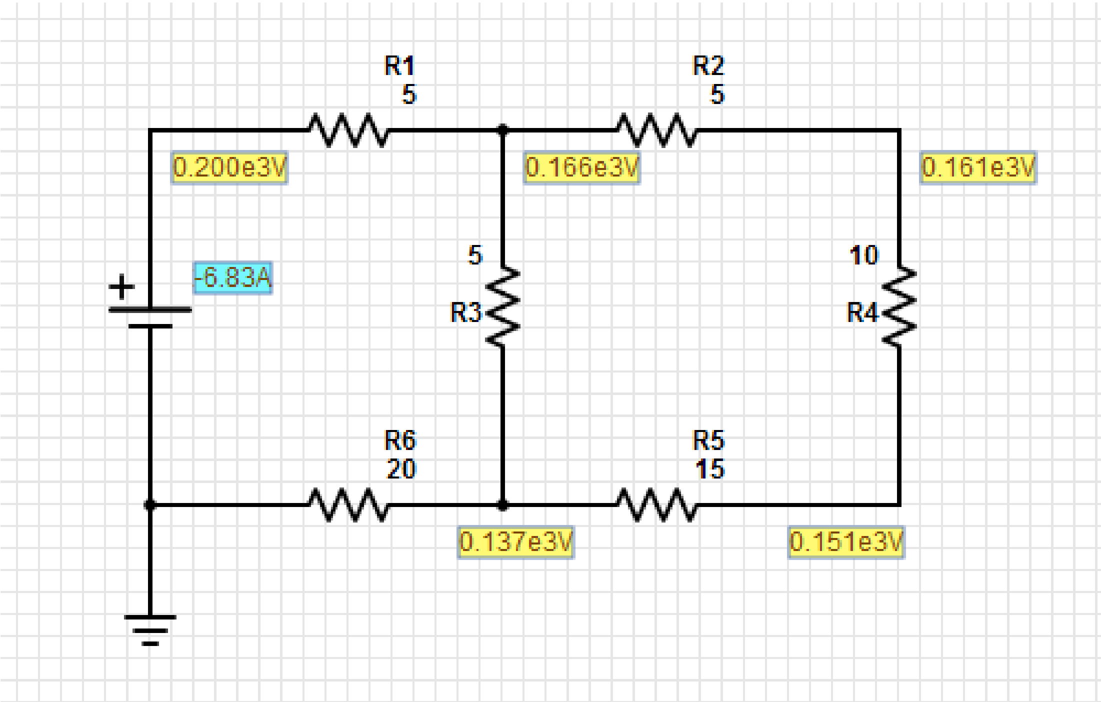
Applications

Introduction to Numerical Problem Solving, Spring 2017

CC BY-NC-SA Sakari Lukkarinen

Helsinki Metropolia University of Applied Sciences

Electric circuits




```
Simulation completed in 0.111 seconds.

DC Operating Point Information

Node Voltages:
net1006                        0.166e3
net1000                        0.200e3
net1002                        0.161e3
net1003                        0.151e3
net1004                        0.137e3

Node Currents:
v1                             -6.83
```

