ROTATION OF HYPERION. I. OBSERVATIONS

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ABSTRACT

Wisdom, Peale, and Mignard (1984) predicted Hyperion to be in a state of chaotic rotation. Simulations indicate that very precise and well-sampled observations over a long period of time are needed to test this prediction (Peale 1986). I have obtained such a dataset after observing Hyperion for 13 weeks (4.5 orbit periods) at observatories in Chile and in Arizona using a CCD camera. Phase-dispersion-minimization analysis of the resulting light curve definitively shows that Hyperion is not in any periodic rotation state, thus implying it is chaotic. Currently, I am attempting to fit this light curve to a dynamical model to determine the principal moments of inertia and the rotation state.

I. INTRODUCTION

Hyperion (SVII) is unique in the solar system. This satellite of Saturn is the only major satellite presently in a nonsynchronous rotation state. In most respects, however, it is an unremarkable object. Hyperion has a semimajor axis of 24.6 Saturn radii, an orbital period of 21.3 days, and an inclination of 0.43°. A 4/3 orbital resonance with Titan forces Hyperion's orbit to remain at a relatively high eccentricity of 0.1042. *Voyager* images revealed the irregular shape of Hyperion, which has radii of $185\times140\times113\pm10$ km (Thomas and Veverka 1985). Hyperion is the most irregularly shaped of the major satellites. This, in conjunction with the large forced eccentricity, led Wisdom *et al.* (1984) to predict that Hyperion would be in a rotation state of chaotic tumbling. Hyperion's nonperiodic rotation state has been conclusively confirmed with observations taken during the summer of 1986 and subsequent analysis.

The rotation state of Hyperion can be determined with well-sampled, precise, ground-based observations. All datasets prior to this work are very undersampled, assuming that Hyperion's rotation is chaotic, as established by Wisdom and Peale (1984) and Wisdom *et al.* (1984). It has also been stressed (Wisdom *et al.* 1984; Peal 1986; Peal and Wisdom 1984; Wisdom and Peale 1984) that the traditional technique of folding the dataset and applying least-squares analysis is futile if the rotation is chaotic and can produce results that can look quasiperiodic. Table I lists all published attempts to resolve Hyperion's rotation state. Andersson (1974) and Goguen *et al.* (1983) used aperture photometers (photomultipliers that give an integrated number of counts for the entire aperture). It is very difficult to subtract the background accurately from data taken with an aperture photometer due to the nonlinear background-light gradient from nearby Saturn. Andersson (1974) presented 13 data points observed over a 742 day interval. After disregarding two of the observations, he presented a light curve "[consistent] with the satellite's brightness being constant." He concluded that Hyperion's magnitude was probably constant or possibly variable at the 0.1 mag level. Andersson did not know of the *Voyager II* results and disregarded two data points as being anomalous. Since his observations were taken with an aperture photometer and were grossly undersampled, Andersson's observations place no constraint on Hyperion's rotation state. Goguen *et al.* (1983) observed Hyperion for 18 nights over a 160 day interval. They folded back the light curve, performed least-squares analysis, and found a 13 day period. However, the fit of their data to this period is not good: observations at identical phase differ by one-half the total amplitude, "many times the measurement error" (Goguen, personal communication 1986). These observations were taken with an aperture photometer and were undersampled as well. Thomas and Veverka (1985) analyzed *Voyager* observations taken over an interval of 61 days. In their first paper (Thomas *et al.* 1984), using 14 low-resolution images and traditional techniques of folding back the light curve and applying least-squares analysis, they reported Hyperion to be in a "coherent 13.1 day spin over [a time span of] 61 days," with the spin axis nearly parallel to the orbital plane. They stated that this "unusual" spin state is consistent with chaotic rotation. Further analysis (Thomas and Veverka 1985) took advantage of the high-resolution images but could not match landmarks using the 13.1 day period: "the net rotation during this time [the sampling period of 61 days] is uncertain by 50%." Binzel *et al.* (1986) presented a light curve of eight observations over a time period of 15 days. It is a light curve with an amplitude of 1.10 mag, twice the amplitude as measured by any other observer. Based on *Voyager* images, however, shape and albedo variations should produce a light-curve amplitude of only 0.4–0.5 mag (Thomas and Veverka 1985). In their analysis, Binzel *et al.* (1986) did not directly mask the light from Saturn or perform any background-sky gradient fitting and subtraction. This probably explains their anomalously large magnitude range. Although observations adequate enough to determine synchronous or periodic rotation states of Hyperion have been attempted, all previous datasets are inadequate to resolve Hyperion's rotation state unambiguously.

This paper is the first of two reports on the investigation of Hyperion's rotation state. This paper addresses only the observations. The second paper will be concerned with fitting the Hyperion observations presented in this paper to a model of chaotic rotation. In the following sections, I will outline my observing procedure and extensively discuss the reduction programs and techniques that I used. After demonstrating that Hyperion is definitely not in any periodic rotation state, I will discuss further work that can be done with this dataset.

[a)] Observations made at McGraw–Hill Observatory, Kitt Peak, operated by the University of Michigan, Dartmouth College, and MIT.

[b)] Visiting Astronomer, Cerro Tololo Inter-American Observatory, National Optical Astronomy Observatories, operated by the Association of Universities for Research in Astronomy, Inc., under contract with the National Science Foundation.

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TABLE I. Hyperion rotation datasets.

Observer	Δm[a]	N[b]	T[c]	Comments
Andersson (1974)	0.10	13	742	Used aperture photometer, omitted two observations.
Goguen *et al.* (1983)	0.48	18	160	Used aperture photometer.
Thomas and Veverka (1985)	0.52	14	61	*Voyager* data.
Binzel *et al.* (1986)	1.10	8	15	No sky fitting/subtraction.
This work	0.53[d]	37	53	38 total observations: one 11 days after main dataset.

[a] Light-curve amplitude.
[b] Actual number of nights on which Hyperion was observed.
[c] Total time interval over which the observations spanned in days.
[d] Omitting observations near opposition "surge."

