厦門大學
XIAMEN UNIVERSITY

Quantum Information and Quantum Computation

Yuanyuan Chen

College of Physical Science and Technology
Xiamen University
Email: chenyy@xmu.edu.cn
http://qolab.xmu.edu.cn

《美国白宫报告书》

政府支持：

- 美国、欧盟、日本等
- 中国也启动了量子调控重大研究计划
- 中国地方政府支持

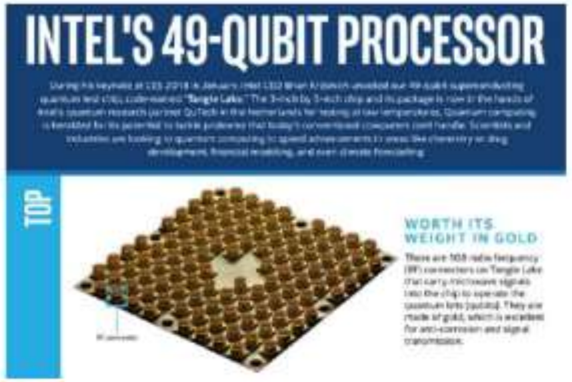
欧盟《量子宣言》2016.05

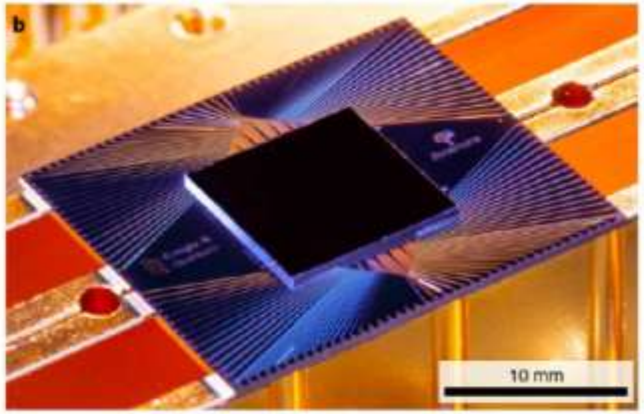
National Quantum Initiative Act

Long title	An Act to provide for a coordinated Federal program to accelerate quantum research and development for the economic and national security of the United States.
Enacted by	the 115th United States Congress
Effective	December 21, 2018

美国《国家量子计划法案》2018.12

欧洲量子旗舰计划
2018.10



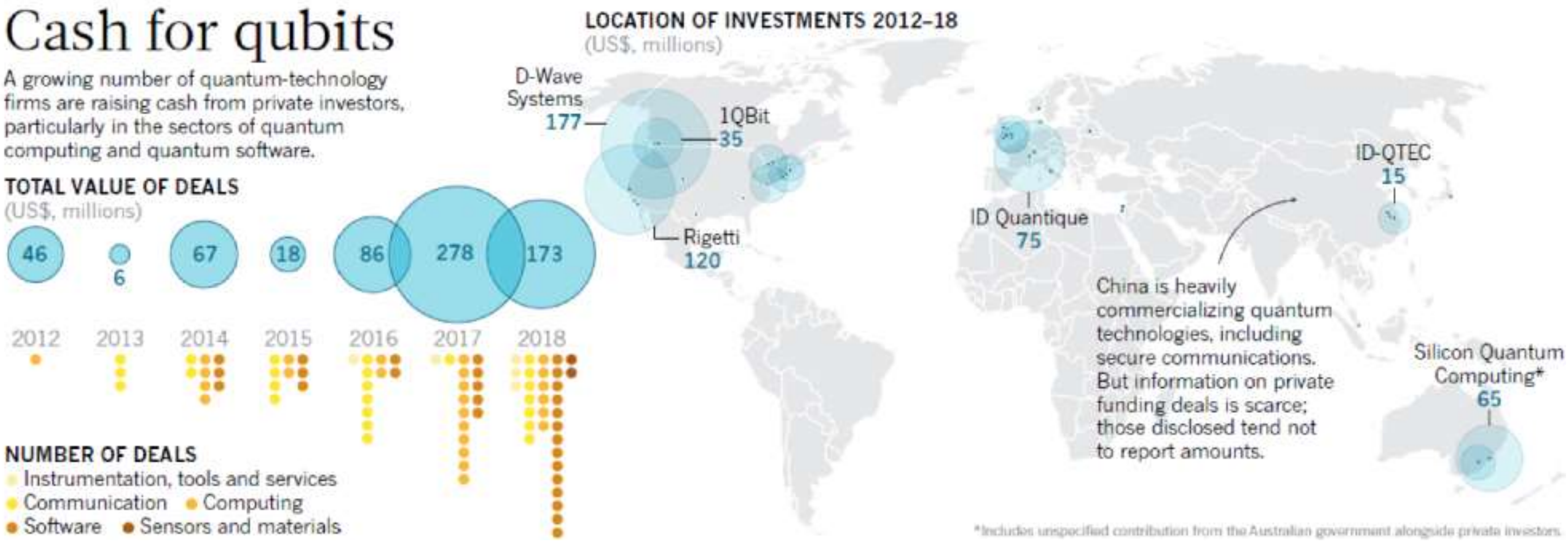


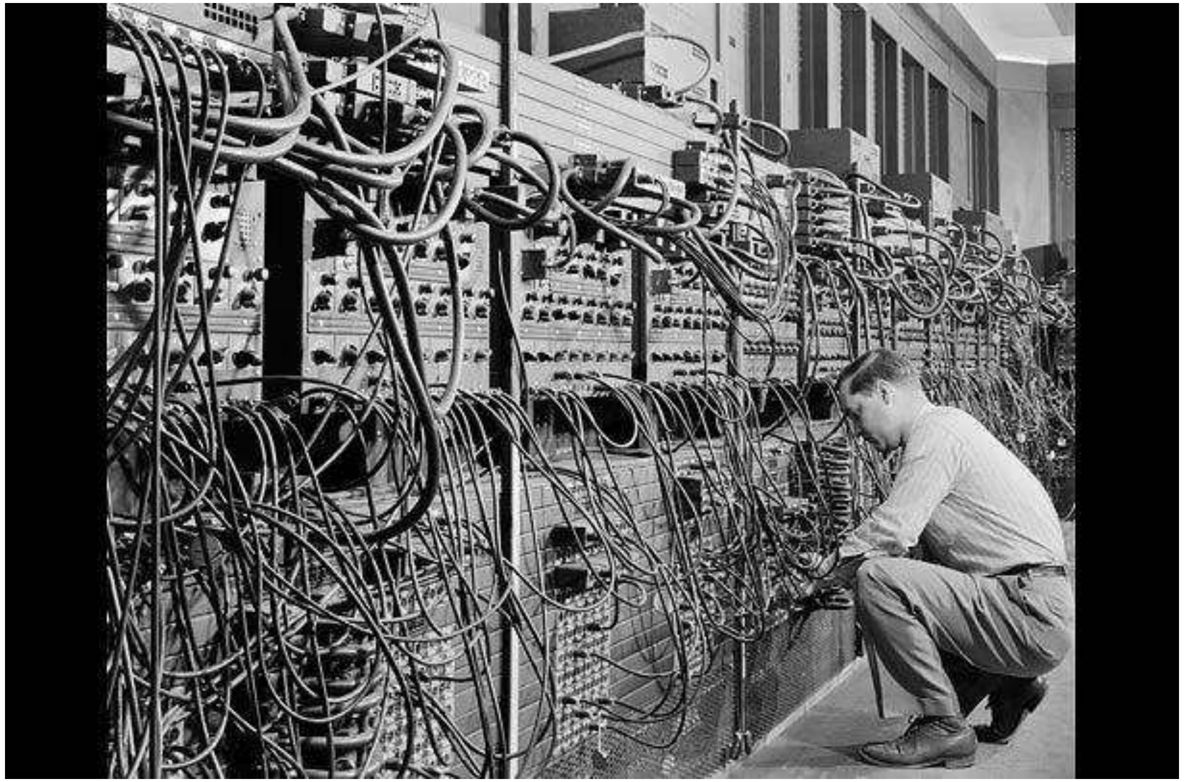
Tencent腾讯 | 腾讯量子实验室

達摩院
ALIBABA DAMO ACADEMY

初创公司（风险投资）：

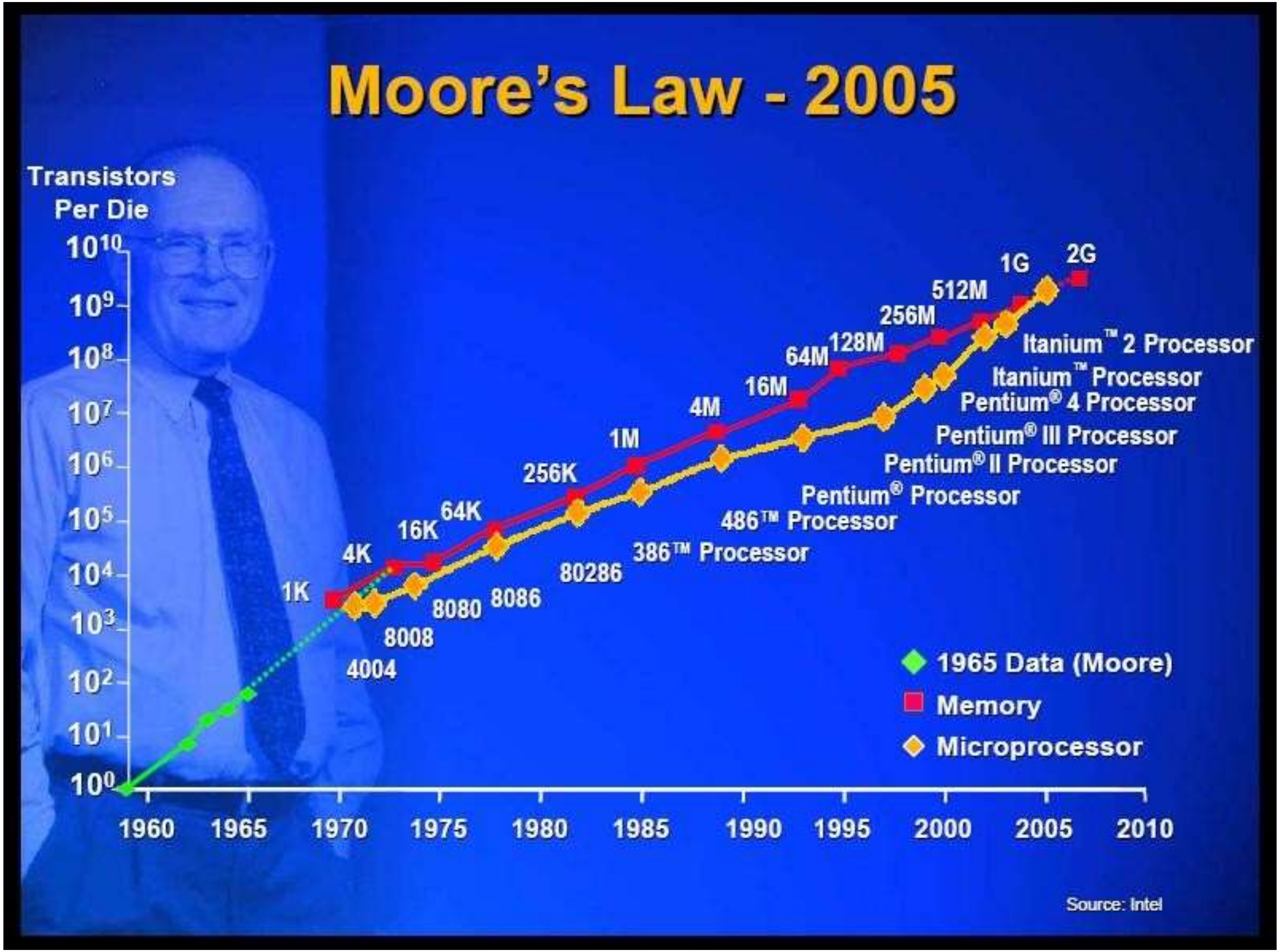
- D-Wave
- Rigetti
- Quantum Circuits, Inc.
- 本源…

