Quantum Information and Quantum Computation

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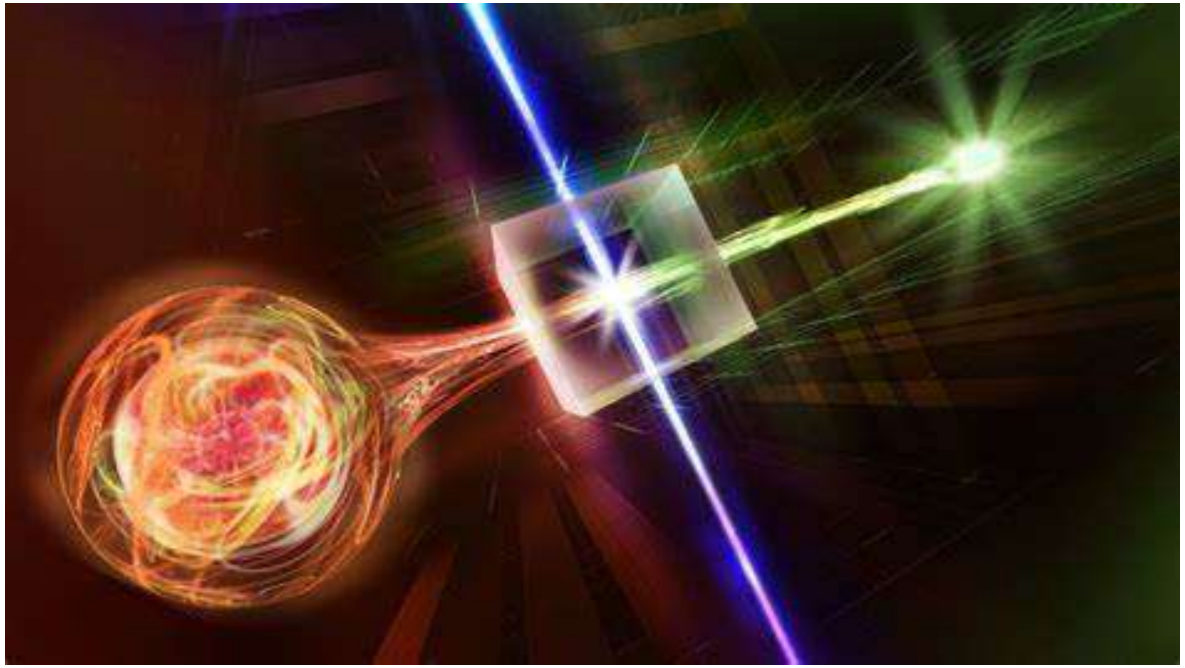
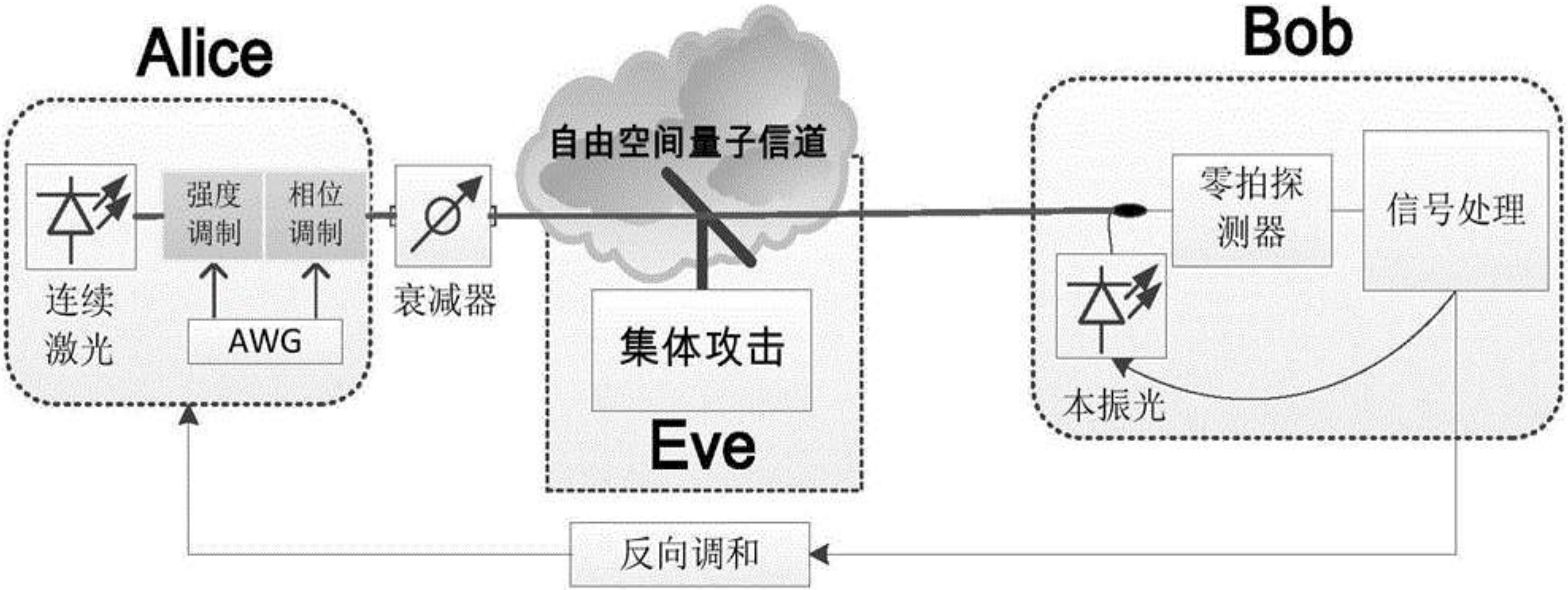
Lecture 5