II. OBSERVATIONS

Hyperion, with an absolute red magnitude $R \approx 14$, has a large light-curve amplitude of $\Delta R \approx 0.5$. An object of this magnitude is generally within the observational capability of a small to moderate sized telescope (diam $\approx$ 1 m) used with an aperture photometer. However, the presence of Saturn, an object almost half a million times brighter and only 1.3–4 arcmin away, causes a large, nonlinear background gradient. Since numerical simulations indicated that nearly nightly sampling and a long baseline were necessary to determine Hyperion's rotation state unambiguously (Peale 1986; Wisdom and Peale 1984), I observed Hyperion as frequently as possible for a period of 13 weeks (91 days), typically half the night for every clear night in this interval (see Table II). My first observations were made at Cerro Tololo Inter-American Observatory (CTIO) one month before Saturn opposition. I then observed for one week at Lowell Observatory and for nine weeks at McGraw-Hill Observatory (MHO). See Table II for a summary of the observations.

At CTIO, I used the 1 m telescope and the Automatic Single-Channel Aperture Photometer (ASCAP) with a 0.7 mm (11.6″) diam aperture. Since there had been published observations of Hyperion made with an aperture photometer, I inferred that background subtraction would not be an insurmountable problem. This turned out not to be the case. I measured the sky using three different methods: (1) I moved the aperture toward, away, and perpendicular to Saturn and then averaged the four measurements; (2) I used an aperture that had four holes offset from the center, allowing me to accomplish nearly the same measurement as described in method (1) without having to move the telescope; and (3) I used an aperture that had two holes offset from the center aligned in the direction perpendicular to Saturn, allowing me to measure the sky background at a number of different radial positions. Then I fit for the sky value at Hyperion's position a procedure similar to using an area-scanning photometer (as described by Franz and Millis 1971). The general observing technique and equipment were adequate to reproduce standard stars to better than 1% (1% $\approx$ 0.01 mag), regardless of the method used to subtract the sky. Although I attempted sky subtraction around Hyperion with these three independent methods, none gave consistent results. The sky gradient was too large and nonlinear to allow accurate measurements of the sky value using the techniques explained above. Numerical simulations indicate that Hyperion observations must be accurate to within about $\pm$ 3% to be useful for model fitting. An accuracy of $\pm$ 5% is one-fifth the light-curve amplitude, and thus questionable for period fitting. Since my reduced CTIO observations of Hyperion had formal and statistical uncertainties ranging from 0.02 to 0.3 mag, it was obvious that I had not achieved the required precision and thus do not include these observations. This work also demonstrates that previous aperture photometry of Hyperion may not be as precise as formal uncertainties indicated.

At Lowell Observatory, I used the Mark IV TI 800×800 CCD camera on the 0.6 m Morgan telescope. Cloudy observing conditions and a large image scale made the photometry noisy (see Table II). As mentioned in Sec. I, accurate photometry requires good subtraction of the background-sky gradient. Background subtraction is much more accurate with a smaller (more expanded) scale. While sky subtractions were performed and the observations reduced to magnitudes, the uncertainties were still relatively large, 0.05–0.10 mag. Since I have only two isolated and noisy observations from the Lowell observing run, which are separated from MHO observations by nine days, I will not include these data in the analysis, therefore avoiding any systematic errors that might otherwise occur. Caution must always be

TABLE II. Observing sites.

Observatory	Scale	Dates (1987)	Nights allocated[a]	Nights photometric[b]
CTIO[c] 1.0 m	16.5″/mm	05/04–05/17	14	8
Lowell 0.6 m	1.5″/pixel	05/20–05/26	7	2
MHO 1.3 m	0.6″/pixel	06/01–06/16	16	12
MHO 2.4 m	0.6″/pixel	06/17–07/06	16	14
MHO 1.3 m	0.6″/pixel	07/07–08/05	30	15

[a] Number of nights on which observing time was allocated at these telescopes.
[b] Number of photometric nights (less than 4% sky variation).
[c] Used aperture photometer.

taken when deciding to discard data, but the Lowell and CTIO datasets were clearly not usable for the determination of Hyperion's rotation state.

The observations from MHO are of high quality. I observed Hyperion from 31 May 1987 through 5 August 1987. Before "monsoon season" began, there were only 11 nights of nonphotometric weather, with most of the other nights being photometric to 1%–2%, as determined from extinction curves. In addition to the nonphotometric nights, I did not obtain data on three nights, one due to equipment failure, one because of high wind, and one when the diffraction spike from Titan was too close to Hyperion. As can be seen from Table I, 37 useful nights of Hyperion data were obtained over a time period of 53 days, with another observation 11 days later during the monsoons.

I observed Hyperion with the 1.3 and 2.4 m telescopes at MHO using focal ratios $f/13.5$ and $f/7.5$, respectively, with the MASCOT CCD (Meyer and Ricker 1980) and the MIS (Multiple Instrument System) CCD finder/guider. On both telescopes, the scale was 0.6″/pixel. All observations were made through standard KPNO filters in the R passband, central wavelength $\lambda_0 = 634$ nm and full width at half-maximum FWHM = 124 nm, and V passband, $\lambda_0 = 548$ nm and FWHM = 117 nm. I used the R filter because the chip is most sensitive in this region of the spectrum and because the extinction is typically less in R than in other passbands. V observations were made on six nights so that accurate color corrections could be made. To reduce the scattered light from Saturn as much as possible, a half-mirror was placed in the optical path such that the direct light from Saturn was never near the CCD chip, optics, or even the sides of the photometer box, and rough black cloth was placed on all flat surfaces inside the photometer box. The MASCOT has a readout noise of approximately 11 electrons and a gain of 13.6 electrons/ADU (Analog-to-Digital Unit). The bias level was ≈ 100 ADUs and varied less than 2% over a given night. The dark current was measured to be less than the uncertainty in the bias level for the longest exposures used. Although the MASCOT is linear to better than 1%, all frames were typically exposed to the same signal level, including the flat frames. The MIS was used first to find the field and then as a guider so that any telescope-tracking errors were insignificant.