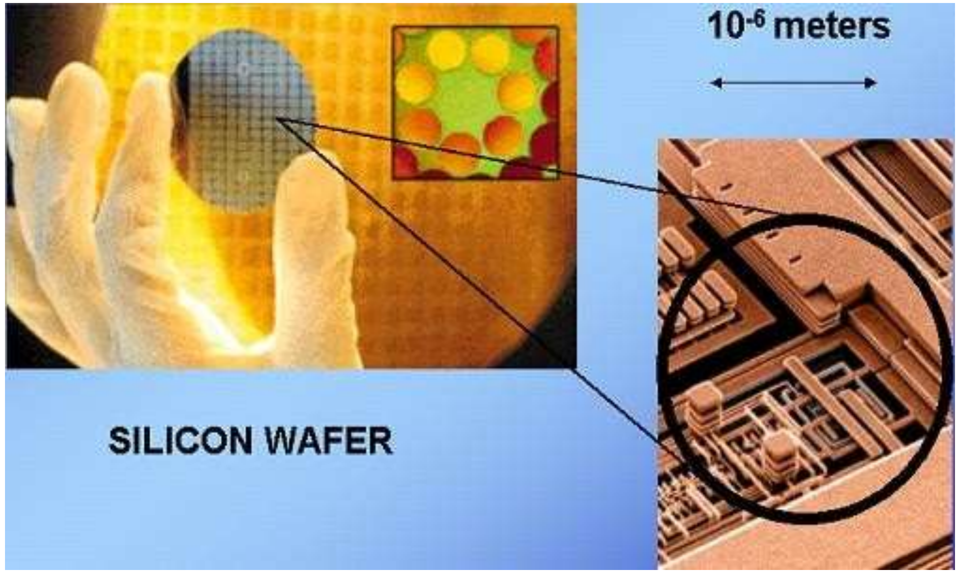


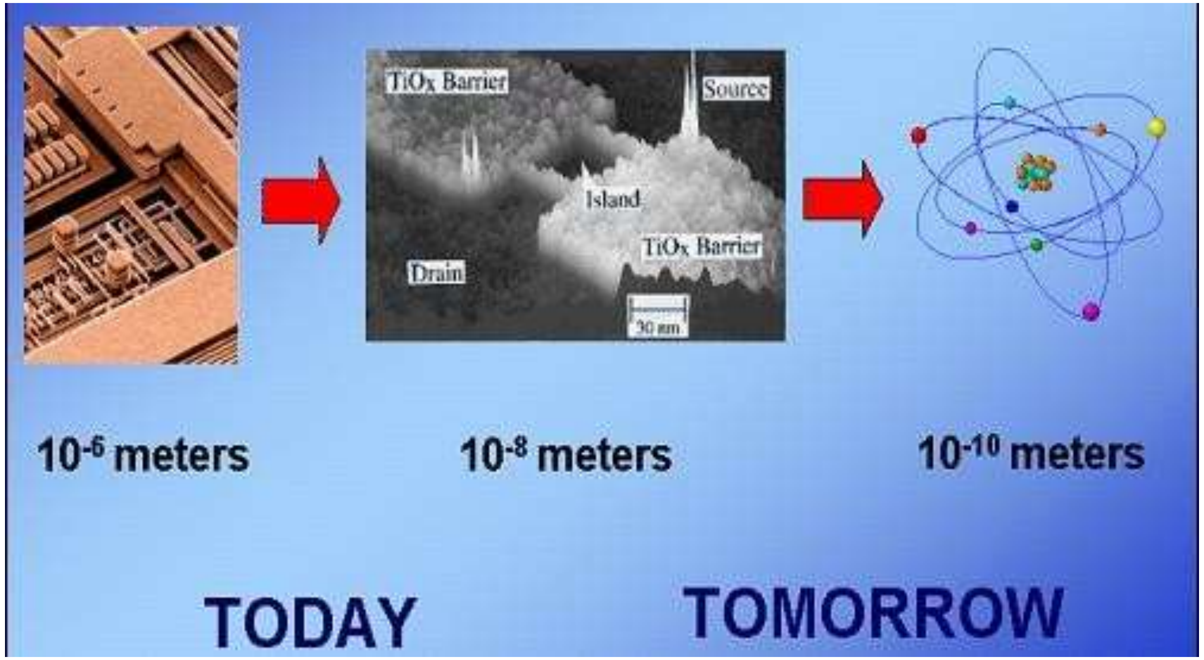


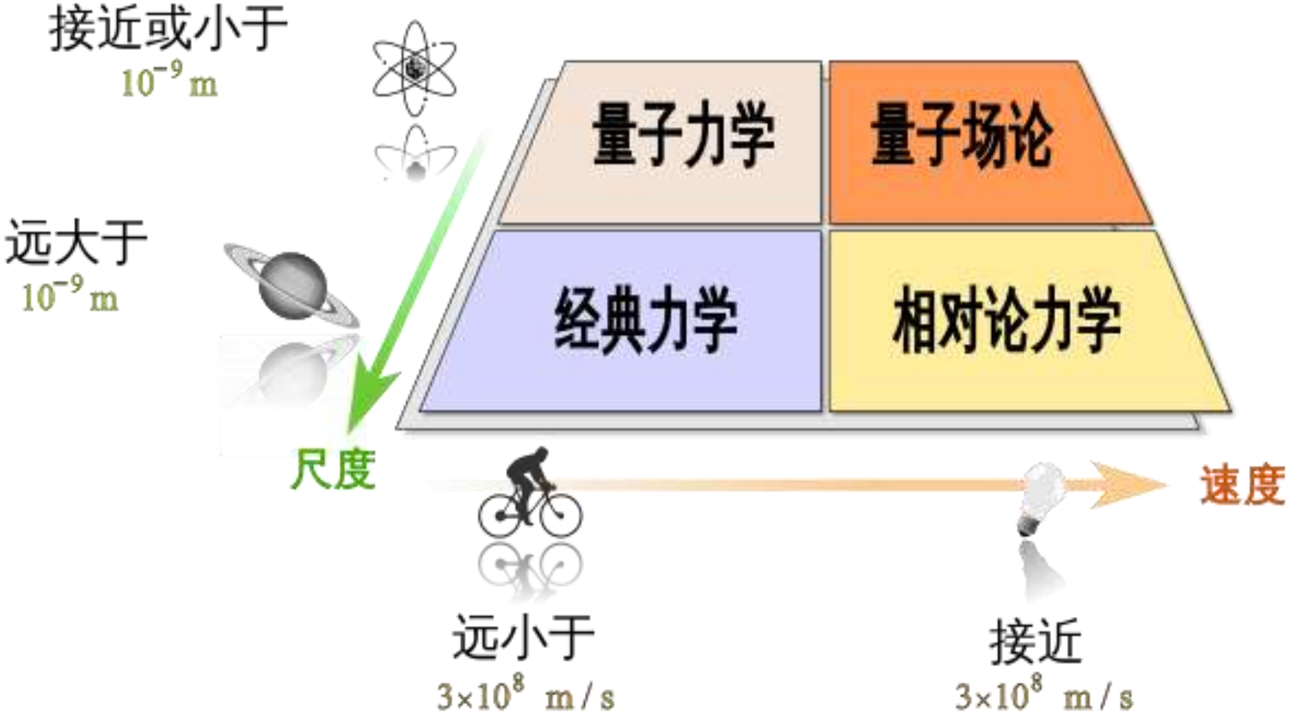
ENIAC·电子数字积分计算机

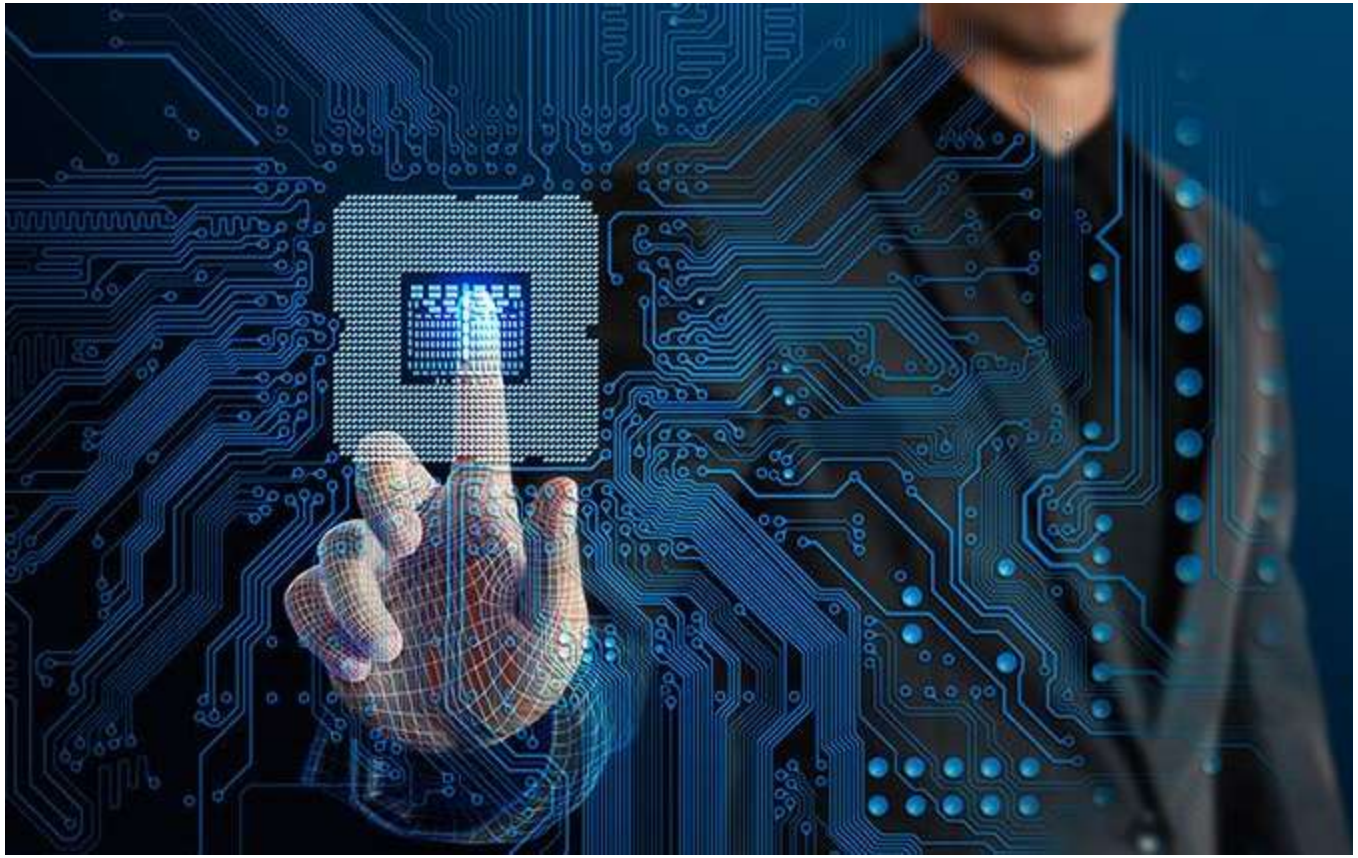
- ◆占地170平
- ◆重达28吨
- ◆费电，150KW
- ◆18600个电子管，只能稳定地工作几个小时


Moore's Law - 2005
Transistors Per Die
10^{10}
10^9
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10^1
10^0
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
1K
4K
16K
64K
256K
1M
4M
16M
64M
128M
256M
512M
1G
2G
4004
8008
8080
8086
80286
386™ Processor
486™ Processor
Pentium® Processor
Pentium® II Processor
Pentium® III Processor
Pentium® 4 Processor
Itanium™ Processor
Itanium™ 2 Processor
1965 Data (Moore)
Memory
Microprocessor
Source: Intel


10-6 meters
SILICON WAFER


TiOx Barrier
Source
Island
TiOx Barrier
Drain
30 nm
10-6 meters
10-8 meters
10-10 meters
TODAY
TOMORROW


接近或小于
10^{-9} m
远大于
10^{-9} m
量子力学
量子场论
经典力学
相对论力学
尺度
速度
远小于
3×10^{8} m/s
接近
3×10^{8} m/s

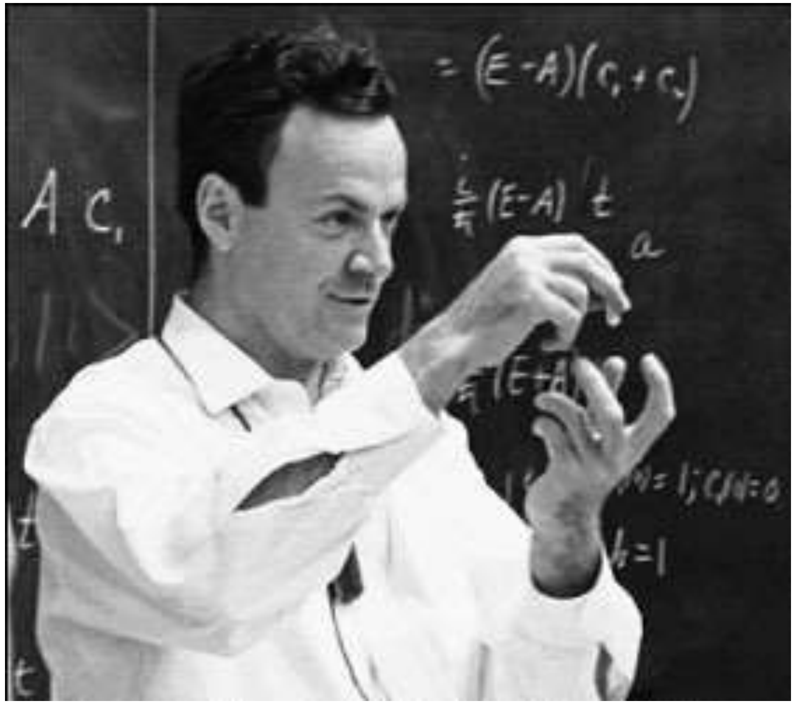


芯片战争2.0：

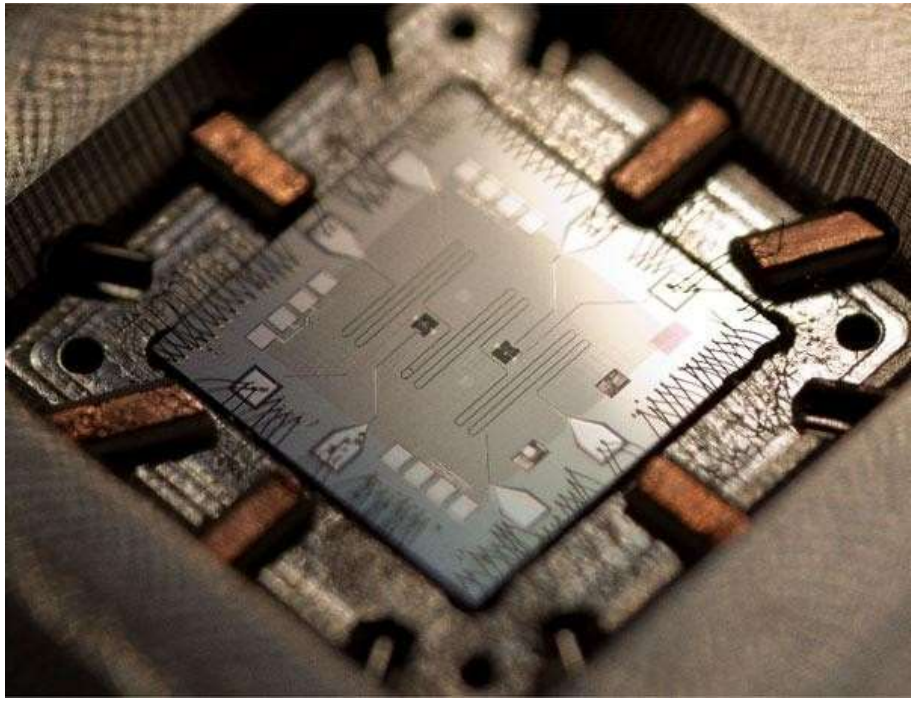
“失效”的摩尔定律

"There is plenty of room at the bottom." (Dec 29, 1959)

