7 Review problems

Exercises adapted from previous exercises.

1. The sine function can be approximated with following series:

$$\sin(x) \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ...$$

   (a) Draw in the same graph the sine function and its 7th order polynomial approximation in range $x \in [-\pi, \pi]$.

   (b) Where (x-value) in the given range does the the 7th order polynomial approximation differ most? How much is the *true error* in that point?

   (c) What is the largest *relative approximation error* (in absolute values) between the 5th order and 7th order polynomials within the given range?

   (d) How many terms we need in the approximation in order that we have the value of $\sin(x)$ correct at least 4 significant figures everywhere in the given range?

2. Use graphical method to solve

$$\begin{aligned} 2x_1 - 6x_2 &= -18 \\ -x_1 + 8x_2 &= 40 \end{aligned}$$

3. Study graphically (do not use any algorithms) the following function

$$f(x) = 10.0e^{-x}\sin(2.0\pi x) + 0.5x$$

   within the range $0 \leq x \leq 10$.

   What is (a) the maximum, (b) the minimum, and (c) the largest root of the function within the given range. Give the answers with four significant figures.

4. The following code implements the forward elimination part of the Gaussian elimination algorithm. Complete the code with the backward substitution part, e.g. implement the following equations

$$x_i = \left(b_i - \sum_{j=i+1}^{n} a_{ij}x_j \right) / a_{ii}$$

   and test and verify your algorithm by solving your choice of 3 x 3 linear equation system.

```python
def GaussianElimination(a, b):

    a = a.copy().astype(float)
    b = b.copy().astype(float)
    n = len(a)

    for k in range(n):
        # Check
        if a[k, k] == 0:
            print('Unable to solve')
            return None
        # Forward eliminate
        for i in range(k+1, n):
            factor = a[i, k]/a[k, k]
            for j in range(k+1, n):
                a[i, j] = a[i, j] - factor*a[k, j]
            b[i] = b[i] - factor*b[k]

    return a, b
```

5. Use the Gauss-Seidel iterative method to solve

$$\begin{bmatrix} -2 & 5 & 9 \\ 7 & 1 & 1 \\ -3 & 7 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ -26 \end{bmatrix}$$

6. The equilibrium equations of the blocks in the spring-block system are

$$\begin{aligned} 3(x_2 - x_1) - 2x_1 &= -80 \\ 3(x_3 - x_2) - 3(x_2 - x_1) &= 0 \\ 3(x_4 - x_3) - 3(x_3 - x_2) &= 0 \\ 3(x_5 - x_4) - 3(x_4 - x_3) &= 60 \\ -2x_5 - 3(x_5 - x_4) &= 0 \end{aligned}$$

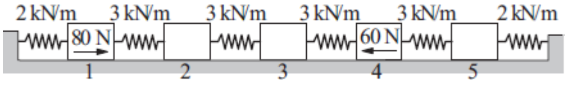

Figure 1: Spring-block system

where x_i are the horizontal displacements of the blocks measured in mm. Solve these equations using any functions of your choice found in *scipy.linalg* package. Show the steps and verify your solution.