Calculus IB: Solutions of Trail Exam

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Question 1 (one-side limit)

Find the limit: $\lim_{x \to 3^-} \dfrac{|x^2 - 9|}{x^2 - 2x - 3}$.

Solution

The procedure $x \to 3^-$ *means we only needs to focus on the case* $x < 3$, *which means* $|x - 3| = 3 - x$. *Then we have*

$$
\begin{aligned}
&\lim_{x \to 3^-} \frac{|x^2 - 9|}{x^2 - 2x - 3} \\
=&\lim_{x \to 3^-} \frac{-|x - 3| \cdot |x + 3|}{(x - 3)(x + 1)} \\
=&\lim_{x \to 3^-} \frac{-(x - 3) \cdot |x + 3|}{(x - 3)(x + 1)} \\
=&\lim_{x \to 3^-} \frac{-|x + 3|}{x + 1} \\
=&\frac{-|3 + 3|}{3 + 1} = -\frac{3}{2}
\end{aligned}
$$

Question 2 (quotient rule, product rule)

Find the limit: $f(x) = \dfrac{xe^x}{1 + \sin x}$. Find the derivative $f'(0)$.

Solution

Using the quotient rule and product rule to find $f'(x)$:

$$\begin{aligned} f'(x) &= \frac{(xe^x)'(1 + \sin x) - xe^x(1 + \sin x)'}{(1 + \sin x)^2} \\ &= \frac{(e^x + xe^x)(1 + \sin x) - xe^x \cos x}{(1 + \sin x)^2}. \end{aligned}$$

Then, we have

$$f'(0) = \frac{(e^0 + 0 \cdot e^0)(1 + \sin 0) - 0 \cdot e^0 \cos 0}{(1 + \sin 0)^2} = 1.$$

Question 3 (substitution rule)

Let f be a function with continuous second derivative f''. Some values of f and its derivative f' are given as follows:

$x =$	0	2
$f(x) =$	2	1
$f'(x) =$	1	2

Evaluate the integral $\int_0^2 2f'(x)f''(x)dx$.

Solution

Let $u = f'(x)$. Consider that $df'(x) = f''(x)dx$, we have

$$\int_0^2 2f'(x)f''(x)dx = \int_0^2 2f'(x)df'(x) = \int_{f'(0)=2}^{f'(2)=1} 2udu = u^2\Big|_2^1 = 2^2 - 1^2 = 3.$$