I determined that there was no systematic variation of the bias level during a night and that the random variations were less than 2 ADUs. Bias measurements were made at the beginning and end of each night. Flat frames were taken nearly every night using a prepared spot inside the dome, the twilight sky, or both. A set of three to five Titan observations of ≈ 1 s exposure were always observed before and after Hyperion to monitor sky conditions over short timescales. Furthermore, Titan was observed throughout the night to obtain an accurate determination of the extinction coefficient. Titan is constant after correction to mean opposition magnitude (see Sec. III). Landolt (1983) standard stars were observed on approximately one-third of the nights and were extensively observed on three nights to accurately determine the color correction and second-order extinction correction (both of which were small, the latter negligible). The Landolt standard stars I observed were 106-700, 106-834, 107-970, HD 149382, 109-747, and 110-340. Of these, 107-970 was found to differ from the tabulated value by 0.316 mag: $R_{\text{meas}} = 10.082 \pm 0.016$ and $R_{\text{Landolt}} = 9.766 \pm 0.013$. Independent observations by John Kruper (1987) confirmed that 107-970 was not at the magnitude tabulated by Landolt. In his paper, Landolt (1983) notes that 107-970 is variable.

Typically, Hyperion was observed for at least one-half the night. No observations were made at greater than 2.5 airmasses, and most observations were at less than 2.0 airmasses. Hyperion was located by calculating offsets to the Saturn ephemeris based on the orbital elements found in the *Astronomical Almanac*. This is more accurate than the tabulated offsets in the *Astronomical Almanac*. Hyperion's motion in the sky was easily detected over a time period of 15 min, confirming its identity. Although most of the light from Saturn around Hyperion was masked, exposure times ranged from 100 to 600 s depending on Hyperion's position in its orbit and its proximity to Saturn, which varied from 80″ to 240″. In addition, the telescope aperture size and the phase and position of the Moon affected the exposure time. For consistency, exposures were taken such that the total signal was approximately 90% of the saturated value of the CCD, typically 300 s. In order to detect nearby stars and cosmic rays as Hyperion moved through the starfield, an average of 11 independent Hyperion observations were made each night. I rejected some of these observations, however, because of background stars or cosmic rays near Hyperion (within about 3″). The actual number of observations used in the final analysis is given in Table III. A large number of independent observations also allowed a check on the consistency of the internal and external uncertainties.

I observed Hyperion for 13 weeks straddling the time of Saturn opposition. The observations were made with minimized scattered light from Saturn. Careful measurements of extinction coefficients and color corrections were made for greater accuracy. Since precise observations are necessary for this project, I have included only observations from MHO because I had consistently clear weather and a good telescope/detector combination. CTIO observations are not included because sky-background subtraction was inaccurate when performed with an aperture photometer. Lowell Observatory observations are not included because of marginal weather. The dataset presented is from observations made only at MHO.

III. DATA REDUCTION AND ANALYSIS

Processing the images to obtain reliable, accurate magnitudes of Hyperion involved the following steps: flattening the raw images, subtracting the background-sky gradient, performing photometry of Hyperion in a crowded starfield, conversion of instrumental magnitudes to Johnson magnitudes, and correcting for phase and geometry.

In general, flattening the frames for an object as bright as Hyperion would not be very critical because the pixel-to-pixel variation is less than a few percent. However, there is a faint ($\leqslant 1\%$) grid pattern on all of the frames, making background fitting susceptible to systematic error. Therefore, all bias-subtracted frames were divided by normalized bias-subtracted flatfields. This eliminated the grid pattern and the frames were flattened to better than 0.1%. On the nights in which I had both dome flats and twilight flats, the final magnitudes of dome-flattened and twilight-flattened images differed by less than 0.003 mag.

From the discussion of sky subtraction with an aperture photometer, it is apparent that sky fitting and subtraction is necessary for accurate photometry of the Hyperion images. Sky subtraction was done using IRAF (Image Reduction and Analysis Facility) (Tody 1986) tasks. Typically, I

TABLE III. MHO observations.

Date	Day[a]	Phase[b]	r[c]	Δ[d]	N[e]	R Mag[f]
06/01	152.4	0.90	10.028	9.023	2	14.107 ± 0.016
06/03	154.4	0.70	10.028	9.020	6	14.360 ± 0.014
06/08	159.3	0.15	10.028	9.014	3	13.952 ± 0.019
06/09	160.2	0.03	10.028	9.014	9	13.826 ± 0.006
06/10	161.3	0.10	10.028	9.014	7	14.101 ± 0.042
06/11	162.4	0.23	10.028	9.014	6	14.124 ± 0.021
06/13	164.4	0.48	10.028	9.016	6	14.046 ± 0.083
06/14	165.3	0.60	10.029	9.017	9	14.095 ± 0.025
06/15	166.2	0.70	10.029	9.019	8	14.151 ± 0.006
06/16	167.2	0.80	10.029	9.021	12	14.131 ± 0.016
06/17	168.4	0.90	10.029	9.023	7	14.419 ± 0.009
06/18	169.2	1.00	10.029	9.025	11	14.431 ± 0.005
06/19	170.3	1.13	10.029	9.028	10	14.445 ± 0.006
06/21	172.4	1.38	10.029	9.035	7	14.146 ± 0.003
06/22	173.4	1.50	10.029	9.038	7	14.236 ± 0.006
06/23	174.4	1.60	10.029	9.042	9	14.216 ± 0.006
06/24	175.2	1.70	10.029	9.046	3	14.355 ± 0.022
06/29	180.2	2.28	10.030	9.072	5	14.315 ± 0.016
06/30	181.2	2.40	10.030	9.077	6	14.261 ± 0.005
07/02	183.3	2.60	10.030	9.089	7	14.568 ± 0.008
07/03	184.3	2.70	10.030	9.098	9	14.733 ± 0.011
07/04	185.3	2.80	10.030	9.102	6	14.490 ± 0.010
07/05	186.2	2.90	10.030	9.112	17	14.405 ± 0.005
07/06	187.2	3.00	10.030	9.119	7	14.560 ± 0.022
07/07	188.3	3.10	10.030	9.127	11	14.491 ± 0.004
07/08	189.2	3.20	10.030	9.136	16	14.559 ± 0.025
07/09	190.2	3.30	10.030	9.144	9	14.691 ± 0.016
07/11	192.2	3.50	10.030	9.161	20	14.312 ± 0.007
07/12	193.3	3.60	10.030	9.170	6	14.385 ± 0.016
07/13	194.2	3.70	10.030	9.179	11	14.485 ± 0.007
07/14	195.2	3.80	10.031	9.189	13	14.485 ± 0.006
07/17	198.2	4.08	10.031	9.219	7	14.430 ± 0.007
07/19	200.2	4.23	10.031	9.240	7	14.514 ± 0.004
07/21	202.2	4.38	10.031	9.262	15	14.587 ± 0.005
07/22	203.2	4.45	10.031	9.273	3	14.567 ± 0.006
07/23	204.2	4.53	10.031	9.284	11	14.414 ± 0.004
07/24	205.2	4.60	10.031	9.297	8	14.343 ± 0.022
08/04	216.2	5.43	10.032	9.438	6	14.572 ± 0.017