Quantum states of light

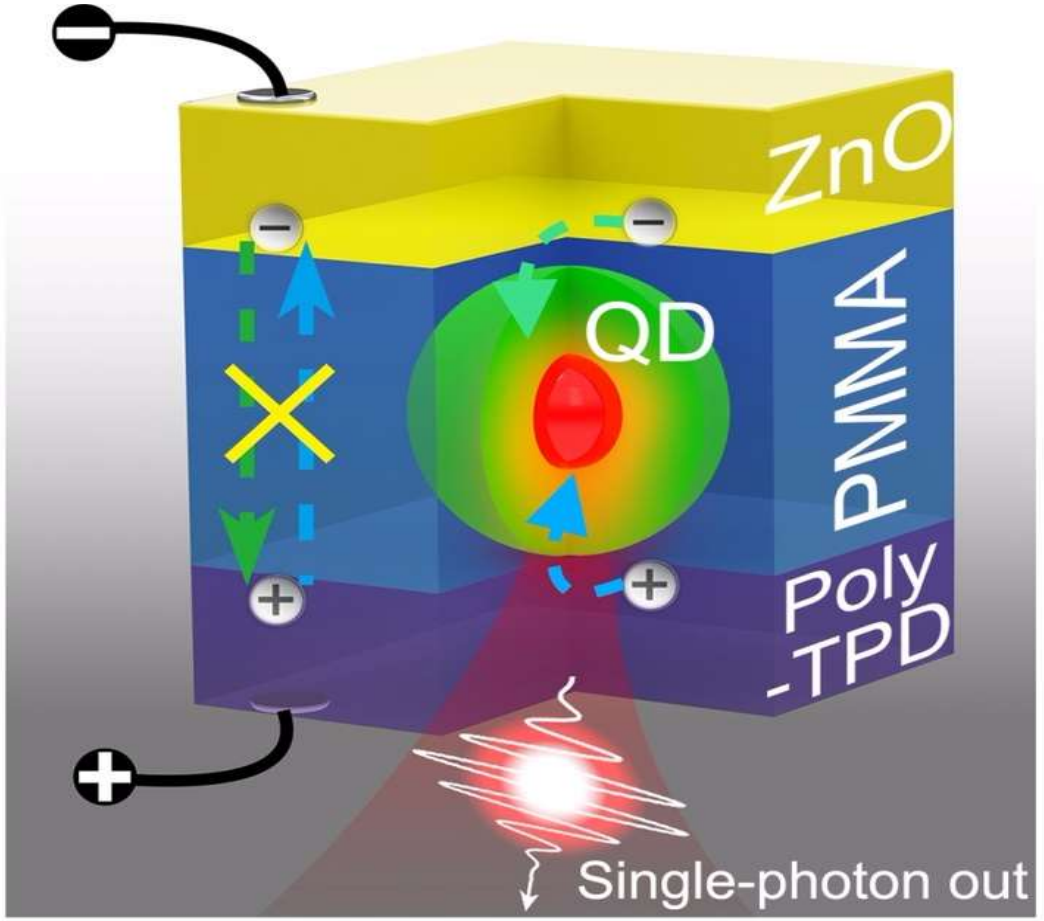


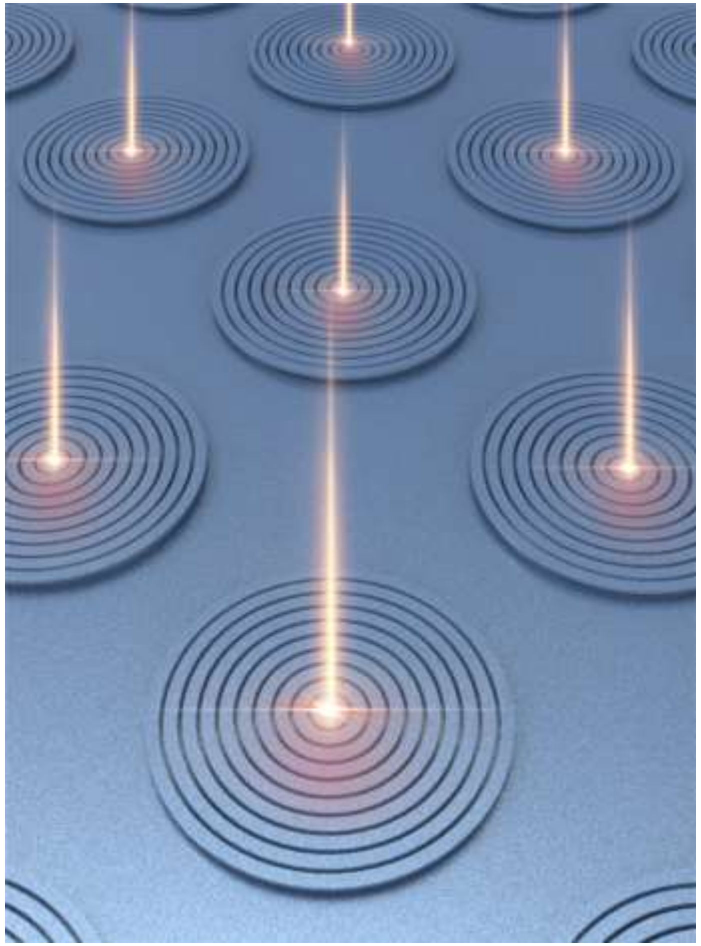
单量子态的制备：衰减



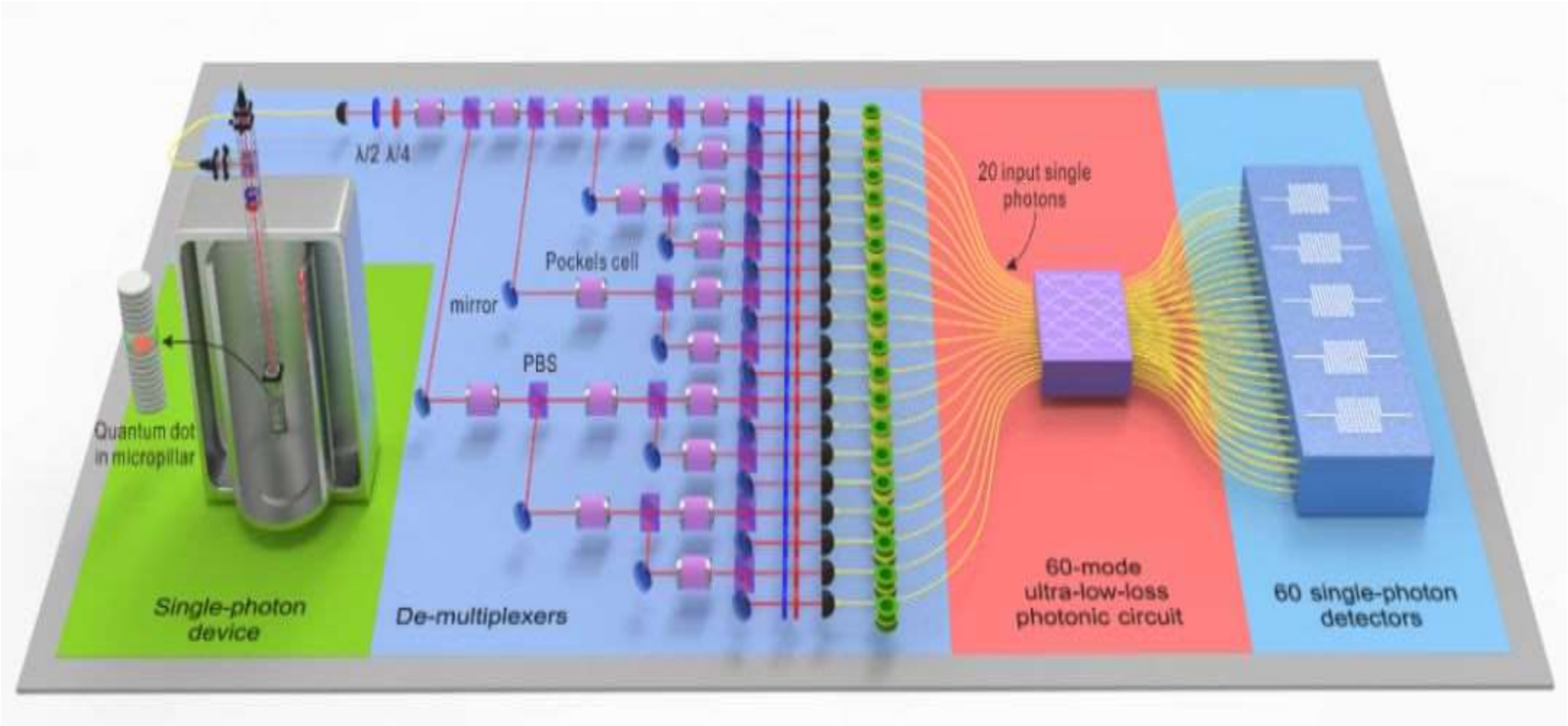
自由空间连续变量量子密钥分发协议的制作方法

单量子态的制备：量子点



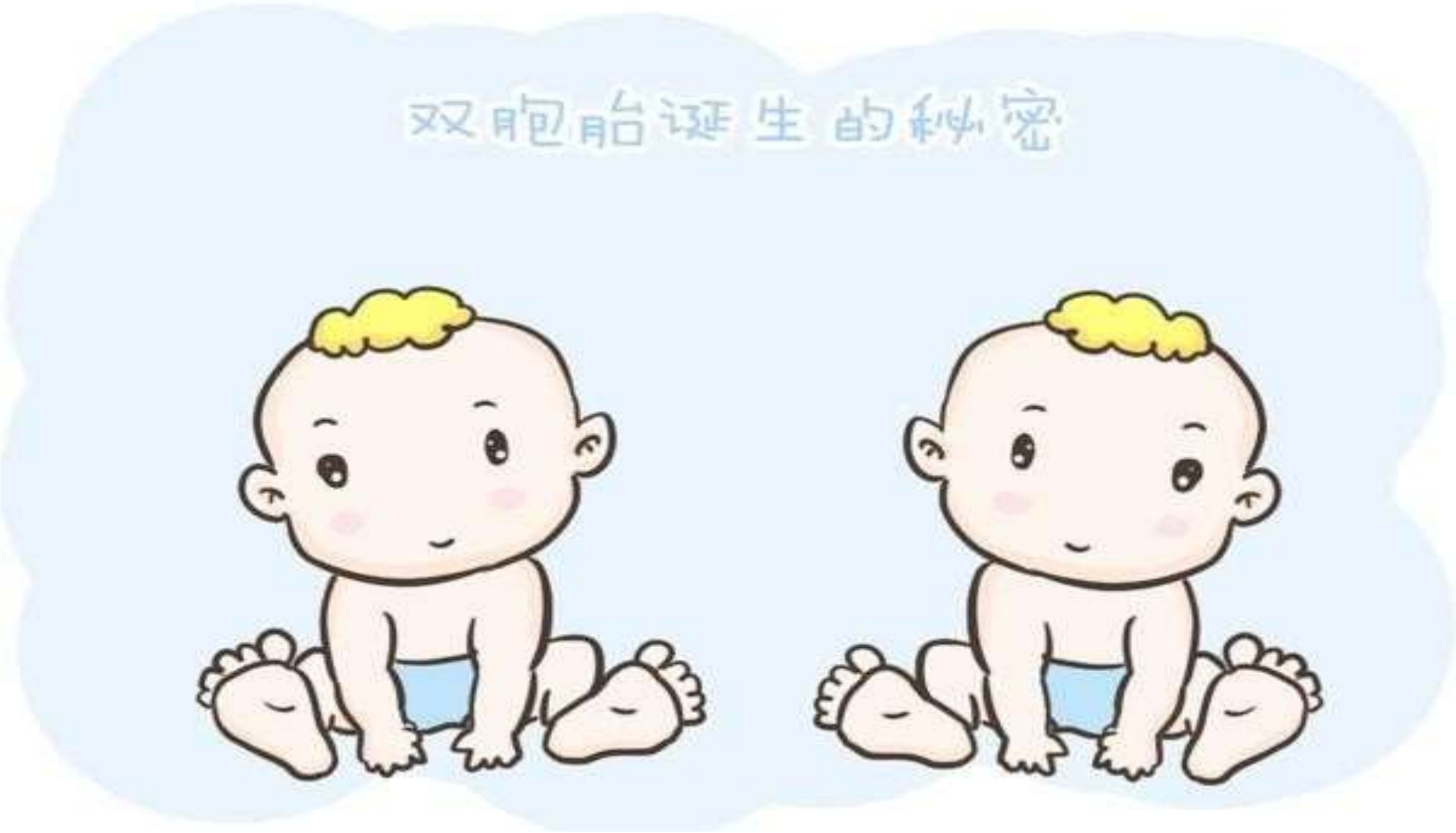


单量子态的制备：量子点



玻色采样

量子纠缠?



