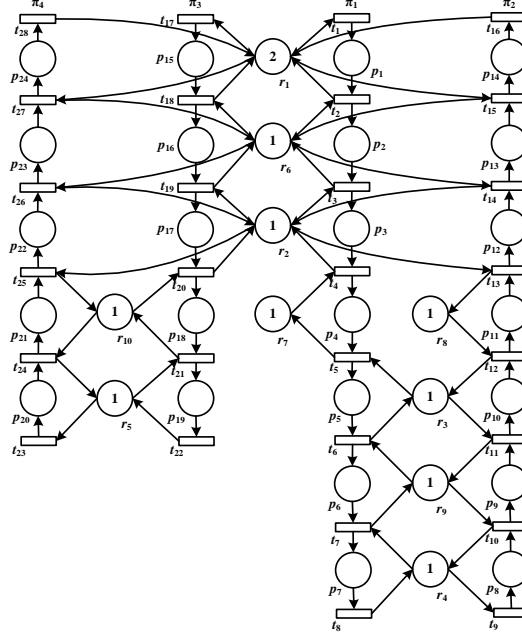


Fig. 7. Petri net model of a railway network system.

Now, we compare MBA with an existing method using the railway network system studied in [9]. It contains ten types of resources (tracks and stations), i.e.,  $r_1$ - $r_{10}$  and four types of trains, i.e.,  $v_1$ - $v_4$ . Their paths are  $\pi_1 = r_1 r_6 r_2 r_7 r_3 r_9 r_4$ ,  $\pi_2 = r_4 r_9 r_3 r_8 r_2 r_6 r_1$ ,  $\pi_3 = r_1 r_6 r_2 r_{10} r_5$ , and  $\pi_4 = r_5 r_{10} r_2 r_6 r_1$ , respectively. Suppose that the capacities of  $r_1$ - $r_{10}$  are 2, 1, 1, 1, 1, 1, 1, 1, 1, and 1, respectively. Note that their capacities are smaller than those given in [9]. This is because the number of reachable markings under those capacities given in [9] is too large to be enumerated. The PN model of the system is given in Fig. 7. The deadlock prevention policy in [9] for this system contains nine constraints, denoted as follows:

- 1)  $M(p_1) + M(p_{13}) + M(p_{15}) + M(p_{23}) \leq 2$ ;
- 2)  $M(p_2) + M(p_{12}) + M(p_{16}) + M(p_{22}) \leq 1$ ;
- 3)  $M(p_5) + M(p_9) \leq 1$ ;
- 4)  $M(p_6) + M(p_8) \leq 1$ ;

$$5) M(p_{17}) + M(p_{21}) \leq 1;$$

$$6) M(p_{18}) + M(p_{20}) \leq 1;$$

$$7) M(p_2) + M(p_{12}) + M(p_{16}) + M(p_{17}) + M(p_{21}) + M(p_{22}) \leq 2;$$

$$8) M(p_5) + M(p_6) + M(p_7) + M(p_8) + M(p_9) + M(p_{10}) \leq 1;$$

$$9) M(p_3) + M(p_{12}) + M(p_{17}) + M(p_{18}) + M(p_{19}) + M(p_{20}) + M(p_{21}) + M(p_{22}) \leq 1.$$

The number of reachable markings under these nine constraints is 11169, while that under MBA is 48622. Obviously, MBA admits much more markings than the deadlock prevention method in [9] does.