

BOUSSINESQ PROBLEM

This document describes an example that is used to verify the elastic deformation capabilities of PLAXIS. The settlement and stress distribution within a semi-infinite, homogeneous and isotropic soil mass with linear stress-strain relationship, under circular uniform pressure, is studied. PLAXIS results are compared with the analytical solution presented by Boussinesq (1885).

Used version:

- PLAXIS 2D - Version 2015.02
- PLAXIS 3D - Anniversary Edition (AE.01)

Geometry: In PLAXIS 2D, an axisymmetric model with 15-noded elements is used. The model is 10 m wide and 10 m deep, as presented in Figure 1. A *Line load* is used to simulate the circular uniform pressure with radius R equal to 0.1 m and magnitude q equal to 10 kN/m/m in vertical direction. A mesh refinement zone is defined with geometry lines, covering an area $1\text{ m} \times 1\text{ m}$ around the load.

In PLAXIS 3D, taking advantage of the model's symmetry, only one-quarter of the geometry is modeled. The model is extended by 10 m in the y-direction. Figure 2 illustrates the model geometry in PLAXIS 3D. A circular surface load with radius R equal to 0.1 m and magnitude q equal to 10 kN/m² in vertical direction is used to simulate the uniform pressure. A mesh refinement zone is defined with geometry surfaces, covering a volume $1\text{ m} \times 1\text{ m} \times 1\text{ m}$ around the load.

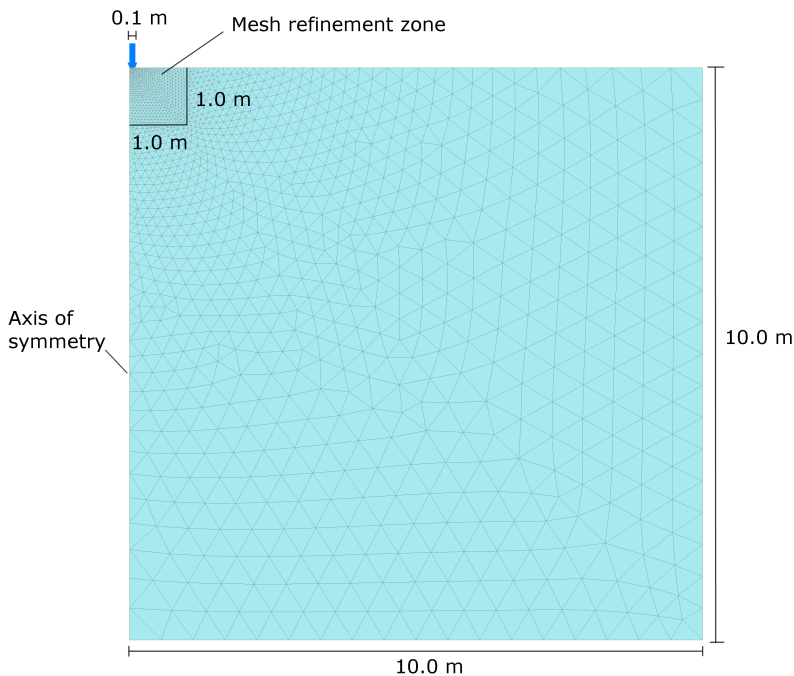


Figure 1 Geometry and generated mesh (PLAXIS 2D)

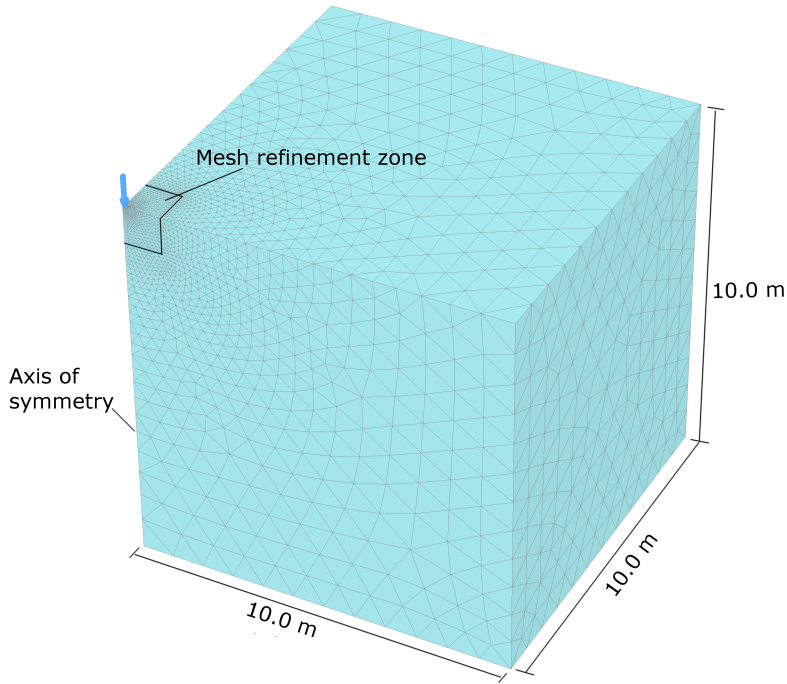


Figure 2 Geometry and generated mesh (PLAXIS 3D)