[a] Day number after January 0, 1987.

[b] Solar phase angle in degrees.

[c] Heliocentric distance (AU).

[d] Geocentric distance (AU).

[e] Total number of independent Hyperion observations.

[f] Measured Johnson R Magnitude (nightly mean).

would use a two-dimensional Chebyshev sixth-order polynomial to fit for the background gradient. On nights when Hyperion was near elongation or conjunction, causing it to be aligned along the telescope spider diffraction spikes, I would use a similar one-dimensional fit for each row or column. All subtractions were good, even when Hyperion was near other bright stars. This is illustrated in Figs. 1(a) and 1(b), in which the results of background fitting and subtraction for a typical Hyperion image are shown. Note that this is a representative case. Background subtraction was performed on every Hyperion image with similar good results.

Photometry of Hyperion was further complicated because it was located near the Milky Way (galactic longitude $\approx 1°$, latitude $\approx 12°$). There were usually ten or more stars nearby, each bright enough to cause a greater than 1% difference in the final value of Hyperion's magnitude. It was possible to subtract accurately the nearby stars and perform the usual aperture photometry using DAOPHOT (Stetson 1987), a crowded-field photometry program. This program was designed to use some of the stars in the frame as a model for the point-spread function (psf), which could then be fitted and/or subtracted. For each Hyperion frame, I identified every star near Hyperion within ≈ 20 pixels (12″). Then, choosing the brightest stars in the entire frame, I obtained a model psf that was scaled and subtracted from all identified stars. At this point, normal aperture photometry could be performed. I chose an aperture size that included greater than 99% of the light, as determined by radial intensity profiles, typically 15 pixels (9″) in radius. Figure 1(c) is a surface plot of the Hyperion frame after subtracting the nearby stars using DAOPHOT. Note that even the very bright star nearby subtracted well. This figure is an example of star subtraction that is again representative of the fields encountered. I used this method on the Titan and standard-star frames to avoid