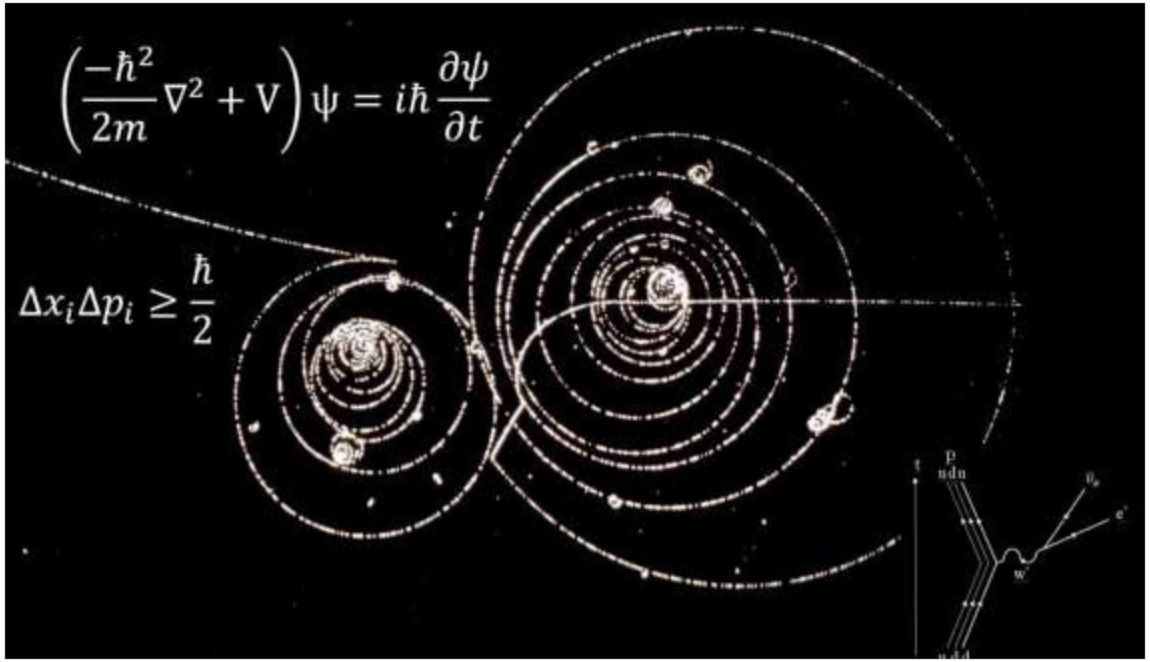
"It seems that the laws of physics present no barrier to reducing the size of computers until bits are the size of atoms, and quantum behavior holds dominant sway."
- - Richard P. Feynman (1985)



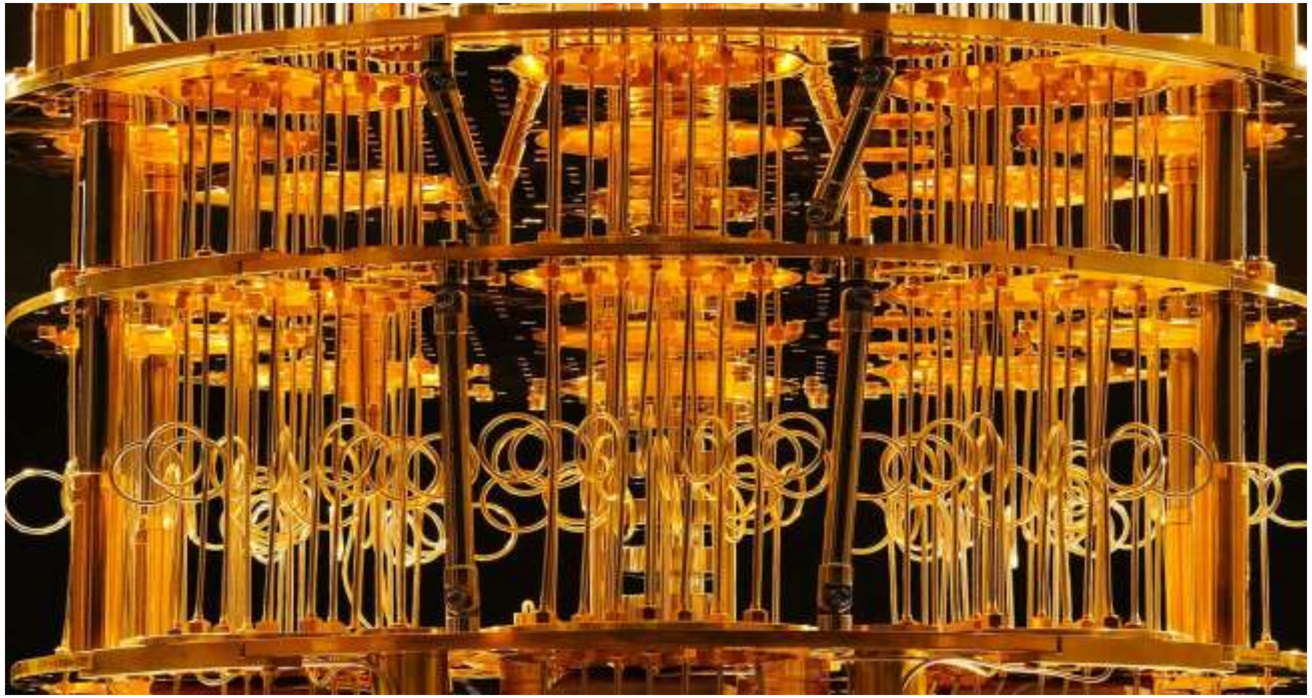
Nobel prize 1965



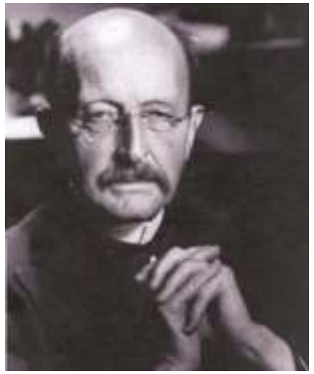
from New Scientist 2011



Quantum computation

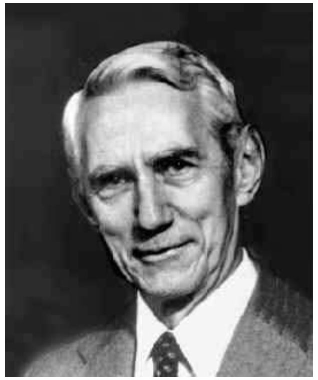


Quantum Information Science



Planck

Turing



Shannon

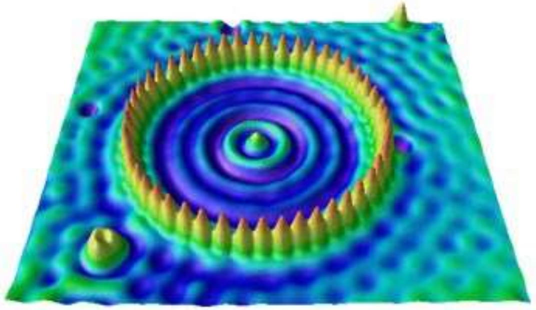
quantum theory

+ computer science

+ information theory

⬇

quantum information science



Brief history of quantum information?

Late 1920 (1927) development of quantum mechanics

1982 Richard Feynman, difficult to simulate quantum by classical
$\Rightarrow$ need quantum

1982 no-cloning theorem (Wooters-Zurek, Dieks, also Yurke)

1984 Charles Bennett and Gilles Brassard, quantum crtyptography
(actually Stephen Wiesner, late 1960s, paper not accepted)

1985 David Deutsch, Universal quantum computer efficiently simulates
arbitrary quantum system, also Deutsch algorithm

1994 Peter Shor, efficient factoring (1995 Kitaev)

1995 Lov Grover, search in unsorted database

1996 Robert Calderbank & Peter Shor, Andrew Steane,
quantum error correction

Since then rapid progress (well funded)

Quantum Information Applications

Quantum sensing
Improving sensitivity and spatial resolution.

Quantum cryptography
Privacy founded on fundamental laws of quantum physics.

Quantum networking
Distributing quantumness around the world.

Quantum simulation
Probes of exotic quantum many-body phenomena.

Quantum computing
Speeding up solutions to hard problems.

Quantum information concepts
Entanglement, error correction, complexity, …

Objectives of this course：

1. Deepen understanding of quantum mechanics

2. Extend your knowledge about the frontiers of quantum information physics

Structure of the course

Overview of quantum mechanics

General structure of a quantum computer, 1-qubit, 2-qubit,
and 3-qubit gates

Usual QC protocol for a function calculation and main trick,
toy problems (algorithms)

EPR, Bell states, quantum teleportation, quantum cryptography

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RSA encryption, Shor's algorithm for factoring

Grover algorithm

Quantum error correction

Computational complexity (very briefly)

Course assessment and requirements

- Attendance (10%) and class performance (20%)
- Midterm exam: Quantum mechanics project (20%)
- Final exam: Dissertation on quantum information science (50%)

参考书

- Michael A. Nielsen and saac L. Chuang,《Quantum Computation and Quantum Information》，Cambridge University Press, 2011年
- 张永德，《量子信息物理原理》，科学出版社，2005年
- A. Yu. Kitaev，A. H. Shen，M. N. Vyalyi，《Classical and Quantum Computation》，Amer Mathematical Society，2002年
- John Preskill的《Lecture notes on quantum information and quantum computation》
- 曾谨言，量子力学，卷I，卷II，科学出版社.
- Introduction to Quantum Mechanics,D. J. Griffiths,机械工业出版社
- 费曼物理学讲义 第三卷

- …

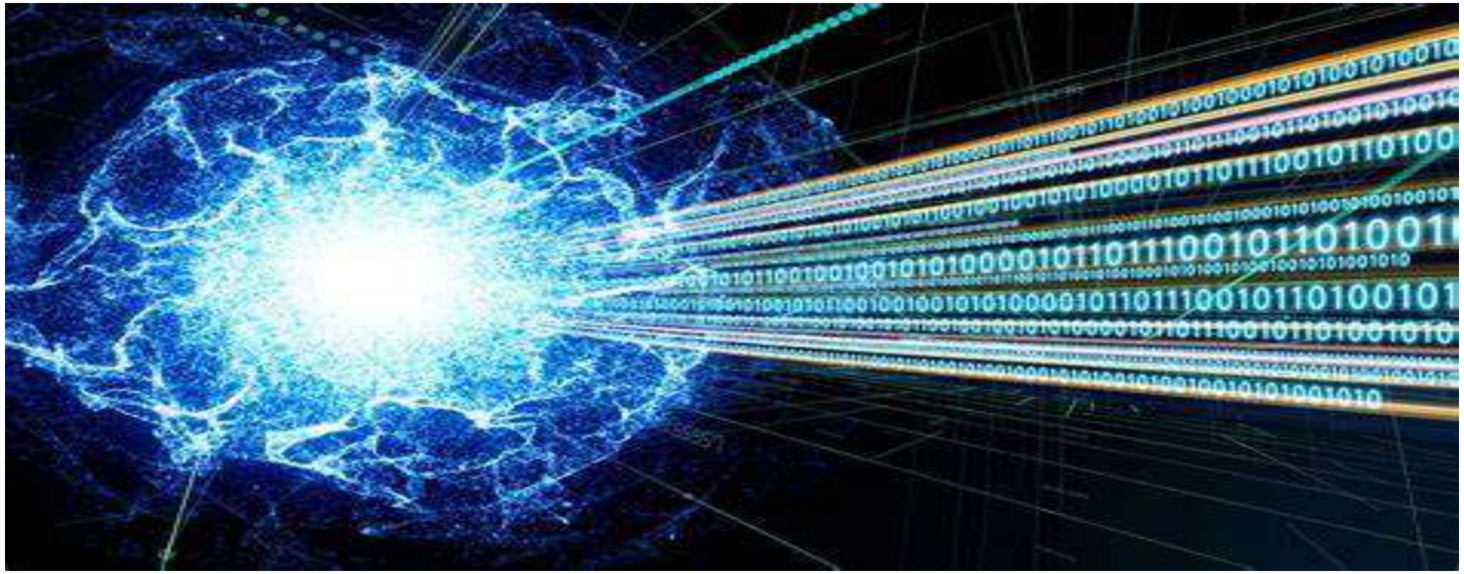
科普书

- 量子力学少年版，曹则贤，中国科学技术大学出版社
- 画说量子力学，陈难先译，清华大学出版社
- 量子力学1小时科普，朱梓忠，清华大学出版社
- 简明量子力学，吴飙，北京大学讲义
- 量子力学的诠释，孙昌璞，物理，46卷（2017年）8期
- 寻找薛定谔的猫，John Gribbin 张广才译 海南出版社
- 量子物理史话:上帝掷骰子吗？，曹天元

Welcome to the magic world of quantum information!

Lecture 1

Introduction to quantum information science



Two fundamental ideas

(1) Quantum complexity

Why we think quantum computing is powerful.

(2) Quantum error correction

Why we think quantum computing is scalable.

