Proof of Lemma 3: Let $(CN[\theta], M_{\theta 0}) = (N, M_0) \otimes (C[\theta], M_\theta)$ be the controlled net by $(C[\theta], M_\theta)$ and $\rho(\theta) = \{p \in P \mid p$ is in some A-paths from $T(\theta)$ to $O(\theta)\}$. Then, $\wp(\theta) \subseteq \rho(\theta)$. In $(CN[\theta], M_{\theta 0})$, the set of places in P_θ and $\rho(\theta)$ constitute a support of a P-invariant Δ_θ, and $\Delta_\theta(p) = 1$ $\forall p \in P_\theta \cup \wp(\theta)$. Let $M \in \mathbb{R}(CN[\theta], M_{\theta 0})$. Since $\Delta_\theta \cdot M = \Delta_\theta \cdot M_{\theta 0} = M_0(R(\theta)) - 1$, $\wp(\theta) \subseteq \|\Delta_\theta\|$, and $\Delta_\theta(p) = 1$ $\forall p \in P_\theta \cup \wp(\theta)$, $(C[\theta], M_\theta)$ can limit the number of tokens in $\wp(\theta)$ such that $M(\wp(\theta)) \leq \Delta_\theta \cdot M = M_0(R(\theta)) - 1$. Thus, $(C[\theta], M_\theta)$ can avoid siphon $\phi(\theta)$ from being emptied. The function of $(C[\theta], M_\theta)$ is to constrain $M(\rho(\theta)) + M(p_\theta) = M_0(R(\theta)) - 1$, while the function of $(C[\theta], M_\theta)$ is to constrain $M(\rho(\theta) \cup N^{-1}(\theta)) + M(p_\theta) = M_0(R(\theta)) - 1$, where $N^{-1}(\theta)$ is the set of places in $N^{-1} \cap \underline{C}[\theta]$. Since $M(N^{-1}(\theta)) \geq 0$ and $M(p_\theta) \geq 0$, $(C[\theta], M_\theta)$ is analogous to $(\underline{C}[\theta], \underline{M}_\theta)$. ■

Proof of Lemma 4: Let $(CN[w], M_{w0}) = (N, M_0) \otimes (C[w], M_w)$ be the controlled net by $(C[w], M_w)$ and $\rho(w) = \{p \in P \mid p$ is in some A-paths from $T(w)$ to $O(\prod_1)\}$. Then, $\wp(\prod_1) \subseteq \rho(w)$. In $(CN[w], M_{w0})$, the set of places in P_w and $\rho(w)$ constitute a support of a P-invariant Δ_w, and $\Delta_w(p) = 1$ $\forall p \in P_w \cup \wp(\prod_1)$. Let $M \in \mathbb{R}(CN[w], M_{w0})$. Since $\Delta_w \cdot M = \Delta_w \cdot M_{w0} = M_0(R(\prod_1)) - 1$, $\wp(\prod_1) \subseteq \|\Delta_w\|$, and $\Delta_w(p) = 1$ $\forall p \in P_w \cup \wp(\prod_1)$, $(C[w], M_w)$ can limit the number of tokens in $\wp(\prod_1)$ such that $M(\wp(\prod_1)) \leq \Delta_w \cdot M = M_0(R(\prod_1)) - 1$. Thus, $(C[w], M_w)$ can ensure w is not semi-saturated and semi-empty at any reachable marking in $\mathbb{R}(CN[w], M_{w0})$. The function of $(C[w], M_w)$ is to constrain $M(\rho(w)) + M(p_w) = M_0(R(\prod_1)) - 1$, while the function of $(\underline{C}[w], \underline{M}_w)$ is to constrain $M(\rho(w) \cup N^{-1}(w)) + M(p_w) = M_0(R(\prod_1)) - 1$, where $N^{-1}(w)$ is the set of places in $N^{-1} \cap \underline{C}[w]$. Since $M(N^{-1}(w)) \geq 0$ and $M(p_w) \geq 0$, $(C[w], M_w)$ is analogous to $(\underline{C}[w], \underline{M}_w)$. ■

Proof of Theorem 2: By Lemmas 3 and 4, we know that the controllers for an A-circuit and a closed Ω-structure in this work are both analogous to those in [39]. Since all the controlled maximal A-circuits and ϖ–structures are the same as those in [39], the controller for APNS defined in this work is analogous to that in [39]. Thus, the controlled net is deadlock-free. ■

Proof of Theorem 3: The correctness of RA means that, for any input chromosome Γ, it can output a repaired chromosome Γ_1 such that $\alpha(\Gamma_1)$ is a feasible sequence of transitions from M_0 to M_f. Since M_{C0} is safe, there is at least one transition that is safe at M_{C0}, and such a transition must be in $\alpha(\Gamma)$. Let g^* be a gene that is first appearing in Γ and can be interpreted as such a transition t^*. Then, the algorithm swaps g^* with $\Gamma[0]$. $M_{C0}[t^*> M_1$, where M_1 is safe.

By induction on the length of Γ, we know that Γ can be repaired into a feasible chromosome Γ_1 such that $M_{C0}[\alpha(\Gamma_1)> M_{Cf}$. Since $\alpha(\Gamma_1)$ can fire in (CN, M_{C0}), it can fire in (N, M_0) and $M_0[\alpha(\Gamma_1)> M_f$. ■