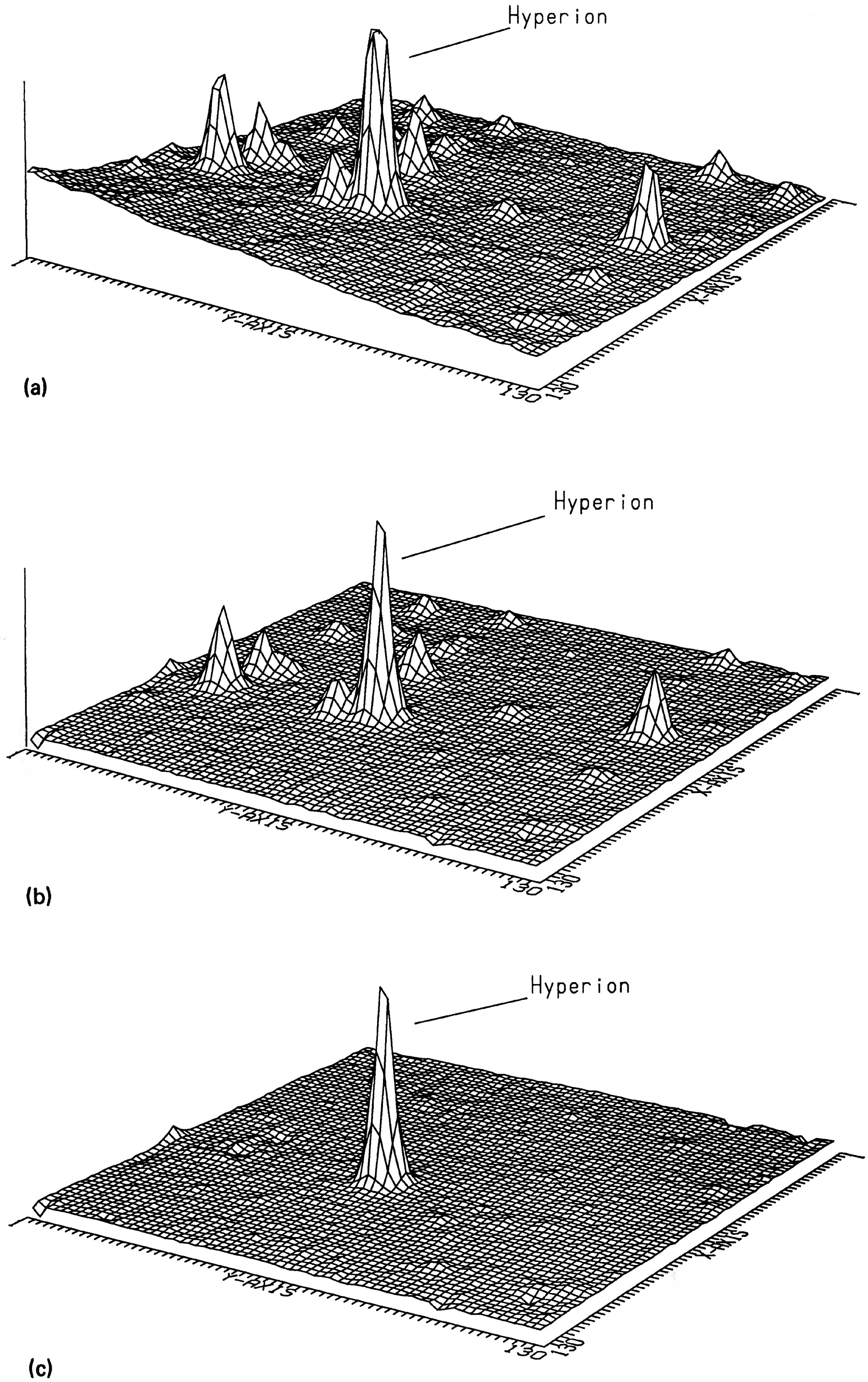

FIG. 1. Reduction of the flattened Hyperion frames. (a) is a surface plot of the flattened Hyperion image. Notice the background gradient. (b) is the same image after fitting the sky gradient and subtraction from (a). (c) is after point-spread-function fitting and subtraction of the surrounding stars. The residual sky variation is less than 1% in the region near Hyperion. Hyperion is the large peak in all images. These are representative of the images I obtained and reduced.

any systematic deviation. However, no sky subtraction or psf fitting was necessary, even when Titan was at conjunction. I used DAOPHOT to do photometry of stars as faint as Hyperion in the same background-sky gradient with very good results. All of these star observations were within ≈ 0.01 mag after correction for extinction.

The instrumental magnitudes were converted to absolute magnitudes using

$$R = -2.5 \log C - kX + \epsilon(V-R) + \zeta,$$

where R is the Johnson R magnitude (similarly for V), C is the number of counts (ADUs) per second, k is the extinction coefficient, X is the airmass, ϵ is the transformation (color) coefficient, and ζ is the zero point (see, for example, Hendon and Kaitchuck 1982). The extinction and zero point were measured each night. All nights were photometric to at least 4%, and most to 1%–2%, as determined from inspection of the extinction curves and calculation of the formal uncertainties. The color transformation was measured three times and was found to be virtually the same for both the 1.3 m and 2.4 m telescopes, which is to be expected because the filters and detector were the same. Second-order extinction coefficients were also calculated but were found to be negligible. The R color transformation coefficient was calculated to be $\epsilon_R = -0.066 \pm 0.009$. Since this is a small coefficient, even a relatively large uncertainty in the color of Hyperion would cause a minor error in the final magnitude, ϵ_V is even smaller: $\epsilon_V = -0.001 \pm 0.012$. The color of Hyperion was measured on six separate nights and was found to be $V-R = 0.41 \pm 0.02$ and constant within the uncertainty. Table IV is a tabulation of these measurements. Landolt standards were used for comparison on these nights, and Titan was used as a standard on other nights. I chose to use Titan as a standard because its proximity to Hyperion allowed me to check the sky variation efficiently while the extinction correction was necessarily the same as for Hyperion. Titan has proven to have a stable magnitude over timescales of months (Andersson 1977). I found no secular change in Titan's magnitude on the nine nights measured over a 2 month period. The rms variation of these Titan magnitudes, after reduction to mean opposition magnitude, is 0.005 mag. Any systematic error due to using Titan as a standard is therefore insignificant. I used a phase coefficient of $\beta_R = 0.0015 \pm 0.0010$ mag/deg based on extrapolation of measurements by Andersson (1974) and comparison of Noland *et al.* (1974) *ubvyr* data; $\beta_V = 0.0036 \pm 0.0012$ (Andersson 1974). I reduced all Hyperion, Titan, and standard-star photometry to R magnitudes, as defined above. Of the average nine independent observations of Hyperion per night (see Table III), I calculated a formal propagated uncertainty as well as the rms uncertainty. Typically, these internal and external uncertainties agreed, but when they did not, I chose the larger uncertainty, and these are the values

TABLE IV. Hyperion $V-R$ color.

Date (1987)	V	$V-R$
06/08	14.01 ± 0.02	0.38 ± 0.02
06/09	14.57 ± 0.04	0.46 ± 0.05
06/17	14.60 ± 0.02	0.40 ± 0.02
07/03	14.60 ± 0.03	0.45 ± 0.04
07/05	14.62 ± 0.03	0.40 ± 0.05
07/18	14.54 ± 0.04	0.43 ± 0.06

listed in Table III. Although some of the uncertainties in my reduced magnitudes are small, < 0.01 mag, they are realistic because the large number of observations provided a check on the results. On no night did I find an unambiguous trend in the brightness variation of Hyperion. Hyperion's brightness was constant over a time period of 6 hr at the 0.01 mag level. Table III lists the raw mean nightly magnitudes of the MHO Hyperion observations. Figure 2 is a plot of the data listed in Table III.