■ 正负电子湮灭会产生一对具有不同偏振方向的光子 （1948年惠勒提出）

EPR correlation of polarization of two photons propagating in opposite directions

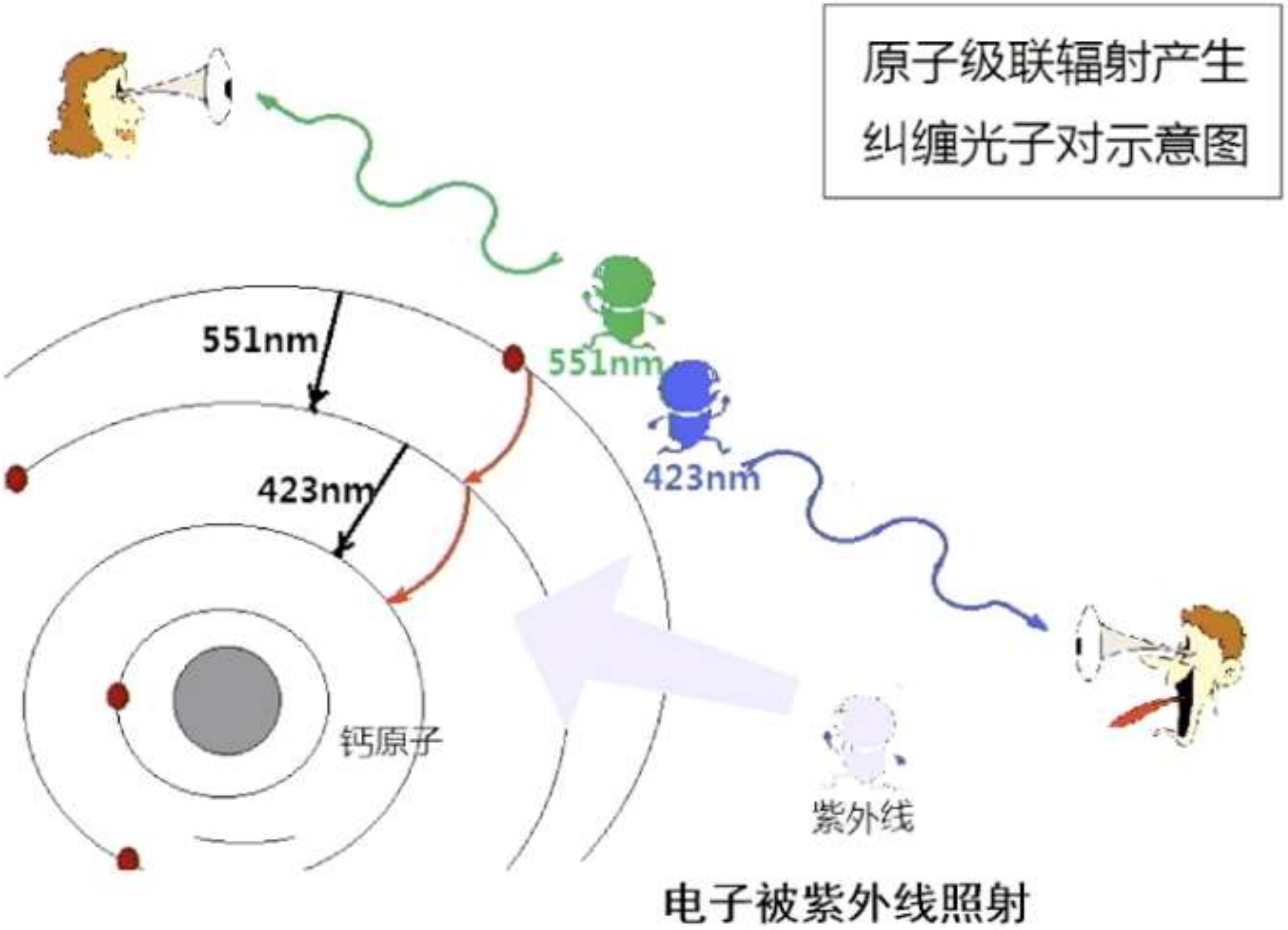
Wu and Shaknov, PRA 77, 136 (1950) Columbia University

吴健雄和萨科诺夫

1950，史上第一对互相纠缠的光子！

实验表明具有零角动量的正、负电子对湮没后发出的两个高能光量子，如狄拉克理论所预料，将互成直角而被极化，也证明正电子与负电子的宇称相反

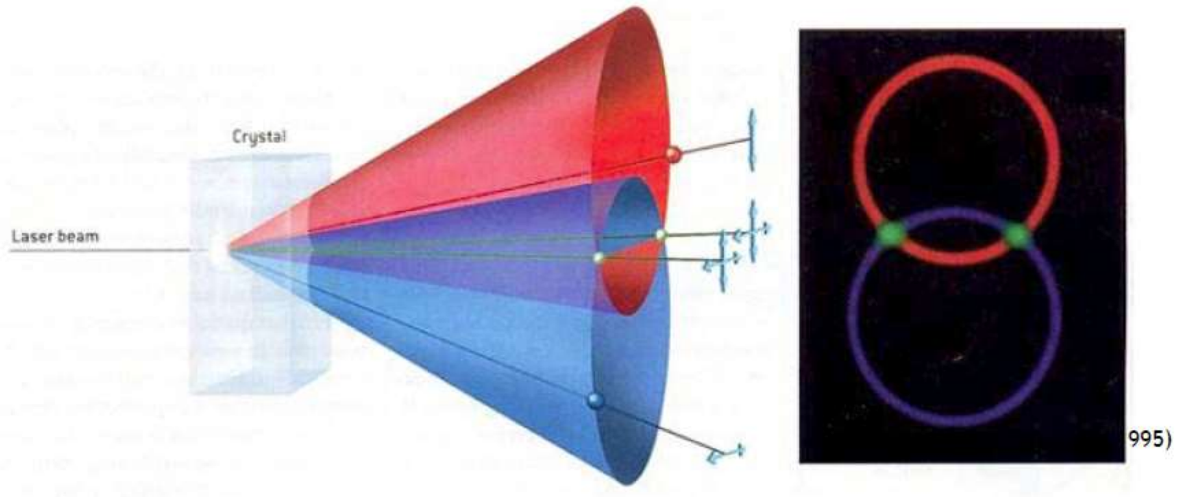
■ 级联光子对 (cascade-photon)

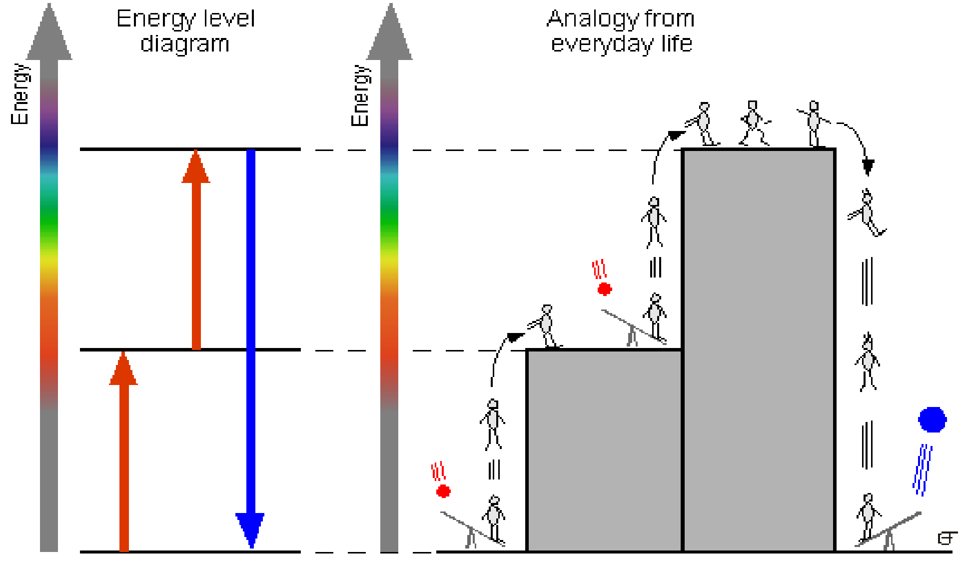


Spontaneous parametric down conversion

Oldie but goodie…

single type II emitter scheme





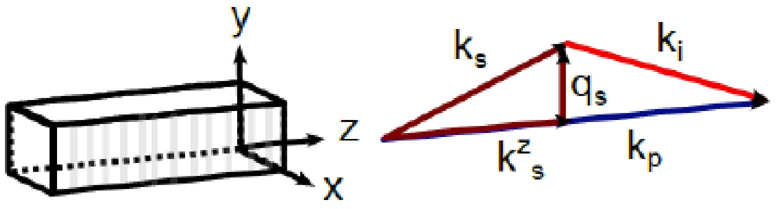
$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_1\,|V\rangle_2 - |V\rangle_1\,|H\rangle_2\right)$$

Kwiat et al, PRL 75, 4337 (1995)

Spontaneous parametric down conversion

PDC interaction Hamiltonian (*assuming only one relevant NL tensor coeffictient*)

$$\hat{\mathrm{H}}_I \propto \int d\boldsymbol{r}\,\chi^{(2)}(\boldsymbol{r}_\perp, z)\,\hat{\mathrm{E}}_p^{+}(\boldsymbol{r}_\perp, z, t)\,\hat{\mathrm{E}}_s^{-}(\boldsymbol{r}_\perp, z, t)\,\hat{\mathrm{E}}_i^{-}(\boldsymbol{r}_\perp, z, t) + c.c$$