Why we think quantum computing is powerful

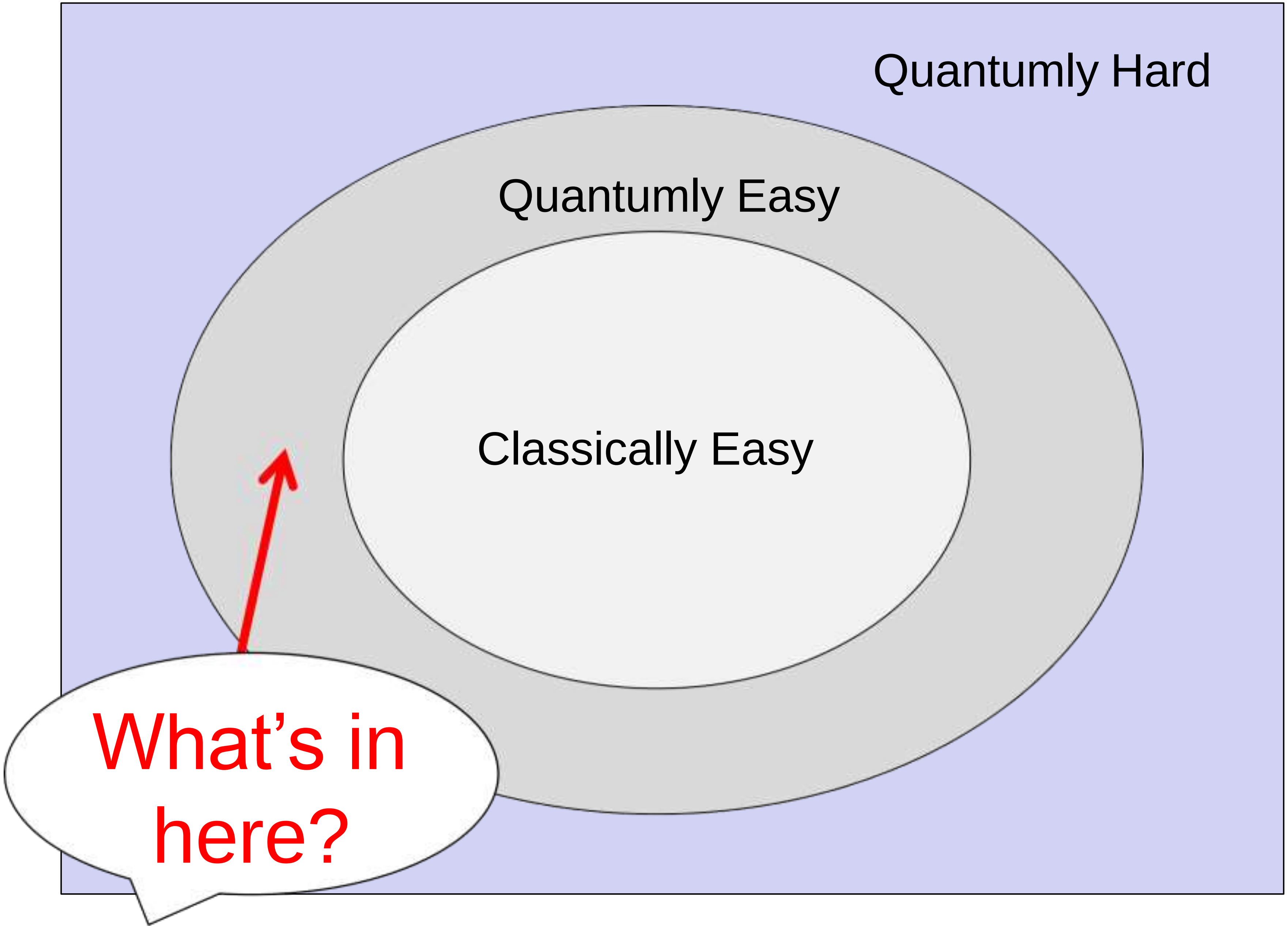
(1)Problems believed to be hard classically, which are easy for quantum computers. Factoring is the best known example.

(2)Complexity theory arguments indicating that quantum computers are hard to simulate classically.

(3)We don't know how to simulate a quantum computer efficiently using a digital ("classical") computer. The cost of the best known simulation algorithm rises exponentially with the number of qubits.

But … the power of quantum computing is limited. For example, we don't believe that quantum computers can efficiently solve worst-case instances of NP-hard optimization problems (e.g., the traveling salesman problem).

Problems



Quantumly Hard
Quantumly Easy
Classically Easy
What's in here?

Why quantum computing is hard

1. We want qubits to interact strongly with one another.

2. We don't want qubits to interact with the environment.

3. Except when we control or measure them.

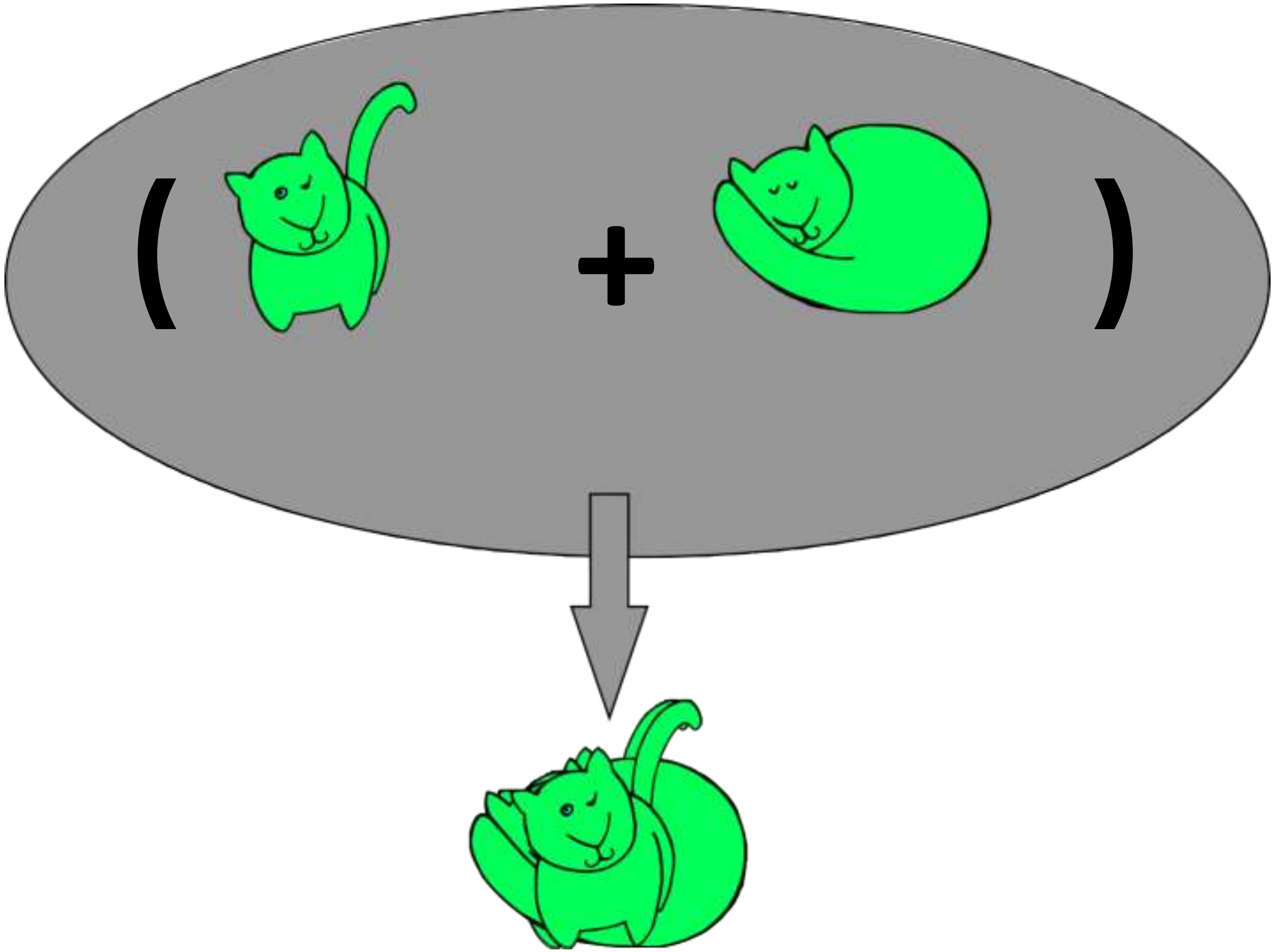
Quantum computing scaling

	Number of qubits	Gate depth	Coherence time
Scaling dimensions	Determines the quantum info that can be processed	Determines number of steps to be executed in an algorithm	Limits the max duration of the algorithm
Current state:	50–60 qubits for gate computing 2,000–5,000 qubits for annealers[1]	Circuit with gate depth 20	Depends on technology
Cases:	Google Sycamore: 54 qubits D-Wave Advantage: 5,000 qubits	Google Sycamore: gate depth 20	Trapped ion: 1–10 seconds Superconductor: Microseconds

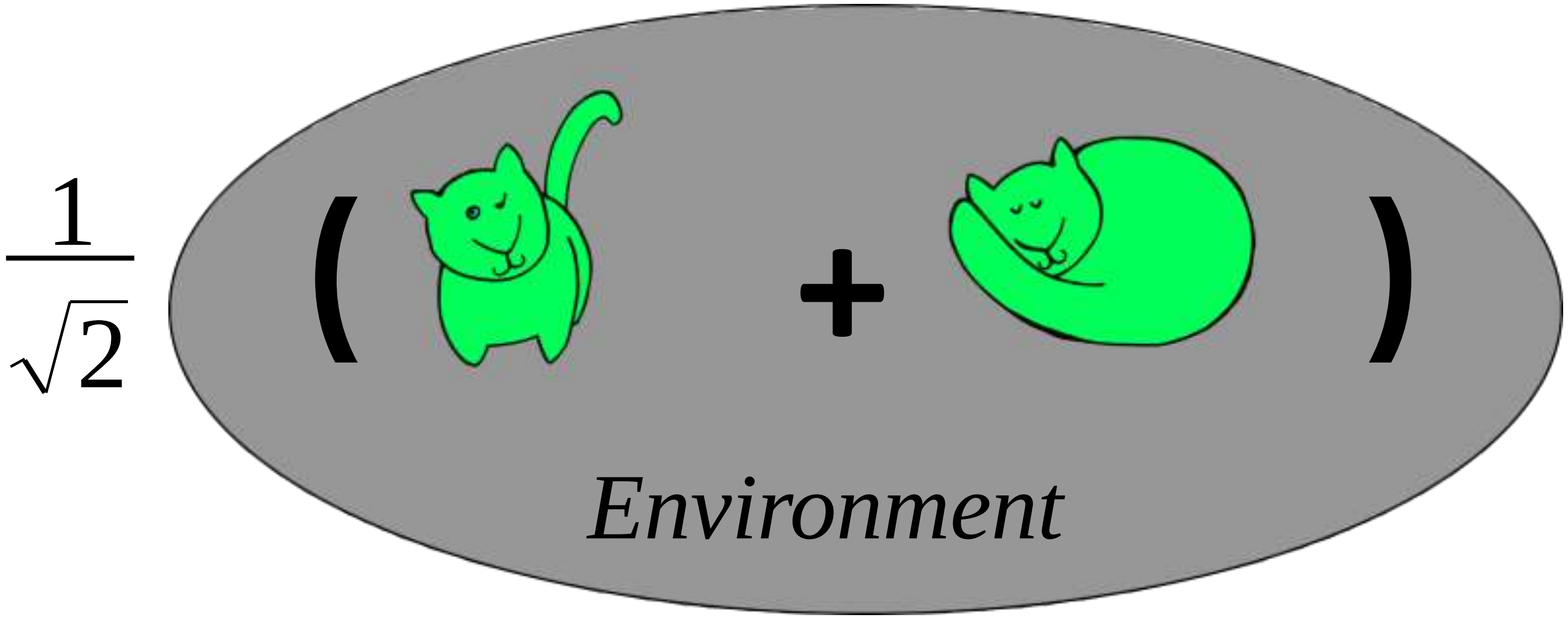
Schrodinger's cat

Decoherence

$$\frac{1}{\sqrt{2}}$$



Decoherence



Decoherence explains why quantum phenomena, though observable in the microscopic systems studied in the physics lab, are not manifest in the macroscopic physical systems that we encounter in our ordinary experience.

ERROR!

How can we protect a quantum computer from decoherence and other sources of error?

ERROR!

To resist decoherence, we must prevent the environment from "learning" about the state of the quantum computer during the computation.

Nature **574**, pages 505–510 (2019), 23 October 2019

Article

Quantum supremacy using a programmable superconducting processor

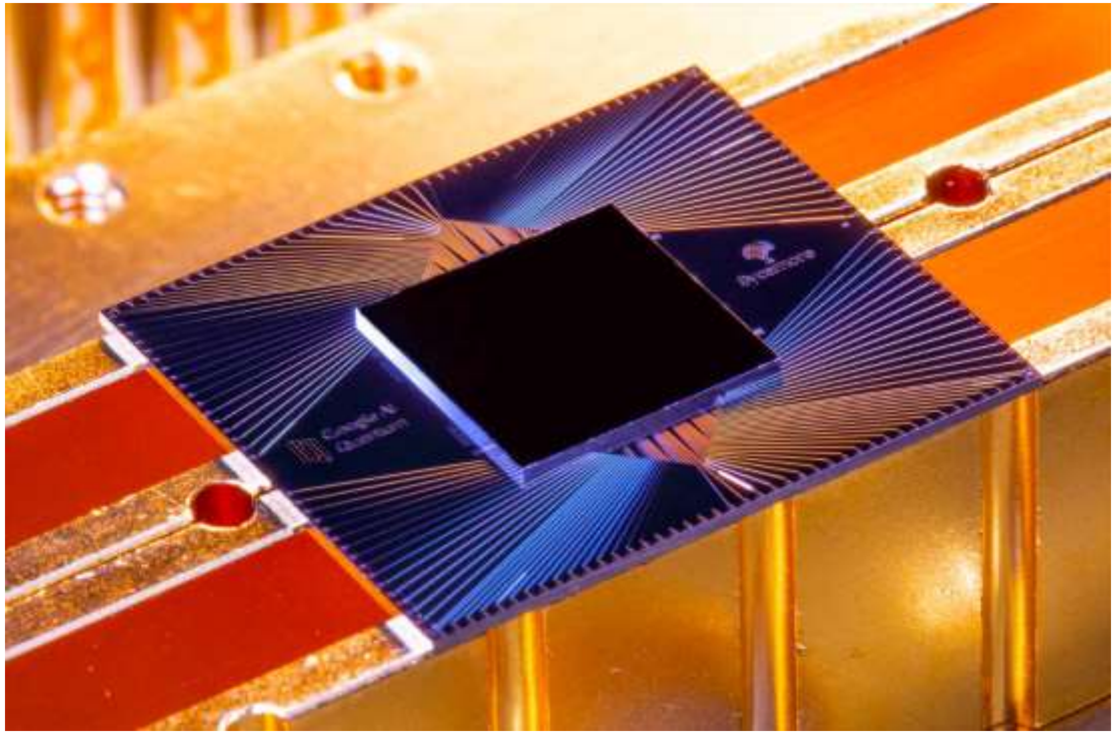
https://doi.org/10.1038/s41586-019-1666-5