Figure 3 is a plot of the measured magnitudes versus solar phase angle. There is an obvious brightening with decreasing phase angle and a surge of 0.2–0.4 mag at phase angles less than $\approx 0.3°$. The exact size of the opposition surge is difficult to determine because of the rotational variation. This variation due to rotation will necessarily make any phase correction inaccurate. Fortunately, the analysis is relatively insensitive to phase correction, as will be shown later. With this in mind, I use the H,G phase model (Bowell, Harris, and Lumme 1987) developed for asteroids and satellites. To do this, I first make the geometric correction to mean opposition distance using the relation

$$R' = R - 5 \log \frac{r\Delta}{r_s \Delta_s},$$

where R' = the mean opposition R magnitude uncorrected for solar phase, r = heliocentric distance, Δ = geocentric distance, $r_s = 9.54$ AU, Saturn's mean heliocentric distance, and $\Delta_s = 8.54$ AU, Saturn's mean geocentric distance. The H,G model is an empirical model written in the form

$$R'(\alpha) = H - 2.5 \log[(1-G)\Phi_1(\alpha) + G\Phi_2(\alpha)],$$

where H equals the reduced magnitude at mean brightness corrected for mean opposition and zero phase, $\alpha = 0°$, G = slope parameter, and $\Phi_i \equiv \exp\left[-A_i(\tan \alpha/2)^{B_i}\right]$, with the following parameters (as given by Bowell, Harris, and Lumme 1987): $A_1 = 3.331$, $B_1 = 0.628$, and $A_2 = 1.866$, $B_2 = 1.217$.

Figure 4 shows the mean opposition magnitudes of the Hyperion nightly means using the least-squares fitted values: $H = 13.81 \pm 0.05$, and $G = 0.056 \pm 0.14$.

For comparison, asteroid 1 Ceres has a slope parameter of $G = 0.083$. The H,G phase model fit for Hyperion is shown in Fig. 3, using the fitted values for the solid line. It is important to note that (1) Hyperion has a rotational variation of about 50%, making the fit inexact, (2) the H,G model may not be adequate for objects that exhibit a large opposition surge (as stated by Bowell *et al.* 1987), and (3) the exact form of the phase correction does not affect the period-fitting analysis. These effects lead to phase corrections which give an approximation to the zero-phase light curve shown in Fig. 4. Next I discuss how the period fitting is insensitive to the exact form of the phase-corrected light curve.

Phase-dispersion-minimization (pdm) techniques give an unbiased best period with no dependence on a fitting function (Stellingwerf 1978). For three methods of phase correction, the results of pdm analysis agree to within a few percent: the H and G phase correction, linear phase correction with some or all of the observations, and no phase correction. The best period for the light curve of the nightly means (Fig. 4) is 6.6 days, as can be seen in the pdm plot in Fig. 5. A measure of the goodness of the 6.6 day period shows it to be statistically insignificant. This is demonstrated by inspection of the phase plot in Fig. 6 in which the data are folded back with the 6.6 day period. Note the large differences at various

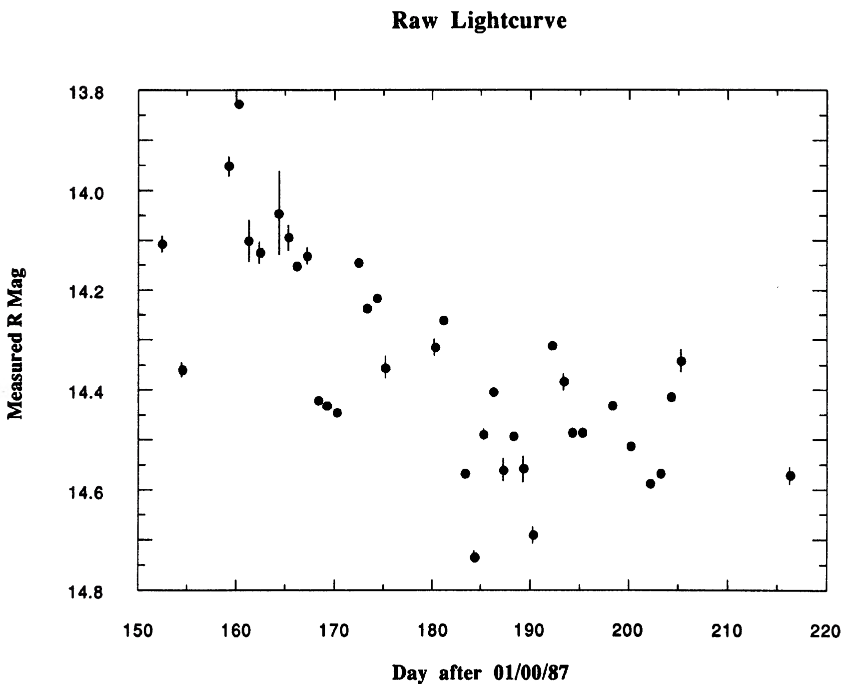

FIG. 2. Hyperion light curve. Measured Johnson R magnitudes uncorrected for mean opposition distance or solar phase angle versus time in days after 01/00/87 (see Table III). Error bars are not shown for points having a formal uncertainty less than 0.01 mag (the size of the dot).

phases, indicating that the fit is not good. Similarly, a period was calculated using the entire dataset instead of the nightly means, which also proved to be statistically insignificant. This shows that short (<1 day) periods are also not fit by the data, which was also demonstrated by the fact that no trends were seen in any of the nightly variations. The phase plot of 13.8 days was similarly unconvincing as a period. In addition, neither the 6.6 nor 13.8 day period is some simple fraction of the orbital period of 21.3 days or some integer multiple of the sampling rate of once per night. The period for the best fit is similar to those periods derived from numerical simulations of a chaotically rotating Hyperion-modeled triaxial ellipsoid using traditional least-squares analysis (Peale 1986; Wisdom and Peale 1984).

Many different phase functions for both the nightly means and the entire dataset were analyzed using the pdm technique. All phase plots for absolute and local minima and corresponding harmonics were plotted. For comparison, the pdm plot for the nightly means with no phase correction is shown in Fig. 7, corresponding to the light curve of Fig. 2.