Transverse field operators (*narrowband*) $\omega = \omega_0 + \Omega$

Transverse momentum

$$\boldsymbol{k} = k^z(\omega, \boldsymbol{q})\boldsymbol{e}_z + \boldsymbol{q}\ , \qquad \boldsymbol{q} = (\mathrm{k_x}, \mathrm{k_y})\ ,$$

Initial State:

$$|\,\Psi_0\rangle = \left|\,E_p(q), s(\omega)\right\rangle \otimes |\,vac\rangle \otimes |\,vac\rangle$$

Two-photon SPDC state

$$\left|\Psi_{SPDC}^{(2-\text{photon})}\right\rangle \propto \int_{T,V} dt\, d\boldsymbol{r}\, \chi^{(2)}(\boldsymbol{r}_\perp, z) E_p(\boldsymbol{r}_\perp, z, t)\, \hat{\mathrm{E}}_s^-(\boldsymbol{r}_\perp, z, t)\, \hat{\mathrm{E}}_i^-(\boldsymbol{r}_\perp, z, t) |\, vac\rangle_{s,i}$$

$$= \int_{T,V} dt\, d\boldsymbol{r}\, \chi^{(2)}(\boldsymbol{r}_\perp, z) \int d\omega_p\, d\boldsymbol{q}_p\, E_p(\boldsymbol{q}_p) \mathrm{s}(\omega_p) \mathrm{e}^{(ik_p^z z + i\, \boldsymbol{q}_p \cdot \boldsymbol{r}_\perp - i\omega_p t)} \times$$

$$\int d\omega_s\, d\boldsymbol{q}_s \mathrm{e}^{-(ik_s^z z + i\, \boldsymbol{q}_s \cdot \boldsymbol{r}_\perp - i\omega_s t)} \int d\omega_i\, d\boldsymbol{q}_i \mathrm{e}^{-(ik_s^z z + i\, \boldsymbol{q}_s \cdot \boldsymbol{r}_\perp - i\omega_s t)} \quad |\omega_s, \boldsymbol{q}_s\rangle |\omega_i, \boldsymbol{q}_i\rangle$$

...

$$\left|\Psi_{SPDC}^{(2-\text{photon})}\right\rangle = \int d\omega_s\, d\boldsymbol{q}_s\, d\omega_i\, d\boldsymbol{q}_i \underbrace{\Phi(\omega_s, \boldsymbol{q}_s, \omega_i, \boldsymbol{q}_i)}_{\textbf{\textit{SPDC biphoton mode function}}} |\omega_s, \boldsymbol{q}_s\rangle |\omega_i, \boldsymbol{q}_i\rangle$$

Two-photon mode function SPDC

$$\Phi(\omega_s\, , \boldsymbol{q}_s\, ,\ \omega_i\, , \boldsymbol{q}_i) = \int_{T,V}\ dt\, d\boldsymbol{r}_\perp dz\, \chi^{(2)}(\boldsymbol{r}_\perp, z) \int d\omega_p\, d\boldsymbol{q}_p\, E_p(\boldsymbol{q}_p) \mathrm{s}(\omega_p) \mathrm{e}^{-i(\omega_p-\omega_s-\omega_i)t} \mathrm{e}^{i(k_p^z-k^z{}_s-k_i^z)z}\ \mathrm{e}^{i(\boldsymbol{q}_p-\boldsymbol{q}_s-\boldsymbol{q}_i)\cdot\boldsymbol{r}_\perp}$$

Crystal transversally homogeneous with large aperture:

$$\chi^{(2)}(\boldsymbol{r}_\perp, z) \to \chi^{(2)}(z) \qquad \int d\boldsymbol{r}_\perp \mathrm{e}^{i(\boldsymbol{q}_p-\boldsymbol{q}_s-\boldsymbol{q}_i)\cdot\boldsymbol{r}_\perp} \to \delta(\boldsymbol{q}_p - \boldsymbol{q}_s - \boldsymbol{q}_i)$$

Long interaction time:

$$\int dt\, \mathrm{e}^{-i(\omega_p-\omega_s-\omega_i)t} \to \delta(\omega_p - \omega_s - \omega_i)$$

Crystal length: L

$$\int_{-\frac{L}{2}}^{\frac{L}{2}} dz\, \mathrm{e}^{i(k_p^z-k^z{}_s-k_i^z)z} \ \to\ sinc\left(\frac{L}{2}\Delta k^z\right)$$

Two-photon mode function SPDC

$$\Phi(\omega_s\,, \boldsymbol{q}_s\,,\ \omega_i\,, \boldsymbol{q}_i) = \int_{T,V}\ dt\, d\boldsymbol{r}_\perp dz\, \chi^{(2)}(\boldsymbol{r}_\perp, z) \int d\omega_p\, d\boldsymbol{q}_p\, E_p(\boldsymbol{q}_p) s(\omega_p) \mathrm{e}^{-i(\omega_p-\omega_s-\omega_i)t}\, \mathrm{e}^{i(k_p^z-k^z{}_s-k_i^z)z}\ \mathrm{e}^{i(\boldsymbol{q_p}-\boldsymbol{q_s}-\boldsymbol{q_i})\cdot \boldsymbol{r}_\perp}$$

Crystal transversally homogeneous with large aperture:

$$\chi^{(2)}(\boldsymbol{r}_\perp, z) \rightarrow \chi^{(2)}(z) \qquad \int d\boldsymbol{r}_\perp \mathrm{e}^{i(\boldsymbol{q_p}-\boldsymbol{q_s}-\boldsymbol{q_i})\cdot \boldsymbol{r}_\perp} \rightarrow \delta(\boldsymbol{q}_p - \boldsymbol{q}_s - \boldsymbol{q}_i)$$

Transverse momentum conservation

$$\boldsymbol{q}_p = \boldsymbol{q}_s + \boldsymbol{q}_i$$

Long interaction time:

$$\int dt\ \mathrm{e}^{-i(\omega_p-\omega_s-\omega_i)t} \rightarrow \delta(\omega_p - \omega_s - \omega_i)$$

Energy conservation

$$\omega_p = \omega_s + \omega_i$$

Crystal length: L

$$\int_{-\frac{L}{2}}^{\frac{L}{2}} dz\ \mathrm{e}^{i(k_p^z-k^z{}_s-k_i^z)z} \rightarrow\ sinc\left(\frac{L}{2}\Delta k^z\right)$$

Longitudinal momentum conservation

$$k_p^z = k^z{}_s + k_i^z$$

Two-photon mode function SPDC

$$\left| \Psi_{SPDC}^{(2-\mathrm{photon})} \right\rangle = \int d\omega_s \, d\boldsymbol{q}_s \, d\omega_i \, d\boldsymbol{q}_i \Phi(\omega_s, \boldsymbol{q}_s, \omega_i, \boldsymbol{q}_i) \, |\omega_s, \boldsymbol{q}_s\rangle |\omega_i, \boldsymbol{q}_i\rangle$$

$$\Phi(\omega_s, \boldsymbol{q}_s, \omega_i, \boldsymbol{q}_i) = \sigma L \, E_p(\boldsymbol{q}_s + \boldsymbol{q}_i) \, \mathrm{s}(\omega_s + \omega_i) \, sinc\left(\frac{L}{2} \Delta k^z\right)$$

Pump contribution: $E_p(\boldsymbol{q}_s + \boldsymbol{q}_i) \, \mathrm{s}(\omega_s + \omega_i)$

Phase matching function: $\Delta k^z(\omega_s, \omega_{i,} \boldsymbol{q}_s, \boldsymbol{q}_i) = k_p^z(\omega_s + \omega_i, \boldsymbol{q}_s + \boldsymbol{q}_i) - k_s^z(\omega_s, \boldsymbol{q}_s) - k_i^z(\omega_i, \boldsymbol{q}_i)$

Two-photon mode function SPDC

$$\left|\Psi_{SPDC}^{(2-\mathrm{photon})}\right\rangle = \int d\omega_s\, d\boldsymbol{q}_s\, d\omega_i\, d\boldsymbol{q}_i \Phi(\omega_s\,, \boldsymbol{q}_s\,,\ \omega_i\,, \boldsymbol{q}_i)\, |\omega_s, \boldsymbol{q}_s\rangle |\omega_i, \boldsymbol{q}_i\rangle$$

$$\Phi(\omega_s\,, \boldsymbol{q}_s\,,\ \omega_i\,, \boldsymbol{q}_i) = \sigma L\, E_p(\boldsymbol{q}_s + \boldsymbol{q}_i)\, \mathrm{s}(\omega_s + \omega_i)\, sinc\left(\frac{L}{2}\Delta k^z\right)$$

Pump contribution: $E_p(\boldsymbol{q}_s + \boldsymbol{q}_i)\, \mathrm{s}(\omega_s + \omega_i)$

Phase matching function: $\Delta k^z(\omega_s, \omega_{i,} \boldsymbol{q}_s, \boldsymbol{q}_i) = k_p^z(\omega_s + \omega_i, \boldsymbol{q}_s + \boldsymbol{q}_i) - k_s^z(\omega_s, \boldsymbol{q}_s) - k_i^z(\omega_i, \boldsymbol{q}_i)$