Received: 22 July 2019

Accepted: 20 September 2019

Published online: 23 October 2019

Frank Arute[1], Kunal Arya[1], Ryan Babbush[1], Dave Bacon[1], Joseph C. Bardin[1,2], Rami Barends[1], Rupak Biswas[3], Sergio Boixo[1], Fernando G. S. L. Brandao[1,4], David A. Buell[1], Brian Burkett[1], Yu Chen[1], Zijun Chen[1], Ben Chiaro[5], Roberto Collins[1], William Courtney[1], Andrew Dunsworth[1], Edward Farhi[1], Brooks Foxen[1,5], Austin Fowler[1], Craig Gidney[1], Marissa Giustina[1], Rob Graff[1], Keith Guerin[1], Steve Habegger[1], Matthew P. Harrigan[1], Michael J. Hartmann[1,6], Alan Ho[1], Markus Hoffmann[1], Trent Huang[1], Travis S. Humble[7], Sergei V. Isakov[1], Evan Jeffrey[1],

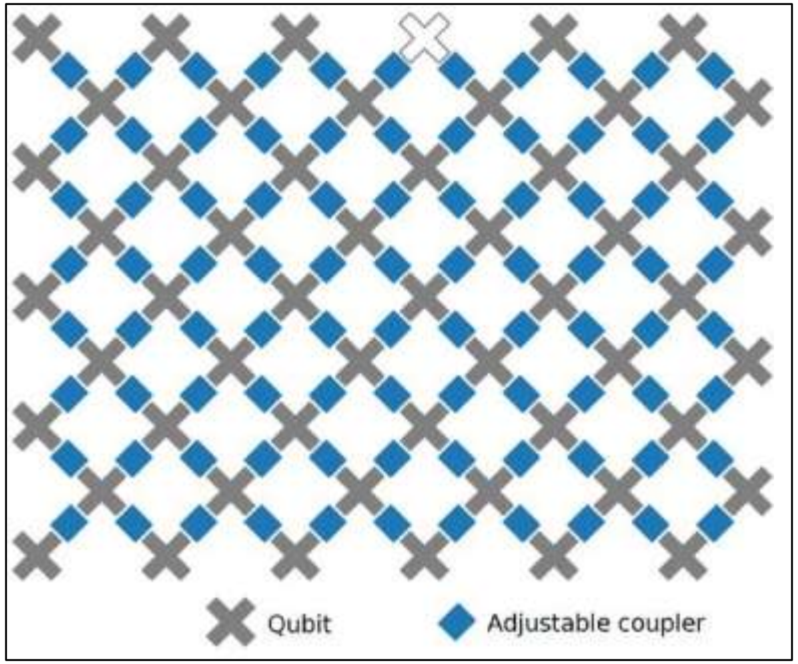


Credit: Erik Lucero/Google

About Google Sycamore

"Quantum David vs. Classical Goliath"

A fully programmable circuit-based quantum computer. *n*= 53 working qubits in a two-dimensional array with coupling of nearest neighbors.



A circuit with 20 layers of 2-qubit gates *can be executed millions of times in a few minutes*, yielding verifiable results.

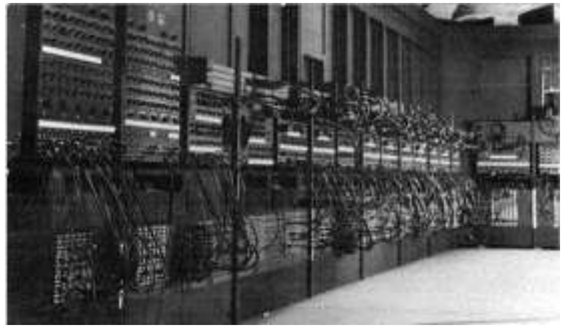
Simulating this quantum circuit using a classical supercomputer is hard. *It would take at least days*, possibly much longer.

Furthermore, the cost of the classical simulation grows exponentially with the number of qubits.

Conclusion: the quantum hardware is working well enough to produce meaningful results in a regime where classical simulation is very difficult.

What quantum computational supremacy means

"Quantum David vs. Classical Goliath"



1. It's a programmable circuit-based quantum computer.
2. An impressive achievement in experimental physics and a testament to ongoing progress in building quantum computing hardware.
3. We have arguably entered the regime where the extravagant exponential resources of the quantum world can be validated.
4. This confirmation does not surprise (most) physicists, but it's a milestone for technology on planet earth.
5. Building a quantum computer is merely *really, really hard*, not *ridiculously hard*. The hardware is working; we can begin a serious search for useful applications.
6. But the specific task performed by Sycamore to demonstrate quantum computational supremacy is not particularly useful.

“Quantum computing is
a marathon not a sprint”

Chris Monroe (UMD/IonQ)

“假如一个人不为量子论感到困惑，那他是没有明白量子论.”

N. Bohr

“I can safely said (that) no body understands quantum theory.”

R.P. Feymann

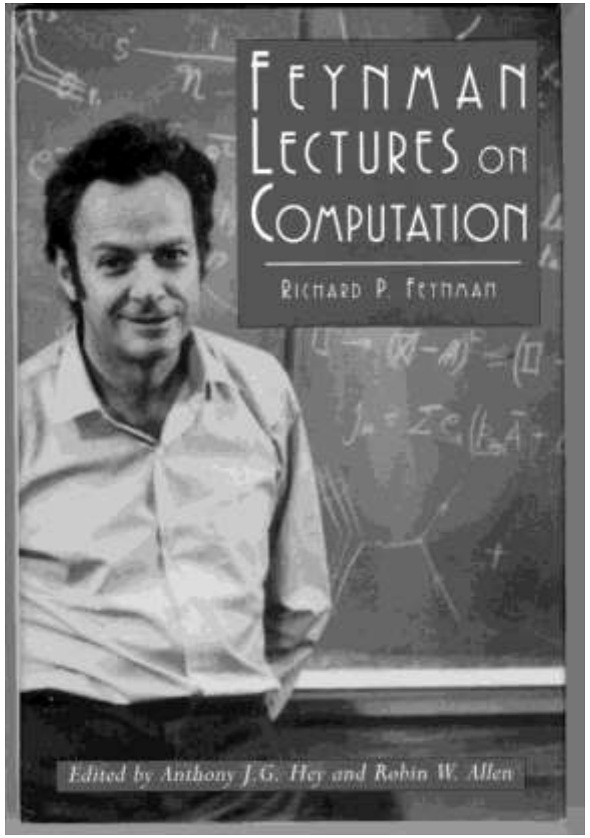
"The theory of everything?"

"The Theory of Everything is not even remotely a theory of every thing …

We know this equation is correct … However, it cannot be solved accurately when the number of particles exceeds about 10. No computer existing, or that will ever exist, can break this barrier because it is a catastrophe of dimension

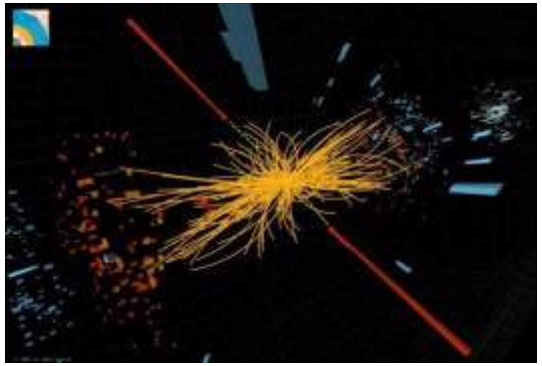
… We have succeeded in reducing all of ordinary physical behavior to a simple, correct Theory of Everything only to discover that it has revealed exactly nothing about many things of great importance."

R. B. Laughlin and D. Pines, PNAS 2000.

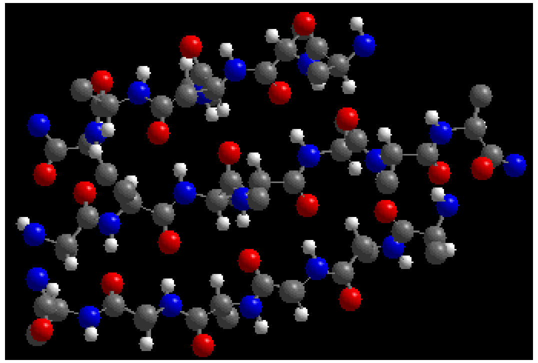


"Nature isn't classical, dammit, and if you want to make a simulation of Nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem because it doesn't look so easy."

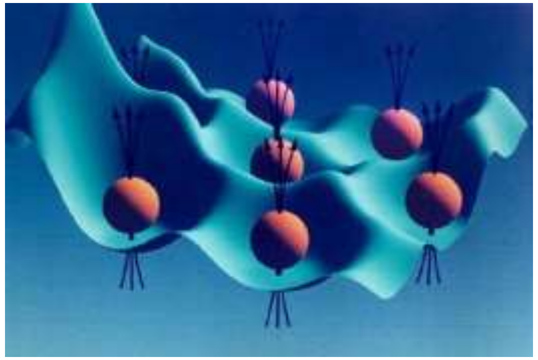
R. P. Feynman, 1981



particle collision



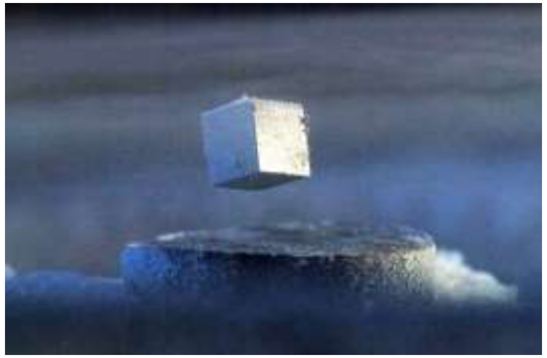
molecular chemistry



entangled electrons

A quantum computer can simulate efficiently any physical process that occurs in Nature.

(Maybe. We don’t actually know for sure.)