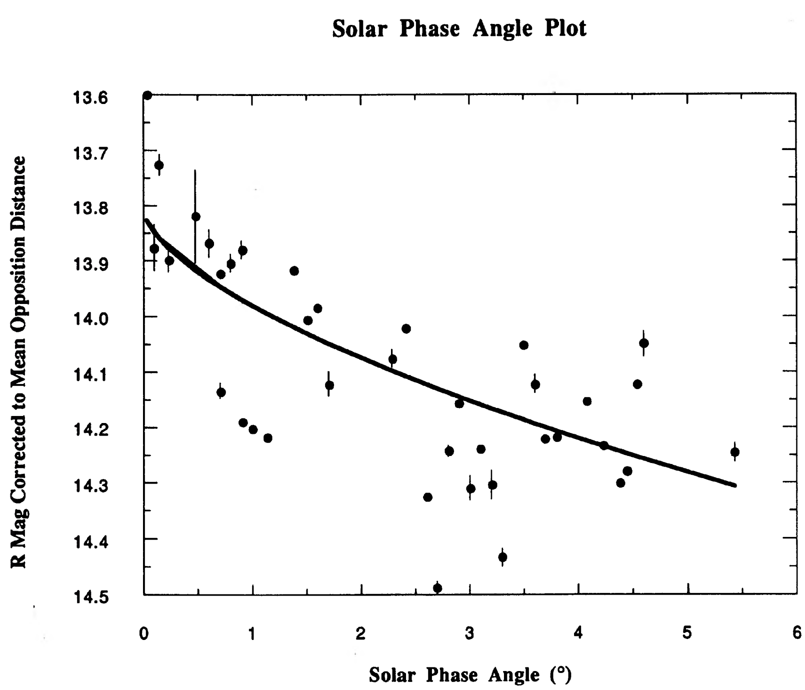

FIG. 3. Hyperion R magnitudes corrected to mean opposition distance versus solar phase angle in degrees (see Table III). The solid line is the least-squares fit to the H,G phase model: $H=13.81$, $G=0.56$. Error bars are not shown for points having a formal uncertainty less than 0.01 mag (the size of the dot).

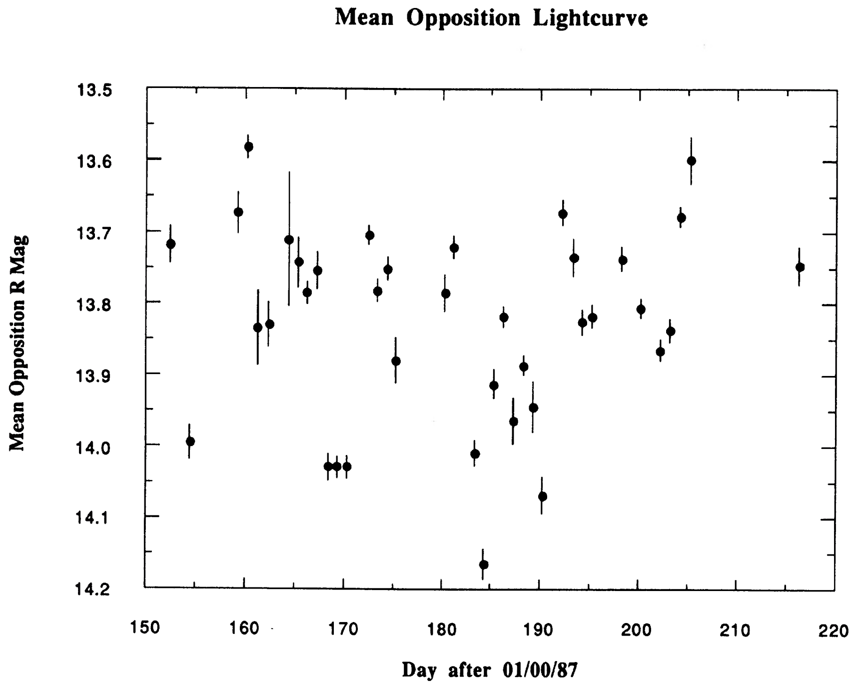

FIG. 4. Hyperion light curve corrected to zero solar phase angle ($\alpha = 0$) and mean opposition distance ($r_s = 9.54$ AU and $\Delta_s = 8.54$ AU). Note the change in scale from Figs. 2 and 3.

The absolute minimum value of the pdm plot shown in Fig. 7 agrees with that value using the *H,G* model phase correction, shown in Fig. 5, and the overall structure is similar. No well-defined period was found for any of the minima. All of the phase plots for these trials looked as scattered as Fig. 6. No period from 1 hr to 50 days even comes close to fitting the dataset satisfactorily.

IV. DISCUSSION

Voyager information on Hyperion's shape led Wisdom *et al.* (1984) to predict that Hyperion would be in a rotation state of chaotic tumbling. All previous published observations (Andersson 1974; Goguen *et al.* 1983; Thomas and Veverka 1985; Binzel *et al.* 1986) were too undersampled to make any definitive conclusions concerning Hyperion's rotation state. Peale (1986), Wisdom *et al.* (1984), Wisdom and Peale (1984), and Peale and Wisdom (1984) warn of the dangers of traditional methods of folding back the light curve and performing least-squares analysis on data sampled less than about once every 1.5 days for a chaotically rotating Hyperion. Goguen *et al.* (1983) and Thomas and Veverka (1985) find best-fit periods using this technique. Both fits, however, produce inconsistent results that are possibly consistent with chaotic rotation. None of the previous datasets have been able to constrain Hyperion's rotation state.

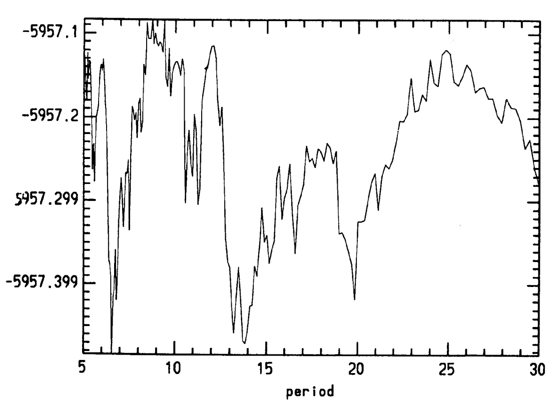

FIG. 5. Phase-dispersion-minimization (pdm) plot of the light curve with *H,G* phase correction corresponding to Fig. 4. Best period is 6.6 days. Ordinate is a relative measure of the dispersion significance of the period; values near zero are good fits.

