3 Graphical analysis

Exercises selected and adapted from Croft, Davison, Hargreaves, (2001), "Engineering Mathematics", Third edition, chapter 2 Engineering functions and chapter 3 The trigonometric functions.

1. Obtain graphs of the following functions. Locate the roots, local minimums, and local maximums graphically within two significant numbers and mark them into the graphs.

   (a) $y = 3x^3 - x^2 + 2x + 1$, when $-2 \leq x \leq 2$

   (b) $y = x^4 + \frac{x^3}{3} - \frac{5x^2}{2} + x - 1$, when $-3 \leq x \leq 2$

   (c) $y = x^5 - x^2 + 2$, when $-2 \leq x \leq 2$.

2. (a) Draw $y = x^3$ and $y = 4 - 2x$ using the same axes. Note the x coordinate of the point of intersection within three significant figures accuracy.

   (b) Draw $y = x^3 + 2x - 4$. Note the coordinate of the point where the curve cuts the x axis. Compare your answer with that from (a). Explain your findings.

3. (a) Draw $y = 2x^2$ and $y = x^3 + 6$ using the same axes. Use your graphs to find approximate solutions to $x^3 - 2x^2 + 6 = 0$.

   (b) Add the line $y = -3x + 5$ to your graph. State approximate solutions to

      i. $x^3 + 3x + 1 = 0$

      ii. $2x^2 + 3x - 5 = 0$

4. Draw the following rational functions. State any asymptotes and draw or mark them in the graphs.

   (a) $f(x) = \dfrac{(2x+1)}{(x-3)} \qquad -4 \leq x \leq 4$

   (b) $g(s) = \dfrac{s}{(s+1)} \qquad -3 \leq x \leq 3$

   (c) $h(z) = \dfrac{z}{(z^2+1)} \qquad -3 \leq x \leq 3$

   (d) $y(x) = \dfrac{x+1}{x} \qquad -3 \leq x \leq 3$

   (e) $r(x) = \dfrac{2x}{(x-1)(x-2)} \qquad -3 \leq x \leq 3$

5. Plot $y = e^{kx}$ for $k = \{-3, -2, -1, 0, 1, 2, 3\}$ and $-3 \leq x \leq 3$ in the same graph.

6. Plot $y = ke^{x}$ for $k = \{-3, -2, -1, 0, 1, 2, 3\}$ and $-3 \leq x \leq 3$ in the same graph.

7. Plot $y = 5 - x^2$ and $y = e^x$ for $-3 \leq x \leq 3$ in the same graph. Based on the graphics, state for which values of x is $e^x < 5 - x^2$?

8. Plot $y = x^4$ and $y = e^x$ for $-1 \leq x \leq 9$. For which values of x is

   (a) $e^x < x^4$

   (b) $e^x > x^4$

9. Draw $y = \log(kx)$ for $0.5 \leq x \leq 50$ for $k = \{1, 2, 3, 4\}$ in the same graph.

10. Draw $y = \ln(x)$ and $y = \ln(\frac{1}{x})$ for $0.5 \leq x \leq 20$. What do you observe? Can you explain your observation using the laws of logarithms?

11. Draw $y = \ln(x)$ and $y = 1 - \dfrac{3}{3}$ for $0.5 \leq x \leq 4$. From your graphs state an approximate solution to $\ln(x) = 1 - \dfrac{x}{3}$.

12. Draw the following hyperbolic functions and locate the minimums and maximums and infinities.

    (a) $y = \sinh(x)$

    (b) $y = \cosh(x)$

    (c) $y = \tanh(x)$ for $-5 \leq x \leq 5$.

13. Draw graphs of $y = \sinh(x)$, $y = \cosh(x)$, and $y = \dfrac{e^x}{2}$ for $0 \leq x \leq 5$. What happens to the three graphs as x increases? Can you explain this algebraically?

14. Draw

    (a) $y = |x|$

    (b) $y = |-x + 6| + 3$

    (c) $y = |(x - 5)^2 - 10|$

15. Draw the following functions step-functions

(a) $f(t) = u(t-1)$

(b) $f(t) = u(t-2) - u(t-6)$

(c) $f(t) = 2u(t+1) - u(t-1)$

16. Draw the impulse train given by

(a) $f(t) = \delta(t-1) + 2\delta(t-2)$

(b) $f(t) = 3\delta(t) + 4\delta(t-2) + \delta(t-3)$

17. Draw $y = \sin(x)$ and $y = \cos(x)$ for $0 \leq x \leq 4\pi$ using the same axes. Use your graphs to find approximate solutions to equation $\sin(x) = \cos(x)$.

18. Draw the graphs of $y = \cos^{-1}(x)$ and $y = \tan^{-1}(x)$.

19. Plot

(a) $y = \sin(2t)$ for $0 \leq t \leq 2\pi$

(b) $y = \cos(3t)$ for $0 \leq t \leq 3\pi$

(c) $y = \sin(t) + 3\cos(t)$ for $0 \leq t \leq 3\pi$. By reading from your graph, state the maximum value of y.

20. Plot $y = 2\sin(3t) - \cos(3t)$ for $0 \leq t \leq 2\pi$.

(a) Use your graph to state the amplitude of $2\sin(3t) - \cos(3t)$.

(b) On the same axes plot $y = \cos(3t)$. Estimate the time displacement of $2\sin(3t) - \cos(3t)$.