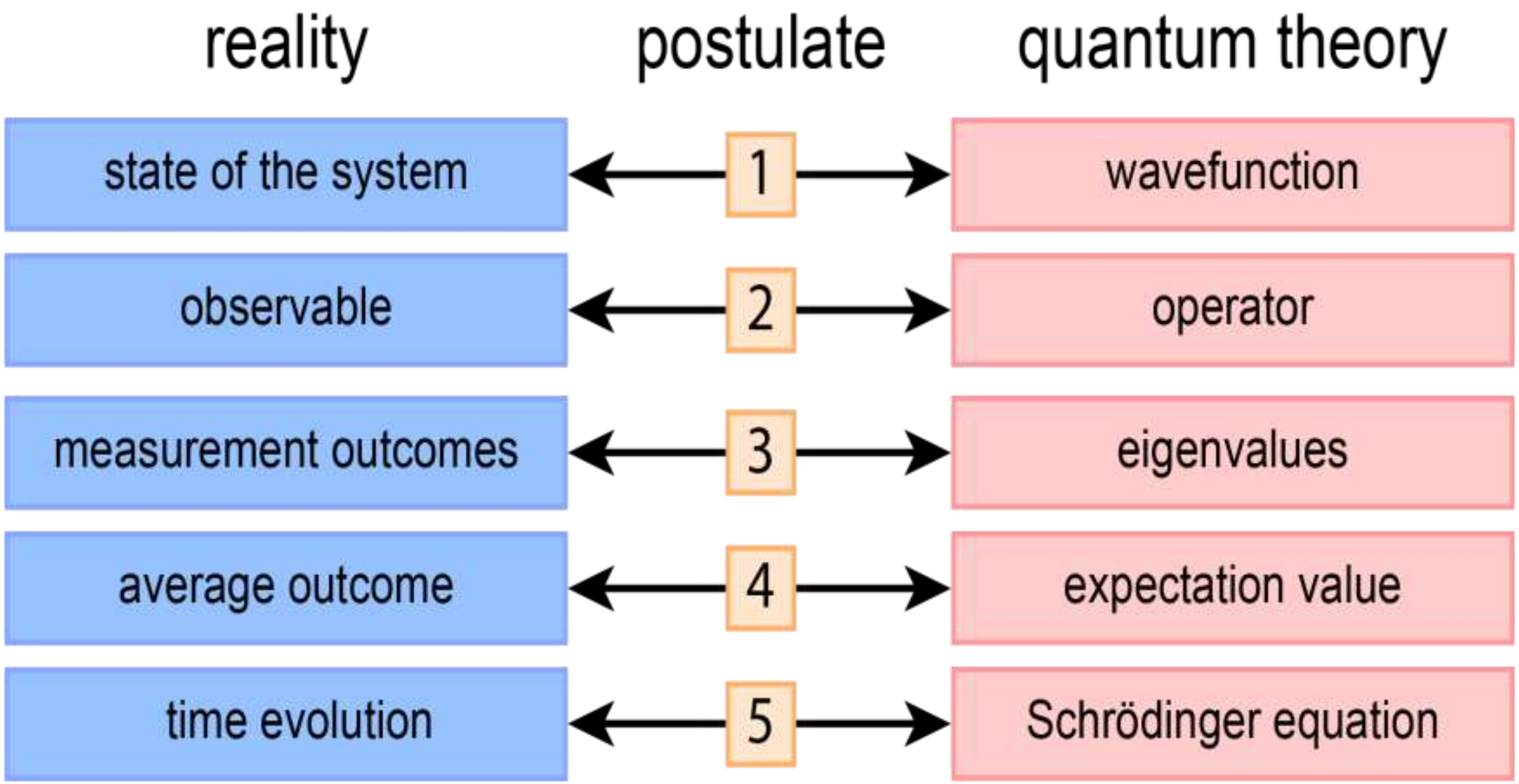
superconductor

black hole

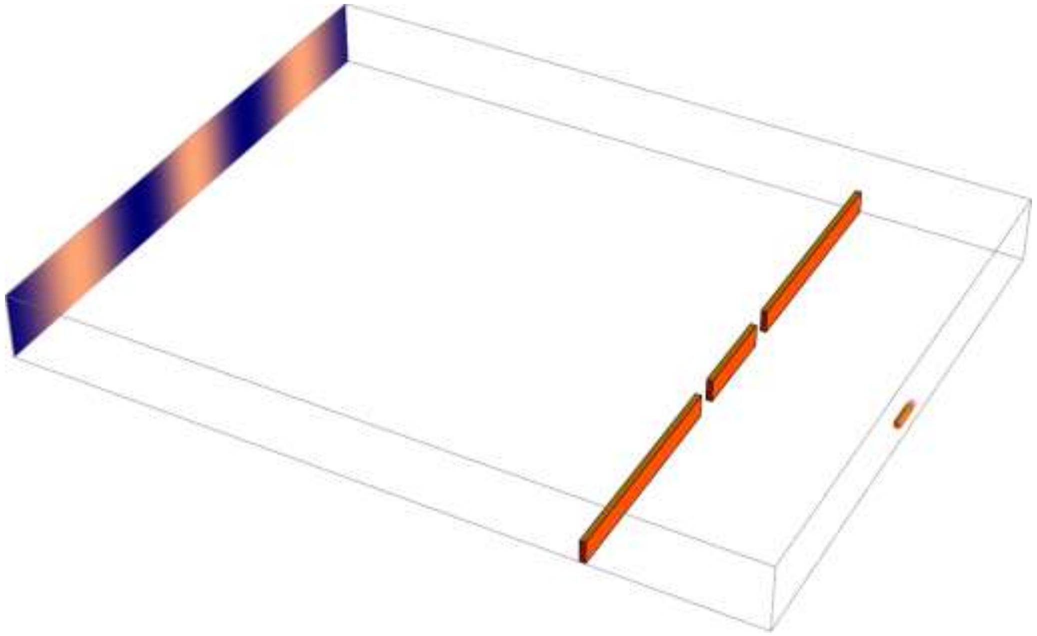
early universe

量子力学的基础和逻辑框架：五大公设

- 波函数公设：微观粒子量子状态可用波函数完全描述。
- 算符公设：量子力学中力学量用厄米算符表示。
- 测量公设：对量子态进行测量，结果必为该力学量算符的本征值之一。
- 动力学演化公设：波函数按Schrodinger方程随时间演化。
- 微观粒子全同性原理公设：全同粒子的交换不改变系统的状态。

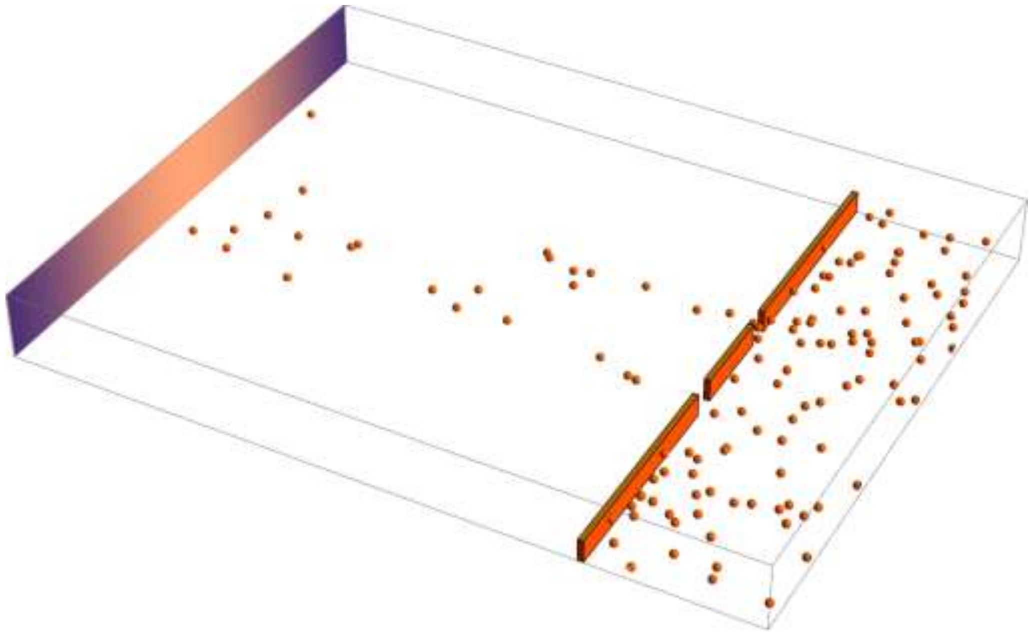

reality
postulate
quantum theory
state of the system
1
wavefunction
observable
2
operator
measurement outcomes
3
eigenvalues
average outcome
4
expectation value
time evolution
5
Schrödinger equation

双缝干涉实验



经典杨氏双缝干涉

VS



20世纪十大最美物理实验之一：
单电子双缝干涉

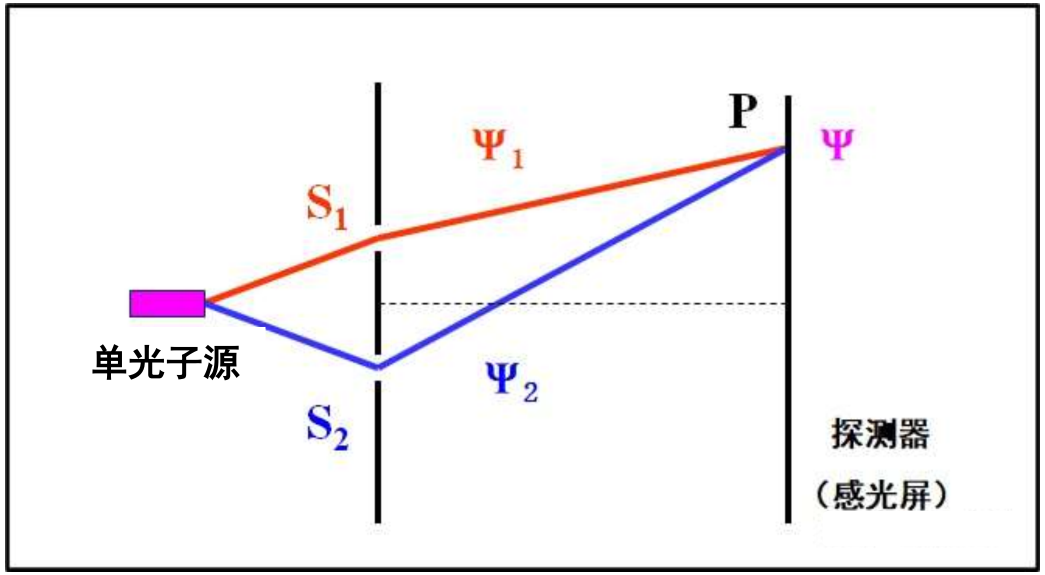
杨氏双缝干涉效应的量子解释

波函数：量子力学中描写微观系统状态的函数。

波恩的统计诠释：波函数的强度正比于粒子出现的概率。

通过缝S_1的粒子的波函数：　ψ_1

通过缝S_2的粒子的波函数：　ψ_2



量子态叠加原理：$\psi = \psi_1 + \psi_2$

P点出现粒子的概率：$|\psi_1 + \psi_2|^2 = |\psi_1|^2 + |\psi_2|^2 + \psi_1^* \psi_2 + \psi_2^* \psi_1$

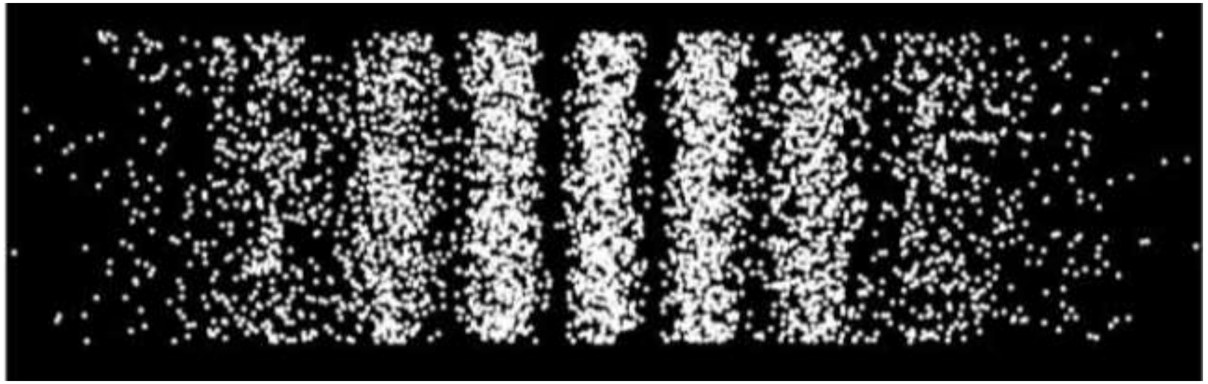
双缝干涉实验的量子解释

P点出现粒子的概率：$|\psi_1+\psi_2|^2=|\psi_1|^2+|\psi_2|^2+\psi_1^*\psi_2+\psi_2^*\psi_1$

$\psi_1=-\psi_2 \Rightarrow |\psi_1+\psi_2|^2=0$　　暗条纹

$\psi_1=\psi_2 \Rightarrow |\psi_1+\psi_2|^2=4|\psi_1|^2$　　明条纹

——形成明暗相间的条纹



在双缝干涉实验中，电子以概率波波动的方式穿过了两条狭缝。波函数的线性叠加是造成观测屏上出现干涉图案的原因。

波函数的归一化

经典波动说：
客观实在的波动

VS

量子波函数：
概率波

本质区别是什么？

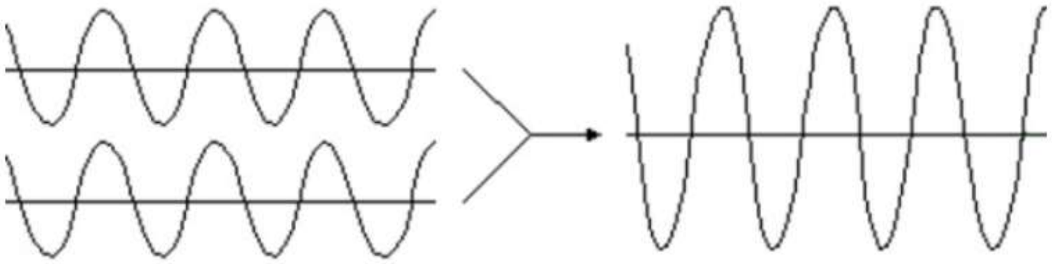
1. 若经典光波的波幅增大一倍，它是否表示相同的波动状态？

2. 若量子波函数的波幅增大一倍，它是否表示相同的波动状态？

2×1=?

波函数的归一化

- 经典电磁场：$E=\varepsilon_0\vec{E}^2/2+\vec{B}^2/2\mu_0$
  能量会增大为原来能量的4倍，因而代表完全不同的波动状态。



- 对于概率分布而言，重要的是相对概率分布。
  $\psi(\vec{r},t)$ 和 $C\psi(\vec{r},t)$描写的是同一个概率波。

$$\frac{|C\psi(\vec{r}_1,t)|^2}{|C\psi(\vec{r}_2,t)|^2}=\frac{|\psi(\vec{r}_1,t)|^2}{|\psi(\vec{r}_2,t)|^2}$$