CW Pump: $\mathrm{s}(\omega_p) = \delta(\omega_p - \omega_{p,0}) \rightarrow \omega_i = \omega_{p,0} - \omega_s$ *Collinear SPDC* $\boldsymbol{q}_s = \boldsymbol{0}, \boldsymbol{q}_i = \boldsymbol{0}$

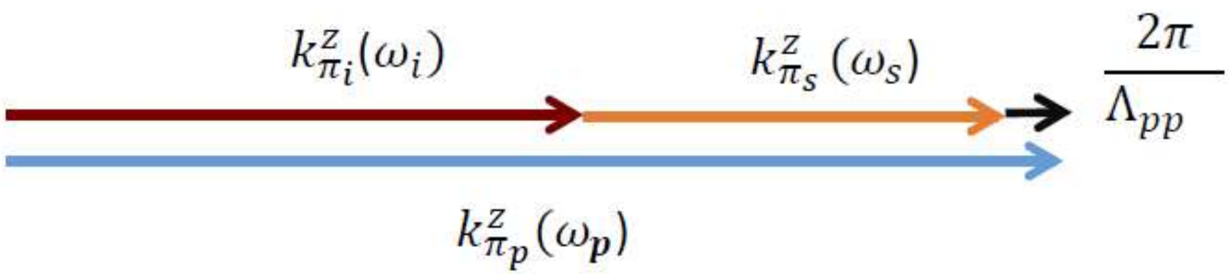
$$\left|\Psi_{SPDC}^{(2-\mathrm{photon})}\right\rangle = \int d\omega_s\, sinc\left(\frac{L}{2}\Delta k^z(\omega_s)\right) |\omega_s\rangle |\omega_{p,0} - \omega_s\rangle$$

Phase-matching

$$\omega_i = \omega_{p,0} - \omega_s$$

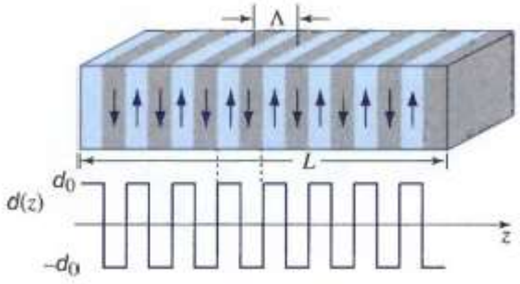
Collinear phase-matching condition : $\Delta k^z(\omega_s) = \frac{2\pi}{c}\left(n_{\pi_p}(\omega_s + \omega_i)\cdot(\omega_s + \omega_i) - n_{\pi_s}(\omega_s)\omega_s - n_{\pi_i}(\omega_i)\omega_i \right) = \mathbf{0}$

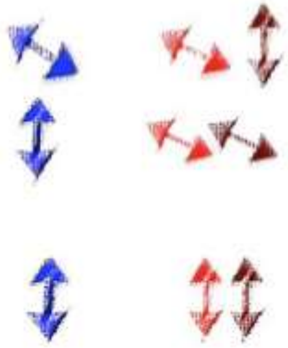
Quasi phase-matching $\chi^{(2)}(z) \rightarrow \chi^{(2)} e^{2\pi \frac{iz}{\Lambda_{pp}}}$



-> Collinear phase-matching condition modified:

$$\Delta k^z \rightarrow k_p - k_s - k_i - \frac{2\pi}{\Lambda_{pp}}$$



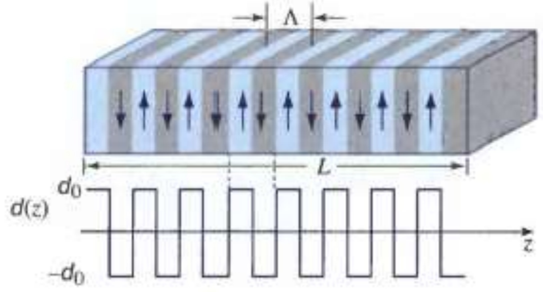


$\pi_p = \pi_s \neq \pi_i$ *type-II*

$\pi_p \neq \pi_s = \pi_i$ *Type-I*

$\pi_p = \pi_s = \pi_i$ *Type-0*

Phase-matching in ppKTP($KTiOPO_4$)

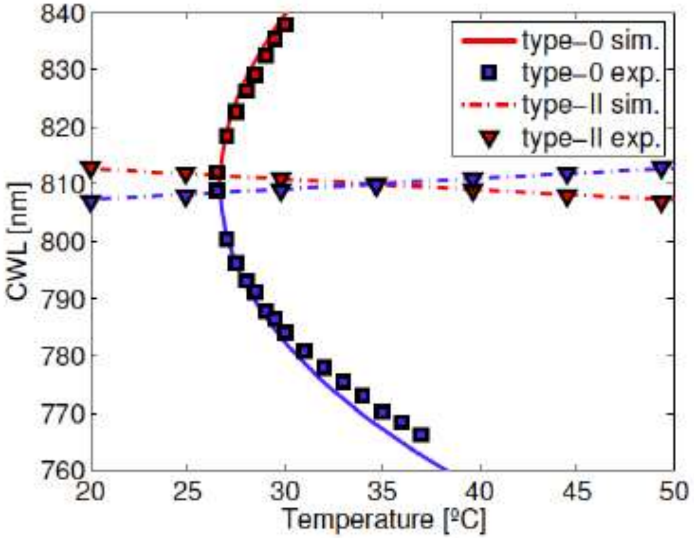


type-0 (z-z-z): Λ≈3.4μm, d_{eff}≈11pm/V

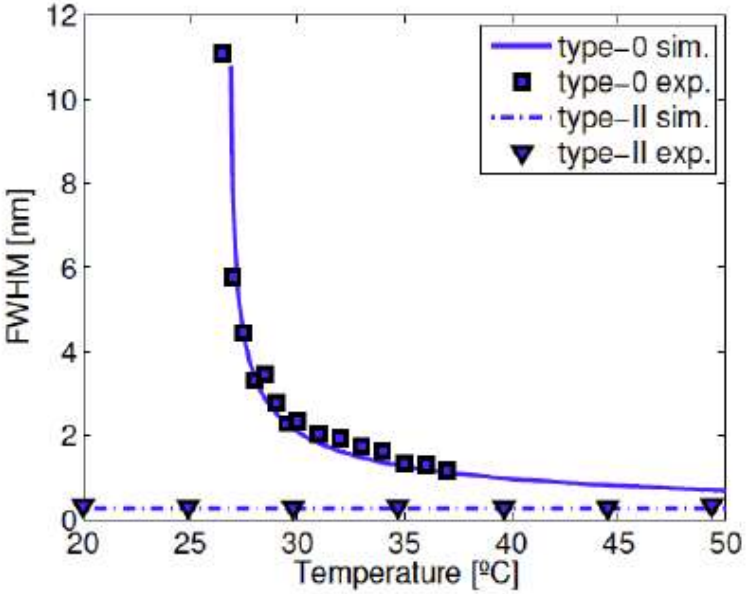
type-I (z-y-y): Λ≈2μm, d_{eff}≈3pm/V

type-II: (y-z-y): Λ≈10μm, d_{eff}≈2pm/V

center wavelength



spectral bandwidth



Beyond polarization qubits

Qubit entanglement

$$|00\rangle + |11\rangle$$

Qudit Entanglement

$$|00\rangle + |11\rangle + |22\rangle + \ldots |dd\rangle$$

Beyond polarization qubits

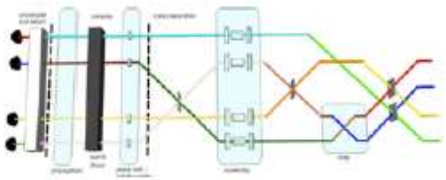
Qubit **entanglement**

$$|00\rangle + |11\rangle$$

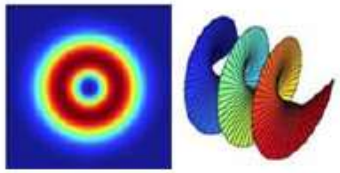
Qudit **Entanglement**

$$|00\rangle + |11\rangle + |22\rangle + \ldots |dd\rangle$$

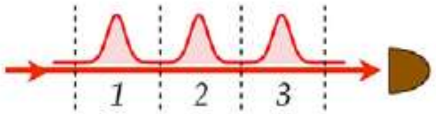
Path



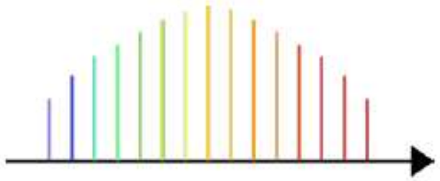
Spatial mode



Time

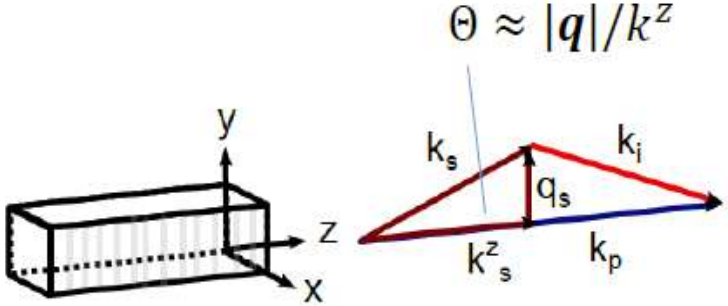


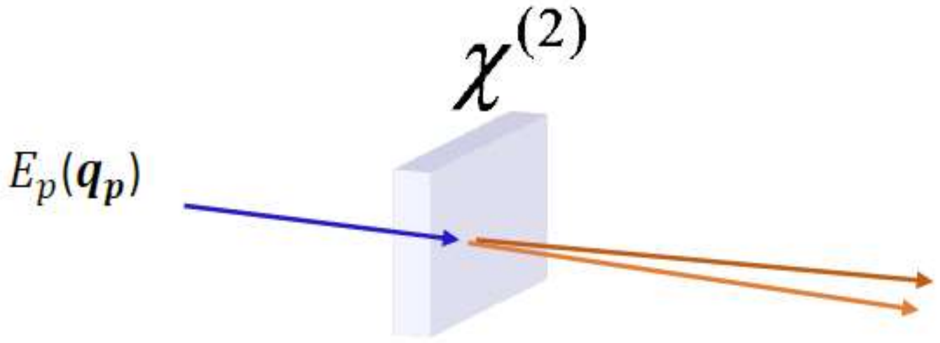
Frequency



We can get high-dimensional qudit entanglement from SPDC...