With an average of nine independent Hyperion observations per night, I obtained a light curve using the MHO 1.3 and 2.4 m telescopes with the MASCOT/MIS detector at the required sampling rate. My final light curve contains 38 nightly means over a period of 64 days (Fig. 4). I found that Hyperion was essentially at a constant brightness over a period of one night and that its color is $V - R = 0.41 \pm 0.02$. The light-curve amplitude, after correction to mean opposition magnitude (Fig. 4), is ≈ 0.6 mag. This is close to what would be expected from the *Voyager 2* knowledge of shape and albedo.

With pdm analysis, I demonstrated that no period from 1 hr to 7 weeks fits the light curve. Although a large part of the discussion in Sec. III concentrated on fitting the *H,G* phase function to my light curve, the essential point is that the analysis is insensitive to the phase correction. This is demonstrated by comparing the pdm plots with phase correction (Fig. 5) and without phase correction (Figs. 7), noting that there is no well-fit period for any of the minima.

CCD photometry of Hyperion over an interval of 64 days shows no evidence of periodic modulation in the light curve. Is this conclusion equivalent to Hyperion being chaotic? Chaos has a very specific definition: chaotic motion is deterministic but unpredictable motion due to exponential divergence of nearby initial conditions (Hénon and Heiles 1964; Wisdom 1987). While this work does not, and cannot, *prove*

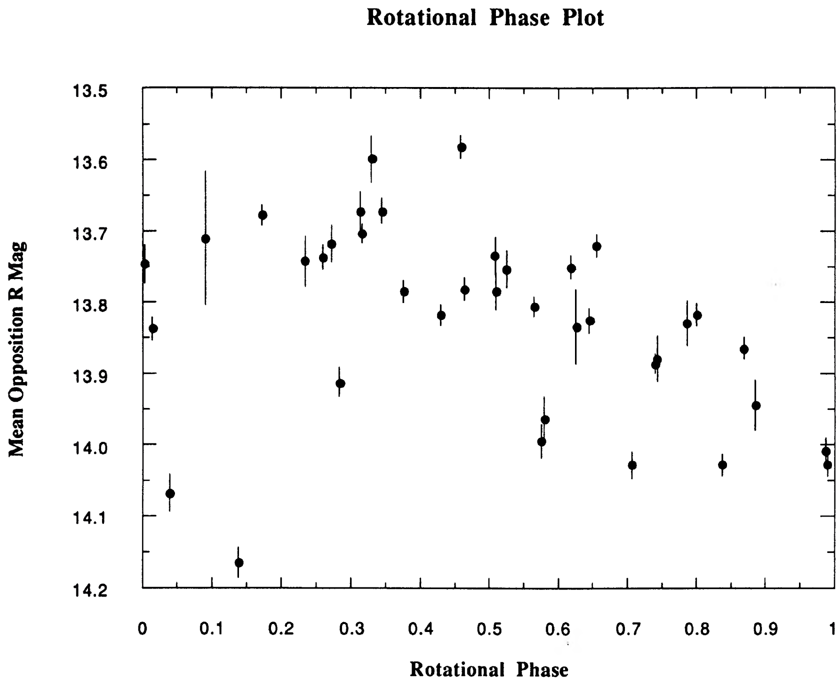

FIG. 6. Light curve folded back (phase plot) using the best-fit period of 6.6 days (see Fig. 5). The large variations at any given phase indicate that this is not a good fit.

that Hyperion is in a chaotic rotation state, it is very strong circumstantial evidence that Hyperion is tumbling chaotically. Furthermore, this is the only dataset from which this can be stated with conviction because of the problems with previous observations/analyses and because this is a high-quality, well-sampled light curve.

My current research goal is a fit for the rotation state of Hyperion based on the present dataset, employing *Voyager* data for shape and albedo information. Assuming that Hyperion is rotating chaotically, it is possible to fit for the initial conditions: the angles describing its position in space, the angular velocities associated with those angles, and the principal moments of inertia. This fitting is possible because the equations describing the spin-orbit coupling of a triaxial-ellipsoid model of Hyperion are well defined (Wisdom *et al.* 1984). Numerical simulations by Chakrabarty (1988, unpublished) indicate that when fitting to chaotic trajectories, an exponential decrease in the uncertainty of the initial conditions is found with increasing number of observations, in contrast with the usual $1/\sqrt{N}$ expected from elementary statistics. Although Chakrabarty used a simple dynamical system, my numerical simulations of a triaxial-ellipsoid model of Hyperion suggest that the moments of inertia and rotation state of Hyperion can be determined to better than 5% from fitting of my MHO light curve, with the largest uncertainty due to inadequate knowledge of Hyperion's shape.

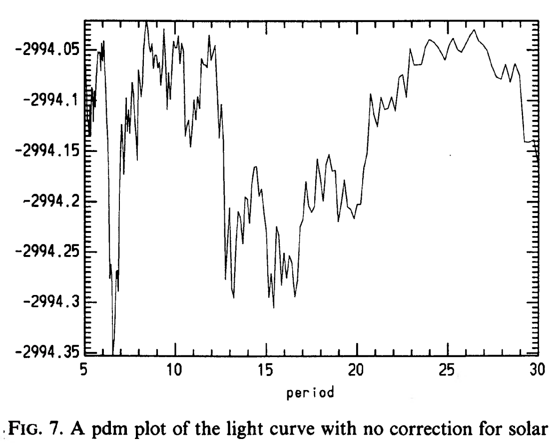

FIG. 7. A pdm plot of the light curve with no correction for solar phase angle corresponding to Fig. 2. Note that this pdm plot is similar to the pdm plot of Fig. 5, indicating that the period fitting is insensitive to the phase correction.

V. CONCLUSIONS

Hyperion is not in any regular/periodic rotation state. Hyperion exhibits a strong brightness variation with phase, including an opposition surge of approximately 0.3 magnitudes at solar phase angles less than about 0.3°.

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