Material: The soil is modeled as *Linear elastic* with unit weight γ equal to zero. The adopted material parameters are:

Soil: Linear elastic (Drained) $E' = 20000 \text{ kN/m}^2$ $\nu' = 0.3$

Meshing: In PLAXIS 2D, the *Fine* option is selected for the *Element distribution*. A *Coarseness factor* equal to 0.03125 is used for the geometry line representing the load, while a *Coarseness factor* of 0.1 is used for the refinement zone. In PLAXIS 3D, same values of the *Coarseness factor* are used for the surface representing the load and the refinement zone. The *Medium* option is selected for the *Element distribution*. The generated mesh is illustrated in Figures 1 and 2 for PLAXIS 2D and PLAXIS 3D respectively.

Calculations: In the Initial phase zero initial stresses are generated by using the *K0 procedure* ($\gamma = 0$). The line/surface load is activated in a separate phase (Phase 1). The calculation type is *Plastic analysis*.

Output: Total displacements obtained in PLAXIS 2D and PLAXIS 3D are presented in Figures 3 and 4 correspondingly.

Verification: The vertical stress σ_{zz} at depth z under the center of a circular uniform pressure q with radius R is given as (Craig, 2013):

$$\sigma_{zz} = q \left\{ 1 - \left[\frac{1}{1 + (R/z)^2} \right]^{3/2} \right\} \quad (1)$$

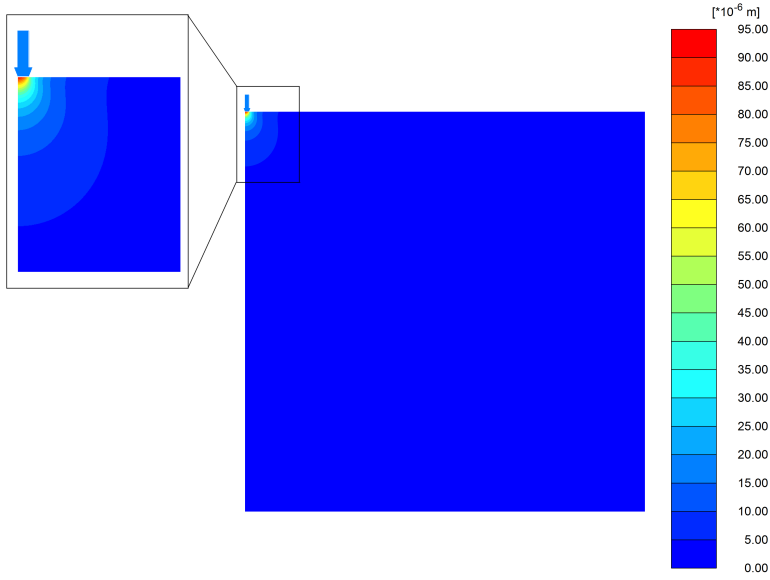


Figure 3 Total displacements (PLAXIS 2D)

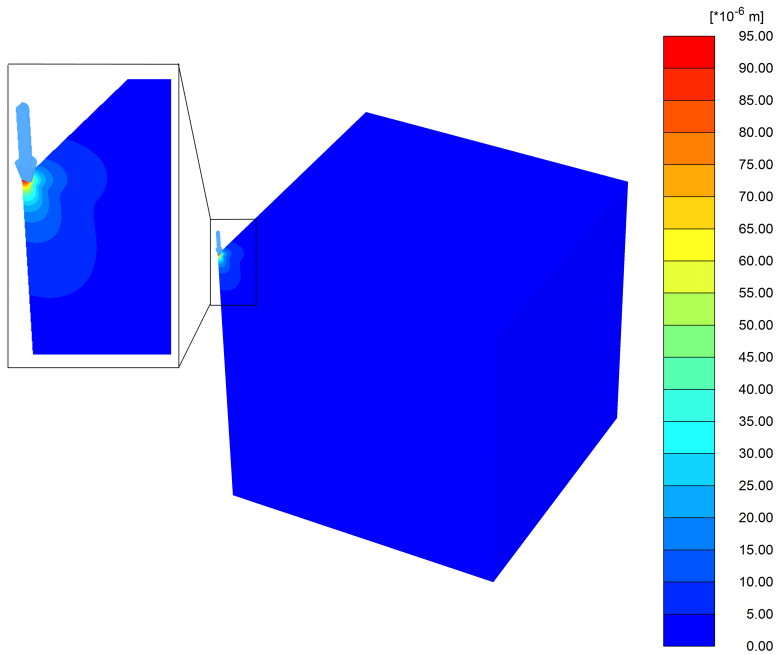


Figure 4 Total displacements (PLAXIS 3D)