Spatial mode function





$$\boldsymbol{q_p} = \boldsymbol{q_s} + \boldsymbol{q_i}$$

$$\Phi(\boldsymbol{q}_s, \boldsymbol{q}_i) \propto E_p(\boldsymbol{q}_s + \boldsymbol{q}_i) sinc\left(\frac{L}{2}\Delta k^z(\boldsymbol{q_s}, \boldsymbol{q_i})\right)$$

Simplification I: Frequencies fixed (narrowband filters)

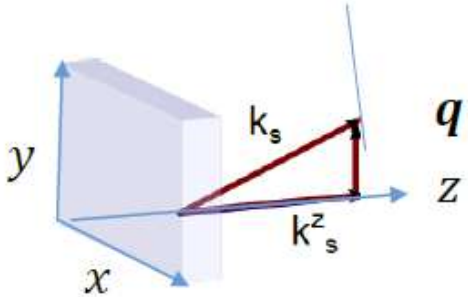
$$\Phi(\omega_s = \omega_{s0}, \boldsymbol{q}_s\,, \omega_i = \omega_{i0}, \boldsymbol{q}_i) \to \Phi(\boldsymbol{q}_s, \boldsymbol{q}_i)$$

$$\omega_{p0} = \omega_{s0} + \omega_{i0}$$

Spatial poperties of the mode function

Gaussian Pump: $\Phi(\boldsymbol{q}_s, \boldsymbol{q}_i) \propto \exp \frac{(\boldsymbol{q}_i + \boldsymbol{q}_s)^2}{4} w_p^2 \quad sinc\left(\frac{L}{2} \frac{|q_s - q_i|^2}{2k_p^z(0)}\right)$

$\Theta \approx |\boldsymbol{q}|/k^z$



Thin crystal limit

far field

Large beam waist:

$$\Phi(\boldsymbol{q}_s, \boldsymbol{q}_i) \propto \delta(\boldsymbol{q}_s + \boldsymbol{q}_i)$$

-> Signal and idler emitted at anti-correlated angles $\boldsymbol{q}_s = -\boldsymbol{q}_i$

Spatial poperties of the mode function

Gaussian Pump: $\Phi(\boldsymbol{q}_s, \boldsymbol{q}_i) \propto \exp\frac{(\boldsymbol{q}_i + \boldsymbol{q}_s)^2}{4} w_p^2 \quad sinc\left(\frac{L}{2}\frac{|q_s - q_i|^2}{2k_p^z(0)}\right)$

$\Theta \approx |\boldsymbol{q}|/k^z$

Thin crystal limit *far field* *near field*

Large beam waist:

$\Phi(\boldsymbol{q}_s, \boldsymbol{q}_i) \propto \delta(\boldsymbol{q}_s + \boldsymbol{q}_i)$ $\Phi(\boldsymbol{x}_s, \boldsymbol{x}_i) \propto \delta(\boldsymbol{x}_s - \boldsymbol{x}_i)$

-> Signal and idler emitted at anti-correlated angles $\boldsymbol{q}_s = -\boldsymbol{q}_i$

$\boldsymbol{x}_s = \boldsymbol{x}_i$ *Signal and idler „born" at same position in the crystal*

Modal entanglement

So far we considered decomposition in transverse momentum modes:

$$|\Psi\rangle = \int d\boldsymbol{q}_s\, d\boldsymbol{q}_i \Phi(\boldsymbol{q}_s, \boldsymbol{q}_i)\, |\boldsymbol{q}_s\rangle |\boldsymbol{q}_i\rangle$$

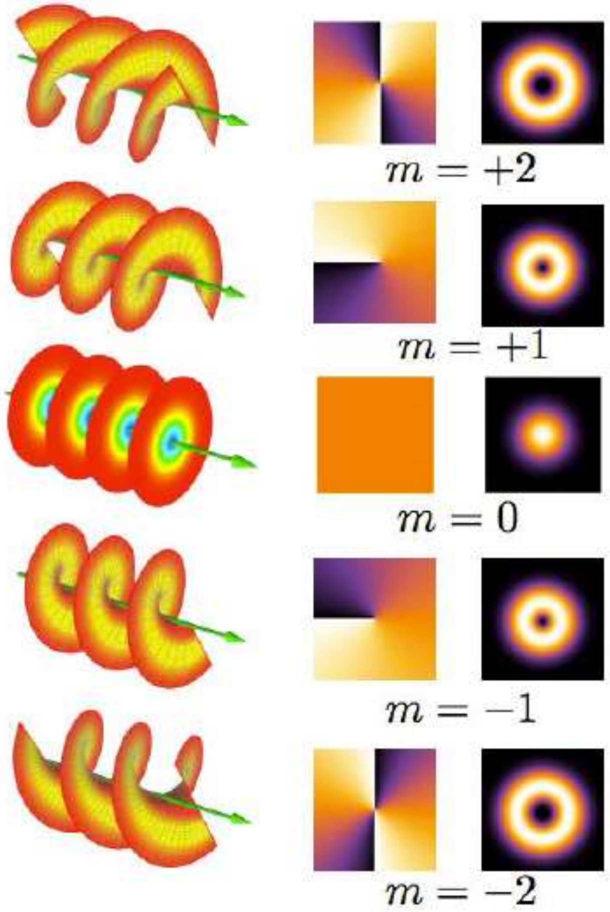
other modal decomposition, e.g. a decomp into Laguerre-Gauss modes

$$|l,p\rangle = \int d\boldsymbol{q}\;\; u_{lp}^{LG}(\boldsymbol{q})|\boldsymbol{q}\rangle$$

$$\hat{L}_z u_{lm}^{LG}(\boldsymbol{q}) = l\,\hbar\;\, u_{lm}^{LG}(\boldsymbol{q})$$

radial mode index **p**
topological winding number **l**

$$u_{lp}^{LG}(\boldsymbol{q}) = \left(\frac{w_0^2 p!}{2\pi(|l|+p)!}\right)^{1/2} \left(\frac{w_0\rho}{\sqrt{2}}\right)^{|l|} L_p^l\left(\frac{\rho^2 w_0^2}{2}\right) \times \exp\left(-\frac{\rho^2 w_0^2}{4}\right) \exp\left[il\phi + \left(p - \left|\frac{l}{2}\right|\right)\pi\right]$$



Mair, A., Vaziri, A., Weihs, G., & Zeilinger, A. (2001). Entanglement of the orbital angular momentum states of photons. Nature, 412(6844), 313.

Modal entanglement

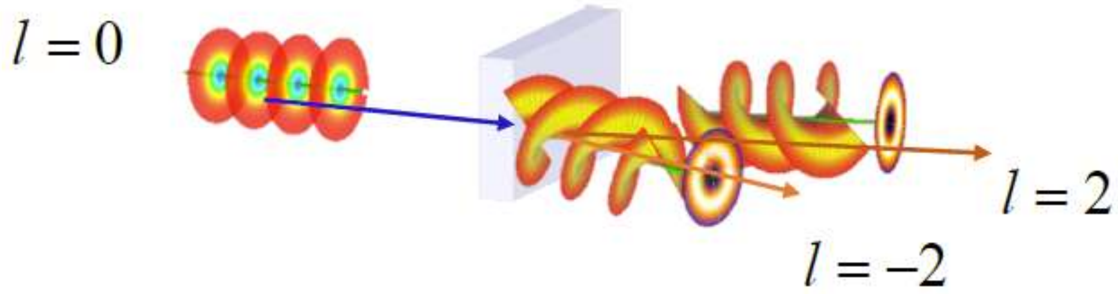
Discrete modal decomposition:

$$|\Psi\rangle = \sum_{p_1,p_2}^{l_1,l_2} C^{l_1,l_2}_{p_1,p_2} |l_1,p_1\rangle_s |l_2,p_2\rangle_i$$