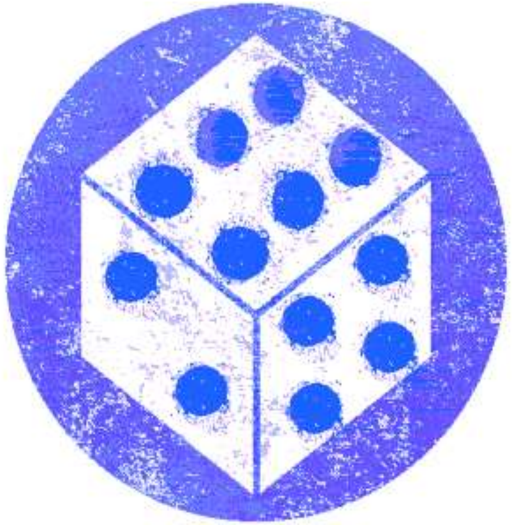
波函数的归一化

经典波函数不可以概率归一化。

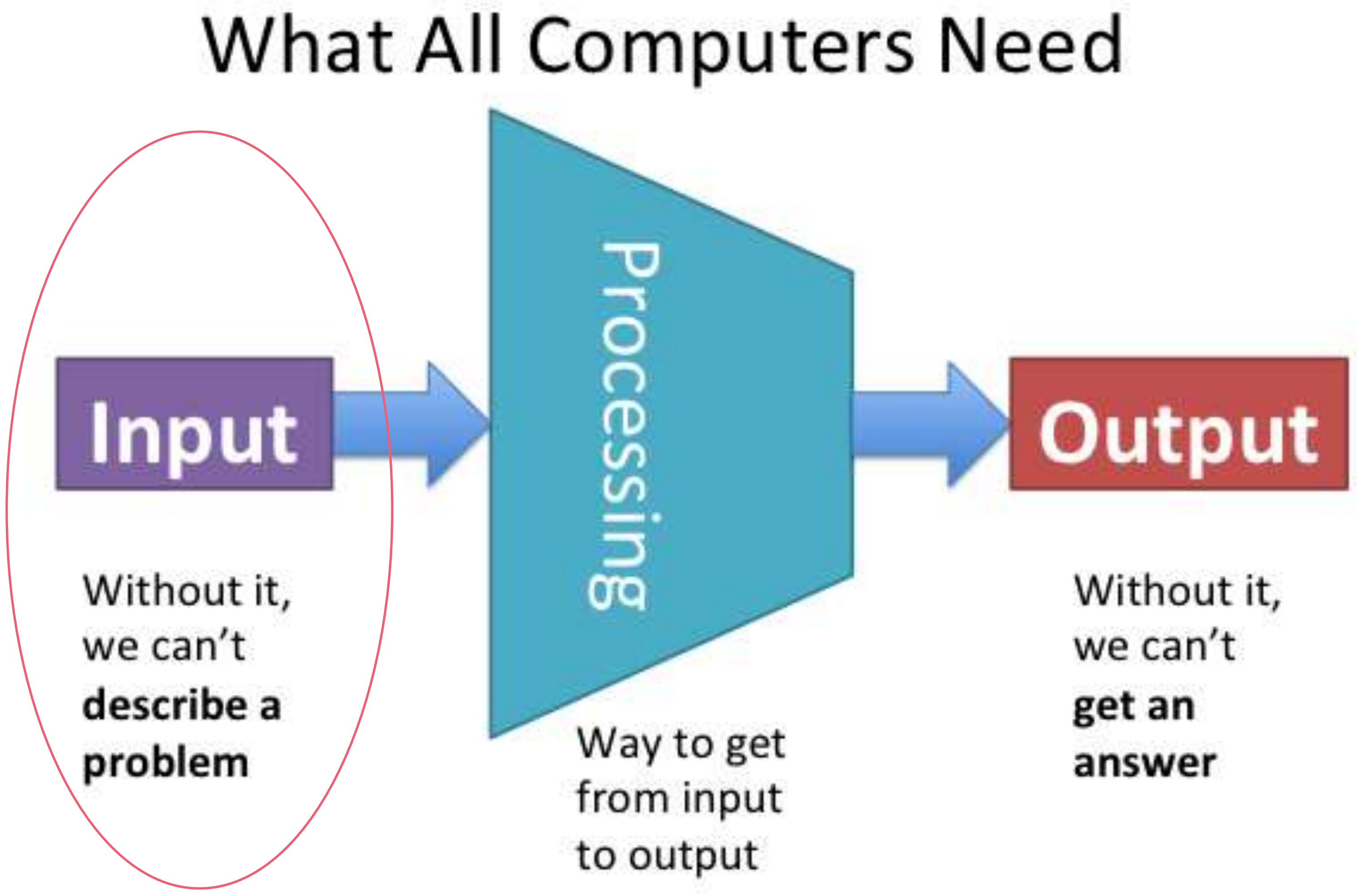
$$E = \varepsilon_0 \vec{E}^2 / 2 + \vec{B}^2 / 2\mu_0$$

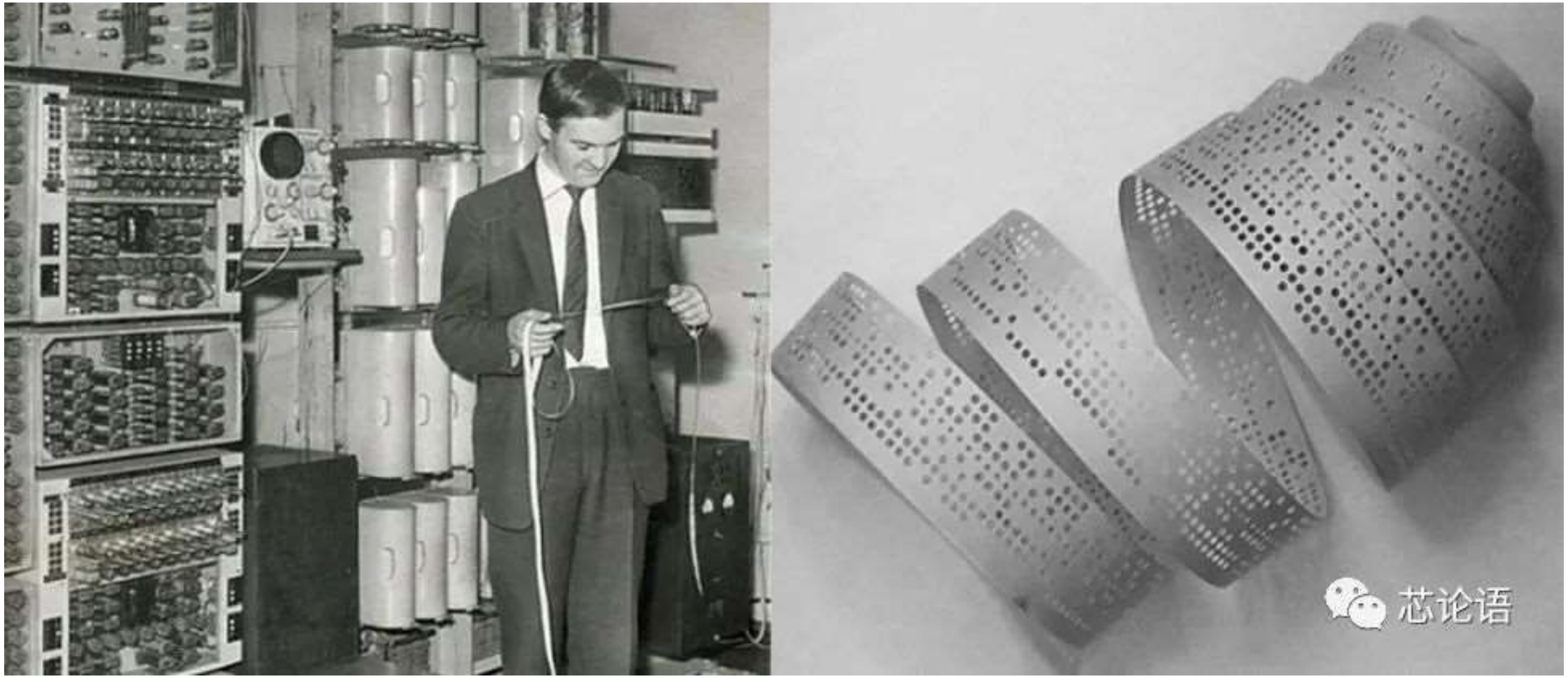
量子波函数可以概率归一化：

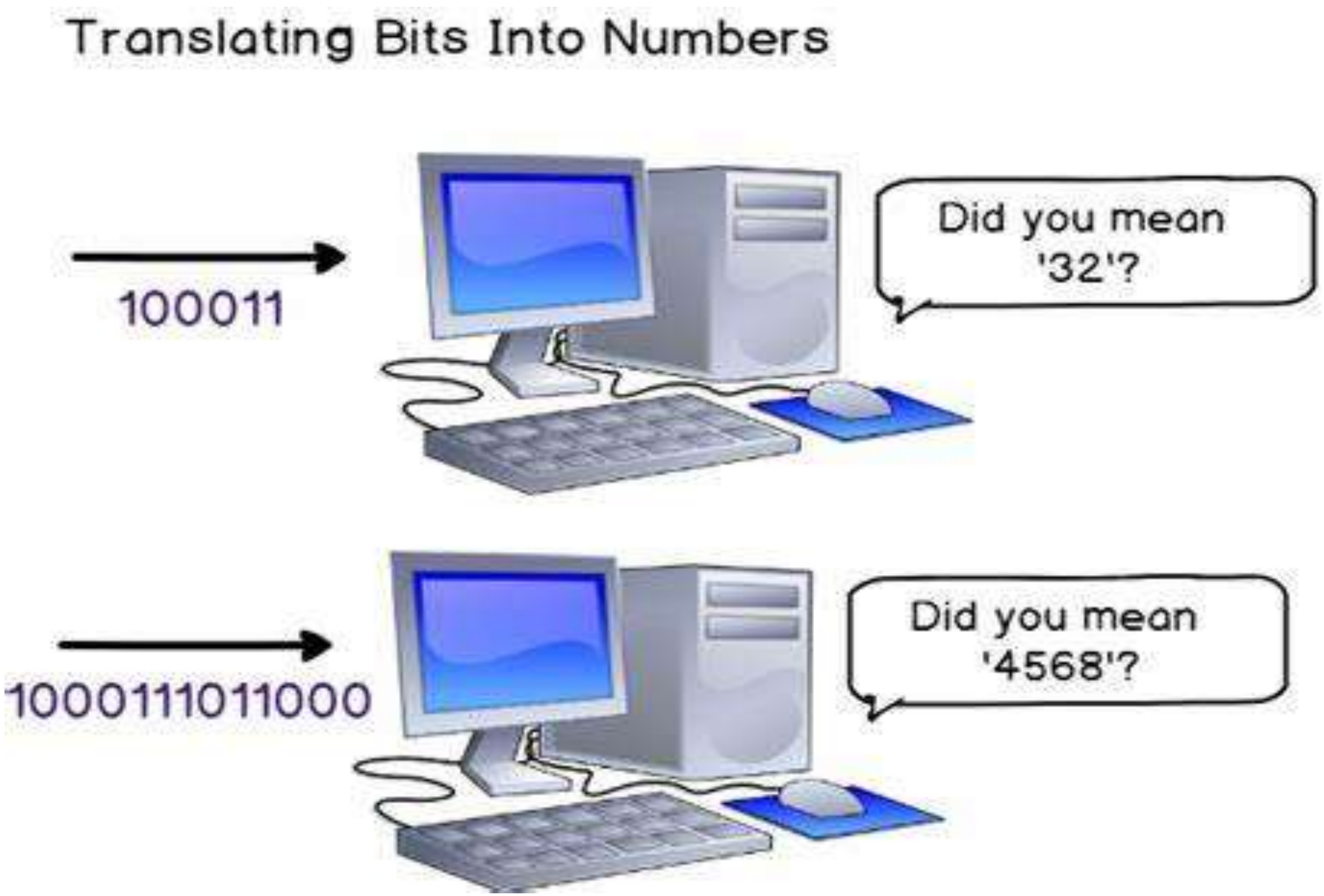
$$\int_{全空间} |\psi(\vec{r},t)|^2 d^3x = 1$$



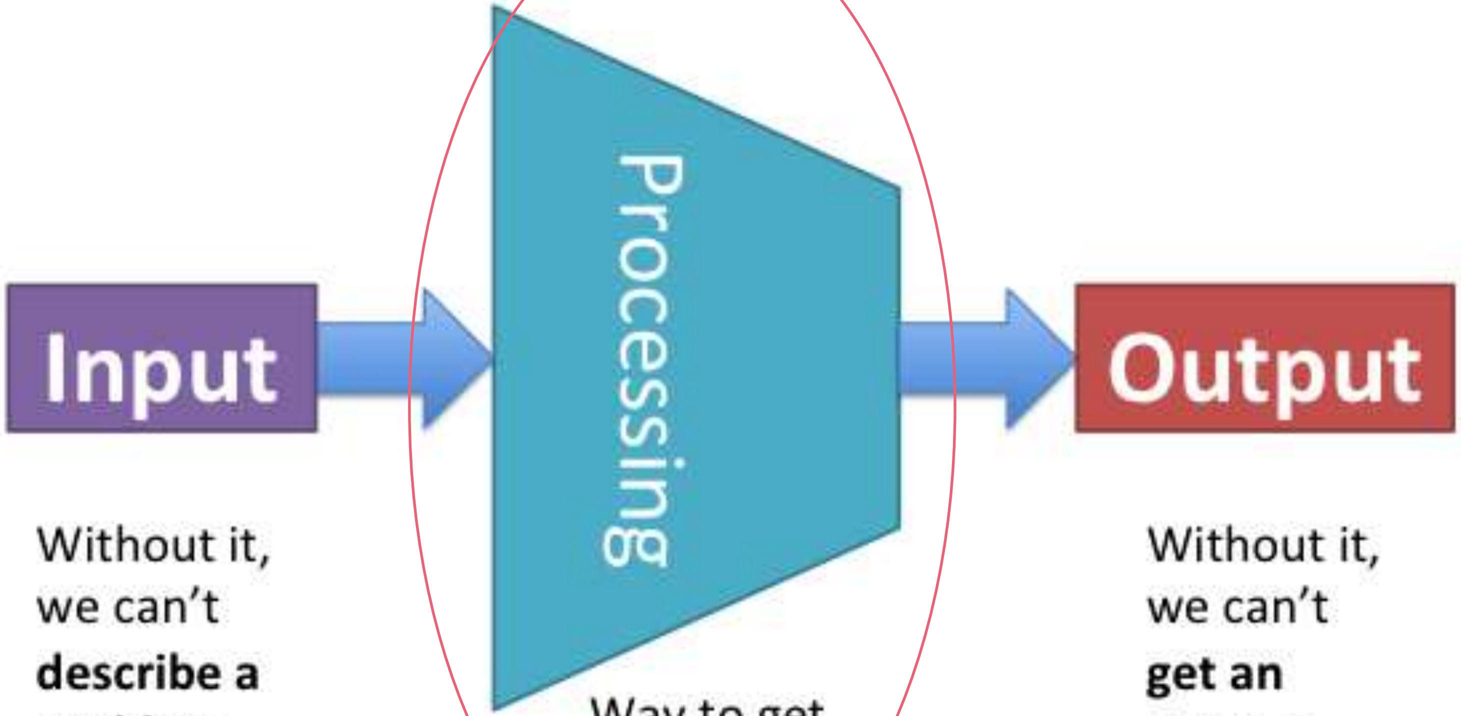
能否归一化是量子波与经典波的本质区别之一。


What All Computers Need
Input
Processing
Output
Without it, we can't **describe a problem**
Way to get from input to output
Without it, we can't **get an answer**


芯论语


Translating Bits Into Numbers
100011
Did you mean '32'?
1000111011000
Did you mean '4568'?

