

$$J(w, b) = \frac{1}{4} \sum_{x \in X} (x^T w + b - f^*(x))^2$$

$$\frac{\partial J(w, b)}{\partial w} = \frac{1}{4} \times 2 \sum_{x \in X} x (x^T w + b - f^*(x))$$

$$= \frac{1}{2} \left[ \left( \sum x x^T \right) w + \left( \sum x \right) b - \sum x f^*(x) \right] = 0 \quad (1)$$

$$\frac{\partial J(w, b)}{\partial b} = \frac{1}{2} \sum (x^T w + b - f^*(x))$$

$$= \frac{1}{2} \left[ \left( \sum x^T \right) w + 4b - \sum f^*(x) \right] = 0 \quad (2)$$

$$(2) \times \sum_{x \in X} x \Rightarrow \frac{1}{2} \left[ \left( \sum x \sum x^T \right) w + 4 \left( \sum x \right) b - \sum x \sum f^*(x) \right] = 0 \quad (3)$$

$$(3) - 4(1) \Rightarrow \frac{1}{2} \left[ \left( \sum x \sum x^T - 4 \sum x x^T \right) w - \sum x \sum f^*(x) + 4 \sum x f^*(x) \right] = 0$$

$$\therefore w = \frac{\sum x \sum f^*(x) - 4 \sum x f^*(x)}{\sum x \sum x^T - 4 \sum x x^T}$$

$$\sum x \sum f^*(x) = \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right) (0+1+1+0) = 2 \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$\sum x f^*(x) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \times 0 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \times 1 + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \times 1 + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \times 0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\therefore \sum x \sum f^*(x) - 4 \sum x f^*(x) = 2 \begin{bmatrix} 2 \\ 2 \end{bmatrix} - 4 \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0$$

$$\therefore w = 0$$

$$b = \frac{1}{4} \left[ \sum f^*(x) - \sum x^T w \right] = \frac{1}{4} [ (0+1+1+0) - 0 ] = 0.5$$