Mode amplitudes related to overlap integral

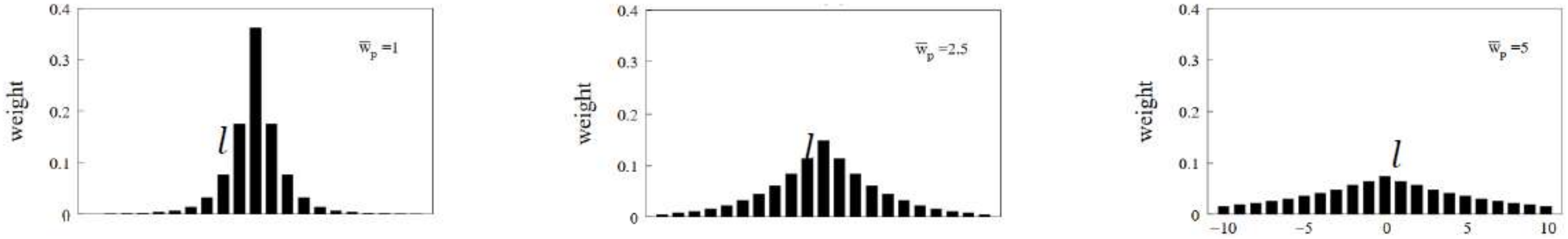
$$C^{l_1 l_2}_{p_1 p_2} = \int d\mathbf{q}_s d\mathbf{q}_i \, \Phi(\mathbf{q}_s,\mathbf{q}_i,) \, U^*_{l_1 p_1}(\mathbf{q}_s) \, U^*_{l_2 p_2}(\mathbf{q}_i)$$

$$l_p = 0 \to \exp i(l_s+l_i)\phi$$



OAM entanglement :

$$|\Psi_{AB}\rangle \propto \sum_l C_l \, |l,-l\rangle$$



Torres, J. P., Alexandrescu, A., & Torner, L. (2003). Quantum spiral bandwidth of entangled two-photon states. *Physical Review A, 68*(5), 050301.

Frequency correlations

Collinear SPDC $\boldsymbol{q}_s = \boldsymbol{0}, \boldsymbol{q}_i = \boldsymbol{0}$ $\omega_{p,0} = \omega_i + \omega_s$

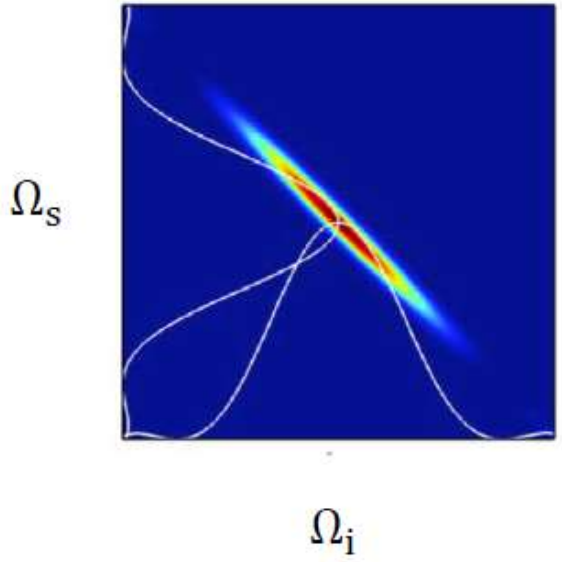
$$\omega_s = \frac{\omega_{p,0}}{2} + \Omega_\mathrm{s} \qquad \omega_i = \frac{\omega_{p,0}}{2} + \Omega_\mathrm{i}$$

$$\Phi(\omega_s\,,\,\omega_i\,) \rightarrow \quad \Phi(\Omega_\mathrm{s},\Omega_\mathrm{i}) = \mathrm{s}(\Omega_\mathrm{s}+\Omega_\mathrm{i})\; sinc\left(\frac{L}{2}\Delta k^z(\Omega_\mathrm{s},\Omega_\mathrm{i})\right)$$

CW Pump: $\mathrm{s}(\Omega_\mathrm{s}+\Omega_\mathrm{i}) = \delta(\Omega_\mathrm{s}+\Omega_\mathrm{i})$

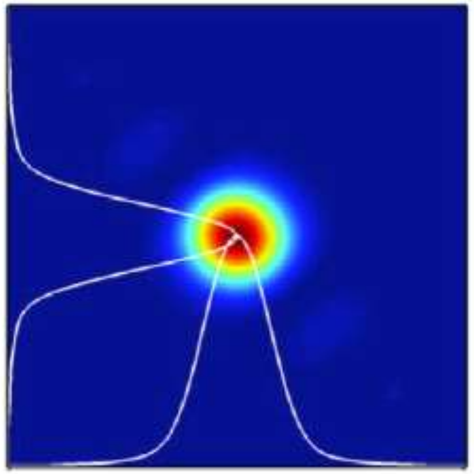
$$|\,\Psi\rangle = \int d\Omega\,\delta(\Omega_\mathrm{s}+\Omega_\mathrm{i})\; sinc\left(\frac{L}{2}\Delta k^z(\Omega)\right)|\Omega_\mathrm{s}\rangle|\Omega_\mathrm{i}\rangle \;\rightarrow \int d\Omega|\Omega\rangle|-\Omega\rangle$$

Anti-correlated joint spectral amplitude



Absence of frequency correlations

Un-correlated joint spectral amplitude



Collinear SPDC $\boldsymbol{q}_s = \mathbf{0}, \boldsymbol{q}_i = \mathbf{0}$ $\omega_{p,0} = \omega_i + \omega_s$

$$\omega_s = \frac{\omega_{p,0}}{2} + \Omega_s \qquad \omega_i = \frac{\omega_{p,0}}{2} + \Omega_i$$

$$\Phi(\omega_s\,,\,\omega_i\,) \rightarrow \quad \Phi(\Omega_s, \Omega_i) = \mathrm{s}(\Omega_s + \Omega_i)\, sinc\left(\frac{L}{2}\Delta k^z(\Omega_s, \Omega_i)\right)$$

pulsed pump: $\mathrm{s}(\Omega_s + \Omega_i) = \exp(-\alpha\, \frac{(\Omega_s+\Omega_i)^2}{\delta\omega_p^2})$

$$\exp(-\alpha\frac{(\Omega_s+\Omega_i)^2}{\delta\omega_p^2})\, sinc\left(\frac{L}{2}\Delta k^z(\Omega_s,\Omega_i)\right) \rightarrow \psi(\Omega_s)\psi(\Omega_i)$$

Un-correlated joint spectral amplitude: signal idler are in pure states → HOM interference

Time-Frequency entanglement

Collinear SPDC $\boldsymbol{q}_s = \boldsymbol{0}, \boldsymbol{q}_i = \boldsymbol{0}$ $\omega_{p,0} = \omega_i + \omega_s$

$$\omega_s = \frac{\omega_{p,0}}{2} + \Omega_\mathrm{s} \qquad \omega_i = \frac{\omega_{p,0}}{2} + \Omega_\mathrm{i}$$

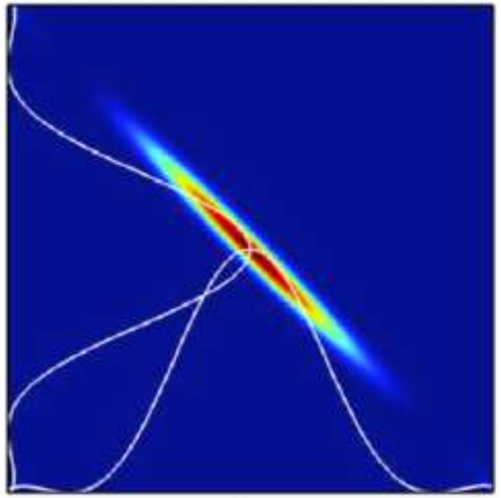
$$\Phi(\omega_s\,,\ \omega_i\,) \rightarrow \quad \Phi(\Omega_\mathrm{s}, \Omega_\mathrm{i}) = \mathrm{s}(\Omega_\mathrm{s}+\Omega_\mathrm{i})\, sinc\left(\frac{L}{2}\Delta k^z(\Omega_\mathrm{s}, \Omega_\mathrm{i})\right)$$

CW Pump: $\mathrm{s}(\Omega_\mathrm{s}+\Omega_\mathrm{i}) = \delta(\Omega_\mathrm{s}+\Omega_\mathrm{i})$

Anti-correlated spectra

$$|\,\Psi\rangle = \int d\Omega\, \delta(\Omega_\mathrm{s} + \Omega_\mathrm{i})\, sinc\left(\frac{L}{2}\Delta k^z(\Omega)\right)|\Omega_\mathrm{s}\rangle|\Omega_\mathrm{i}\rangle \rightarrow \int d\Omega|\Omega\rangle|-\Omega\rangle$$