The radial stress σ_{rr} under the center equals the circumferential $\sigma_{\theta\theta}$, given as:

$$\sigma_{rr} = \sigma_{\theta\theta} = \frac{q}{2} \left\{ (1 + 2\nu') - \frac{2(1 + \nu')}{[1 + (R/z)^2]^{1/2}} + \frac{1}{[1 + (R/z)^2]^{3/2}} \right\} \quad (2)$$

The vertical displacement s under the circular load is expressed as:

$$s = \frac{2qR}{E'} (1 - \nu^2) I_s \quad (3)$$

in which I_s equals 1.00 for the settlement at the center and 0.64 for the settlement at the perimeter.

Figures 5 and 6 present the vertical and radial stress variation with depth, under the center of the circular load. Table 1 gives the vertical displacement under the center and the perimeter of the load. It is concluded that PLAXIS results are in good agreement with the analytical solution.

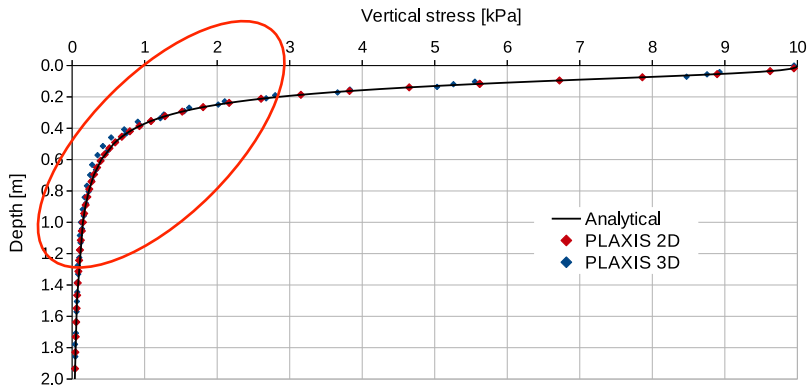


Figure 5 Vertical stress versus depth under the center of the circular load

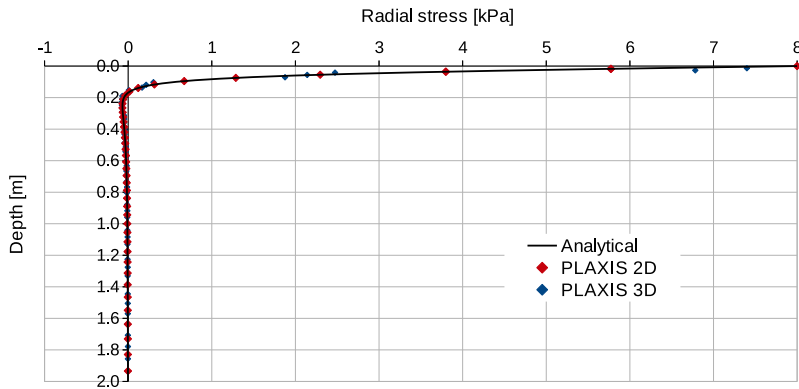


Figure 6 Radial stress versus depth under the center of the circular load

Table 1 Vertical displacement under the circular load

Location	Vertical displacement (mm)			Error	
	Boussinesq	PLAXIS 2D	PLAXIS 3D	PLAXIS 2D	PLAXIS 3D
Center	0.0910	0.0903	0.0904	0.7%	0.6%
Perimeter	0.0582	0.0573	0.0573	1.7%	1.7%

Hint: Boussinesq analytical solution refers to an infinite half space, while the model geometry in PLAXIS is finite. Thus, the further away the model boundaries are set, the closer values to the analytical solution are obtained.

REFERENCES

- [1] Boussinesq, J. (1885). Application des potentiels a l'étude de l'équilibre et du mouvement des solides élastiques: mémoire suivi de notes étendues sur divers points de physique mathématique et d'analyse/par j. boussinesq.
- [2] Craig, F. (2013). Soil mechanics. Springer.

