HW3 — part 1

Cost function $J(w) = \frac{1}{2} \sum_{j=1}^{M} (w^T x^{(j)} - y^{(j)})^2$

$= \frac{1}{2} (Xw - y)^T (Xw - y)$

$\therefore \nabla_w J(w) = \frac{\partial J(w)}{\partial w} = X^T (Xw - y)$

$\therefore H(J)(w) = \frac{\partial (\nabla_w J(w))}{\partial w} = X^T X$

Newton's method:

$w^* = w^{(0)} - H(J)(w^{(0)})^{-1} \nabla_w J(w^{(0)})$

$= w^{(0)} - (X^T X)^{-1} \cdot X^T (Xw^{(0)} - y)$

$= w^{(0)} - X^{-1} (X^T)^{-1} X^T X w^{(0)} + (X^T X)^{-1} X^T y$

$= w^{(0)} - X^{-1} X w^{(0)} + (X^T X)^{-1} X^T y$

$= w^{(0)} - w^{(0)} + (X^T X)^{-1} X^T y$

$= (X^T X)^{-1} X^T y$