Ω_s



Ω_i

correlated emission time

FT: $\rightarrow \int dt|\mathrm{t}\rangle|t\rangle$

Time-frequency entanglement

long pump coherence time

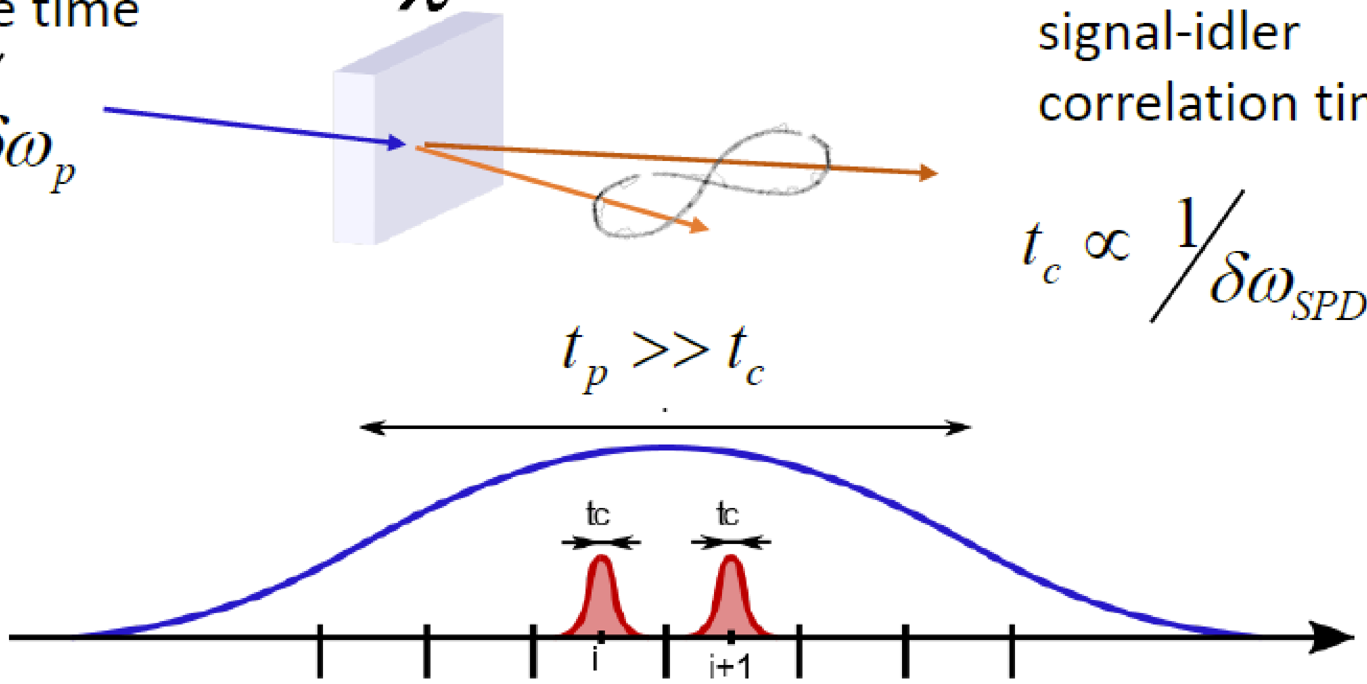
$t_p \propto 1/\delta\omega_p$

$\chi^{(2)}$

signal-idler correlation time

$t_c \propto 1/\delta\omega_{SPDC}$

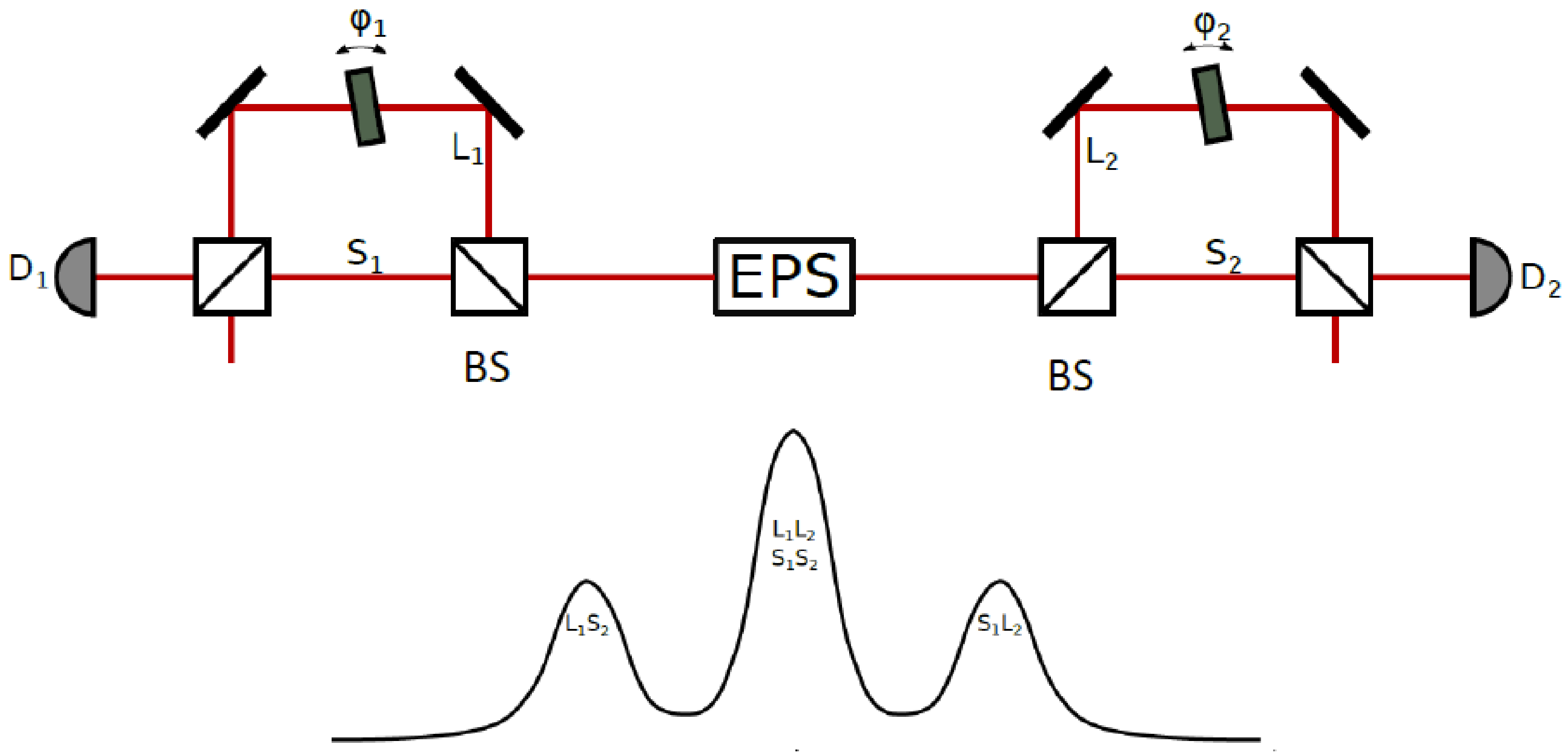
$t_p >> t_c$



Time-energy entanglement: $|\Psi_{AB}\rangle \propto \sum_{i=1}^{N} |t_i, t_i\rangle$ $\quad N \approx \frac{t_p}{t_c}$

Franson interferometer

$$|\Psi\rangle_{AB} \propto \ldots + |t_i, t_i\rangle + |t_{i+1}, t_{i+1}\rangle + \ldots$$

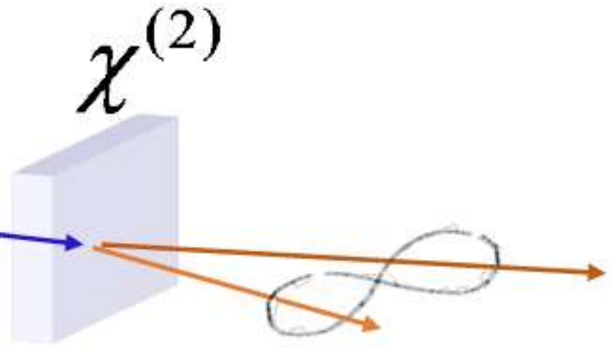


Franson J. D. Bell inequality for position and time. Phys. Rev. Lett. 62, 2205–2208 (1989)

Hyper-entanglement

long pump
coherence time

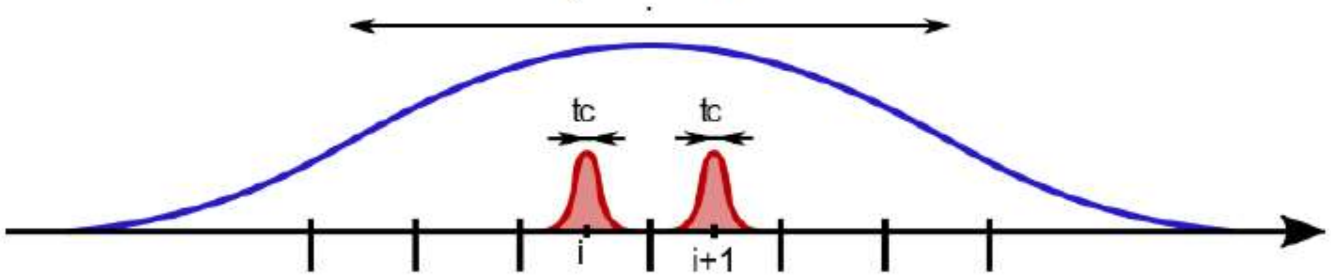
$$t_p \propto {}^{1}\!/\!{}_{\delta\omega_p}$$

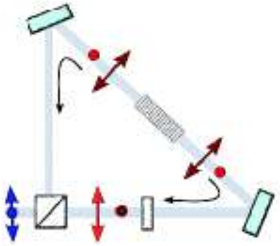


signal-idler
correlation time

$$t_c \propto {}^{1}\!/\!{}_{\delta\omega_{SPDC}}$$

$$t_p >> t_c$$





Polarization & time-energy Entanglement:

$$\sum_i |t_i, t_i\rangle \otimes \left(|HV\rangle + |VH\rangle \right)$$

„hyper entanglement“