What All Computers Need



Without it, we can't **describe a problem**

Way to get from input to output

Without it, we can't **get an answer**

	与	或	非
表达式	$Y = A \cdot B$	$Y = A + B$	$Y = \overline{A}$

电路逻辑

真值表

与

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

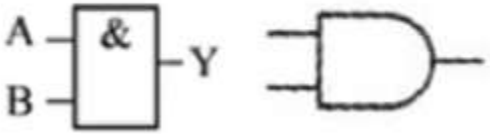
或

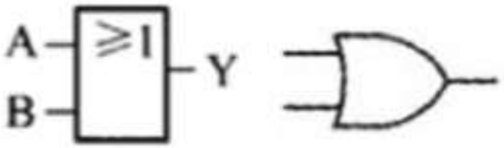
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

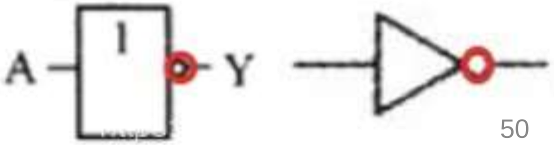
非

A	Y
0	1
1	0

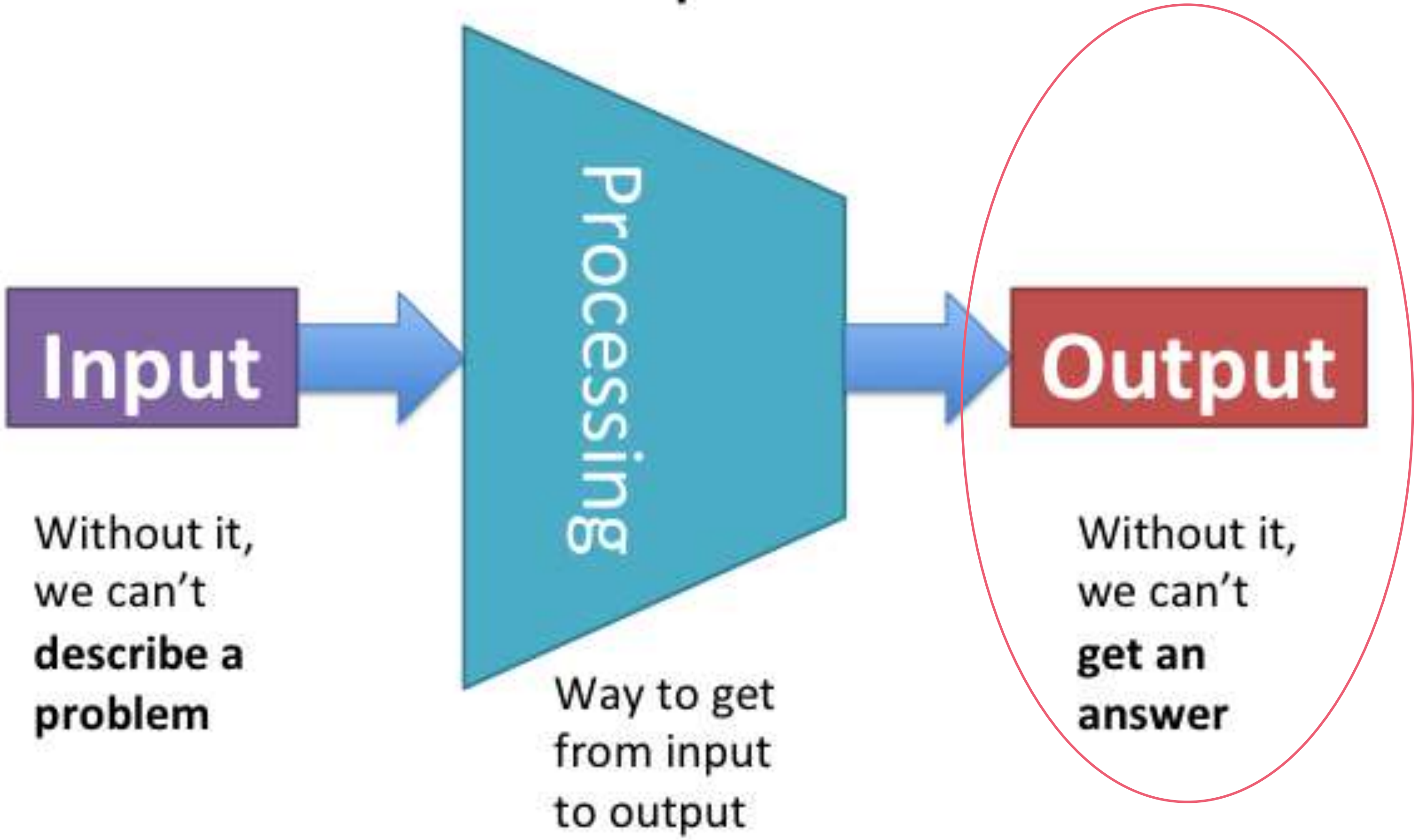
电路符号







What All Computers Need



Postulate 1. State of a quantum system is represented by a vector in a Hilbert space with the norm ("length") of 1.

Notation: $|\psi\rangle$ ("ket-vector")

number and order of postulates not important

Hilbert space: complete inner-product space

Linear space, can introduce basis, orthonormal basis

In 1D or 3D case $|\psi\rangle \leftrightarrow \psi(x)$ or $\psi(\vec{r})$, so Hilbert space is infinite-dimensional.

In QC much simpler, since Hilbert space is always finite-dimensional.

$$|\psi\rangle = \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \cdot\cdot \\ \cdot\cdot \\ \alpha_N \end{pmatrix} \qquad |\phi\rangle = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \cdot\cdot \\ \cdot\cdot \\ \beta_N \end{pmatrix}$$

α_i , β_i are complex numbers

Inner product is

$$\langle\phi|\psi\rangle = \sum_i \beta_i^* \alpha_i$$

(Dirac notation, bra-ket)

Postulate 1 (cont.)

Different bases are possible (e.g., measurement of spin along different directions)

"Computational basis": $|000\rangle, |001\rangle, |010\rangle, \ldots$

$$|\psi\rangle = \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_7 \end{pmatrix} = \alpha_0|000\rangle + \alpha_1|001\rangle + \alpha_2|010\rangle + \cdots + \alpha_7|111\rangle$$

superposition $\quad \sum_i |\alpha_i|^2 = 1$ (normalization)

Most states are entangled: 1 qubit is characterized by 2 complex numbers, so k separate qubits would be characterized by $2k$ complex numbers, but general k-qubit state is characterized by 2^k complex numbers.

2 qubits, "separable state"

$$(\alpha|0\rangle + \beta|1\rangle) \otimes (\gamma|0\rangle + \delta|1\rangle) = \underbrace{\alpha\gamma}_{\alpha_{00}}|00\rangle + \underbrace{\alpha\delta}_{\alpha_{01}}|01\rangle + \underbrace{\beta\gamma}_{\alpha_{10}}|10\rangle + \underbrace{\beta\delta}_{\alpha_{11}}|11\rangle$$

We see that $\alpha_{00}\alpha_{11} = \alpha_{10}\alpha_{01}$, while general 2-qubit wavefunction does not satisfy this condition $\Rightarrow$ most states are not separable (i.e., entangled)

Postulate 2

Postulate 2. Measurable quantities (physical magnitudes, dynamical variables, "observables") are represented by Hermitian operators

Hermitian (self-adjoint) operator: $\hat{B}^\dagger = \hat{B}$

for a matrix, Hermitian conjugate is $B^\dagger_{ij} = B^*_{ji}$

Hermitian matrix: $B_{ij} = B^*_{ji}$ (real on diagonal, complex-conjugate off-diagonal)

Properties of Hermitian operators

- Eigenvalues are real
- Eigenvectors form orthonormal basis (somewhat oversimplified), so each observable defines an orthonormal basis, in which its matrix is diagonal (with real elements)

Postulate 3

Postulate 3. Measurement result is necessarily one of eigenvalues of the corresponding operator (no other results possible).

Measurement result r is generally random, with probability $p_r = |\langle\psi_r|\psi\rangle|^2$, where $|\psi\rangle$ is the state before measurement and $|\psi_r\rangle$ is the normalized eigenvector, corresponding to the eigenvalue r.

If spectrum of the measured operator is degenerate (i.e., a subspace corresponds to the eigenvalue r), then we need to choose a basis $|\psi_{r,j}\rangle$ in this subspace, and $p_r = \sum_j \left|\langle\psi_{r,j}|\psi\rangle\right|^2$.

Another way to think: $p_r = \|\, \widehat{\mathbb{P}}_r|\psi\rangle \,\|^2$

where $\widehat{\mathbb{P}}_r$ is operator of projection onto subspace, corresponding to the eigenvalue r and $\| \,..\, \|$ denotes norm ("length") of a vector.

Postulate 3 (cont.)

Example

$$|\psi\rangle = \alpha_0|000\rangle + \alpha_1|001\rangle + \alpha_2|010\rangle + \cdots + \alpha_7|111\rangle$$

Measure all 3 qubits. 8 possible results.

$$\begin{array}{ll} 0 \leftrightarrow 000 \leftrightarrow |000\rangle & P_0 = |\alpha_0|^2 \\ 1 \leftrightarrow 001 \leftrightarrow |001\rangle & P_1 = |\alpha_1|^2 \\ 2 \leftrightarrow 010 \leftrightarrow |010\rangle & P_2 = |\alpha_2|^2 \\ \ldots & \ldots \\ 7 \leftrightarrow 111 \leftrightarrow |111\rangle & P_7 = |\alpha_7|^2 \end{array}$$

Measured observable

$$\hat{M} = \begin{pmatrix} 0 & & & & & & & \\ & 1 & & & & & \mathbf{0} & \\ & & 2 & & & & & \\ & & & 3 & & & & \\ & & & & 4 & & & \\ & \mathbf{0} & & & & 5 & & \\ & & & & & & 6 & \\ & & & & & & & 7 \end{pmatrix}$$

eigenvalue for corresp. eigenstate (comp.basis)

Now measure only first qubit, results 0 or 1

$$\hat{M}_1 = \begin{pmatrix} 0 & & & & & & & \\ & 0 & & & & & \mathbf{0} & \\ & & 0 & & & & & \\ & & & 0 & & & & \\ & & & & 1 & & & \\ & \mathbf{0} & & & & 1 & & \\ & & & & & & 1 & \\ & & & & & & & 1 \end{pmatrix}$$

eigenvectors: $\begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ for 0 $\quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_7 \end{pmatrix}$ for 1

Postulate 3 (cont.)

Measure qubits 2 and 3

$$\hat{M}_2 = \begin{pmatrix} 0 & & & & & & & \\ & 0 & & & & & & \\ & & 1 & & & & & \\ & & & 1 & & & & \\ & & & & 0 & & & \\ & & & & & 0 & & \\ & & & & & & 1 & \\ & & & & & & & 1 \end{pmatrix} \quad \hat{M}_3 = \begin{pmatrix} 0 & & & & & & & \\ & 1 & & & & & & \\ & & 0 & & & & & \\ & & & 1 & & & & \\ & & & & 0 & & & \\ & & & & & 1 & & \\ & & & & & & 0 & \\ & & & & & & & 1 \end{pmatrix}$$

$$\hat{M} = 4\hat{M}_1 + 2\hat{M}_2 + \hat{M}_3$$

8 eigenvalues: 0, 1, .. 7

Postulate 3’

Postulate 3’. Average (“expectation”) value for measuring operator $\hat{B}$ is

$$\langle \hat{B} \rangle = \langle \psi | \hat{B} | \psi \rangle.$$

Follows from postulate 3, but often a separate postulate

Important in standard quantum mechanics, but not important for QC (except NMR)

$$\langle \psi | \hat{B} | \psi \rangle = (\alpha_0^* \, \alpha_1^* \, .. \, \alpha_N^*) \begin{pmatrix} b_{00} & & \\ & b_{ij} & \\ & & b_{NN} \end{pmatrix} \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ .. \\ \alpha_N \end{pmatrix}$$

$$\langle \psi | \hat{B} | \psi \rangle = \langle \psi | \, (\hat{B} | \psi \rangle) = (\langle \psi | \hat{B}) \, | \psi \rangle$$

Proof

$\hat{B} = \sum_r r \, \hat{\mathbb{P}}_r$ (since Hermitian)

$$\langle \psi | \hat{B} | \psi \rangle = \sum_r r \, \langle \psi | \, \hat{\mathbb{P}}_r | \psi \rangle = \sum_r r \, \langle \psi | \, \hat{\mathbb{P}}_r \, \hat{\mathbb{P}}_r | \psi \rangle = \sum_r r \, \| \, \hat{\mathbb{P}}_r | \psi \rangle \|^2 = \sum_r r \, p_r$$

Postulate 4

Postulate 4. After measurement of $\hat{B}$ with result r, the state abruptly changes:

$$|\psi\rangle \to \frac{\hat{\mathbb{P}}|\psi\rangle}{\| \hat{\mathbb{P}}|\psi\rangle \|} \quad \text{(projected onto subspace and normalized)}$$

Called wavefunction collapse

Examples

$$|\psi\rangle = \alpha_0|000\rangle + \alpha_1|001\rangle + \alpha_2|010\rangle + \cdots + \alpha_7|111\rangle$$

(a) Measure all qubits, get result $3 = 011$, then $|\psi\rangle \to |011\rangle$

(Does not matter what was before!
Cannot get more information on α_i.)

(b) Measure only first qubit, get result 0, then

$$|\psi\rangle \to \frac{\alpha_0|000\rangle + \alpha_1|001\rangle + \alpha_2|010\rangle + \alpha_3|011\rangle}{\sqrt{|\alpha_0|^2 + |\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2}}$$

Postulate 5

Postulate 5. Evolution of a quantum state is described by

the Schrödinger equation $$\frac{d|\psi\rangle}{dt} = -\frac{i}{\hbar}\hat{H}|\psi\rangle\ ,$$

where $\hat{H}$ is the operator of energy (Hamiltonian)

We will not really use it, but important that evolution is described by a unitary operator

$$|\psi(t)\rangle = e^{-\frac{i}{\hbar}\hat{H}t}|\psi(0)\rangle = \hat{U}\ |\psi(0)\rangle$$

Since $\hat{H}$ is Hermitian, $\hat{U}$ is unitary, $\hat{U}^{\dagger} = \hat{U}^{-1}$ $\qquad \hat{U}\hat{U}^{\dagger} = \hat{U}^{\dagger}\hat{U} = \hat{1}$

A unitary operator preserves inner product

$$\langle(\hat{U}\phi)|(\hat{U}\psi)\rangle = \langle\phi|\ \hat{U}^{\dagger}\hat{U}\psi\rangle = \langle\phi|\psi\rangle$$

Unitary operator transforms an orthonormal basis into an orthonormal basis (rotation of a space)