https://www.partsim.com/simulator#

Simulation report

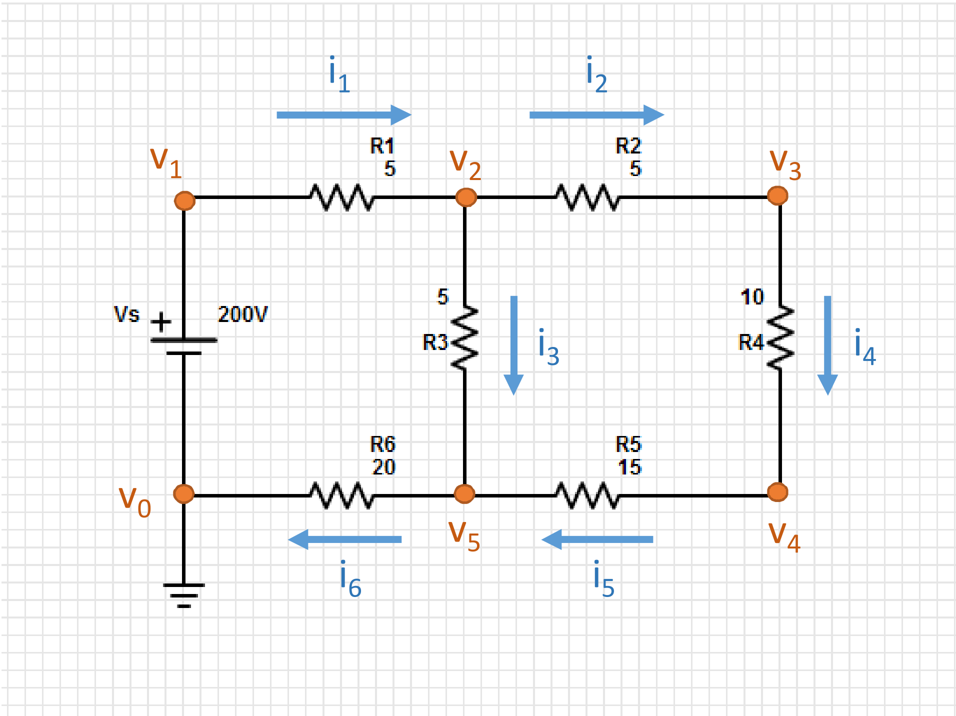
```
No. of Data Rows : 1
Total CPU time: 0.032 seconds.
CPU time since last call: 0.032 seconds.
Total DRAM available = 1694.265625 MB.
DRAM currently available = 211.285156 MB.
Total ngspice program size = 15.574219 MB.
Resident set size = 1.969727 MB.
Shared ngspice pages = 1.737305 MB.
Text (code) pages = 1.392578 MB.
Stack = 0 bytes.
Library pages = 259.000 kB.
0 page faults, 1 vol + 3 invol = 4 context switches.

Number of lines in the deck = 17
Netlist loading time = 0
Netlist parsing time = 0

Nominal temperature = 27
Operating temperature = 27
Total iterations = 3
Transient iterations = 0
Circuit Equations = 19
Circuit original non-zeroes = 47
Circuit fill-in non-zeroes = 18
Circuit total non-zeroes = 65
```

```
Total analysis time = 0
Matrix load time = 0
Matrix synchronize time = 0
Matrix reorder time = 0
Matrix factor time = 0
Matrix solve time = 0
Transient analysis time = 0
Transient load time = 0
Transient sync time = 0
Transient factor time = 0
Transient solve time = 0
Transient trunc time = 0
Transient iters per point = 3
AC analysis time = 0
AC load time = 0
AC sync time = 0
AC factor time = 0
AC solve time = 0
Note: No ".plot", ".print", or ".fourier" lines; no simulati
```

Circuit laws



Ohm’s law:

$$\Delta U = IR$$

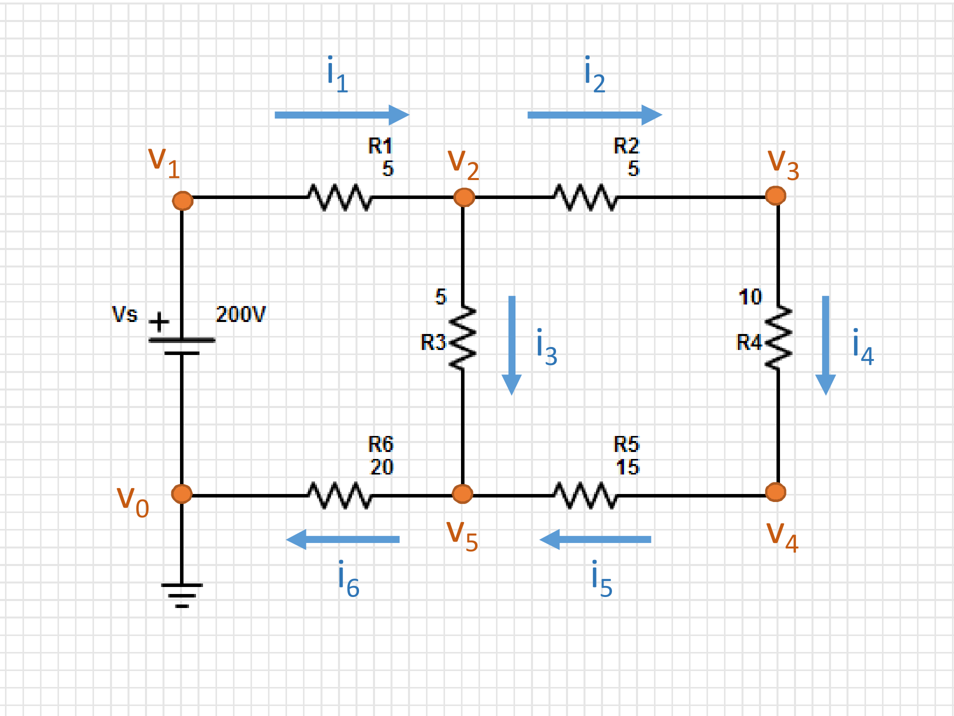
Kirchhoff current (point) law:

$$\sum_n i_n = 0$$

Kirchoff voltage (loop) law:

$$\sum_n v_n = 0$$

Applying circuit laws to solve currents

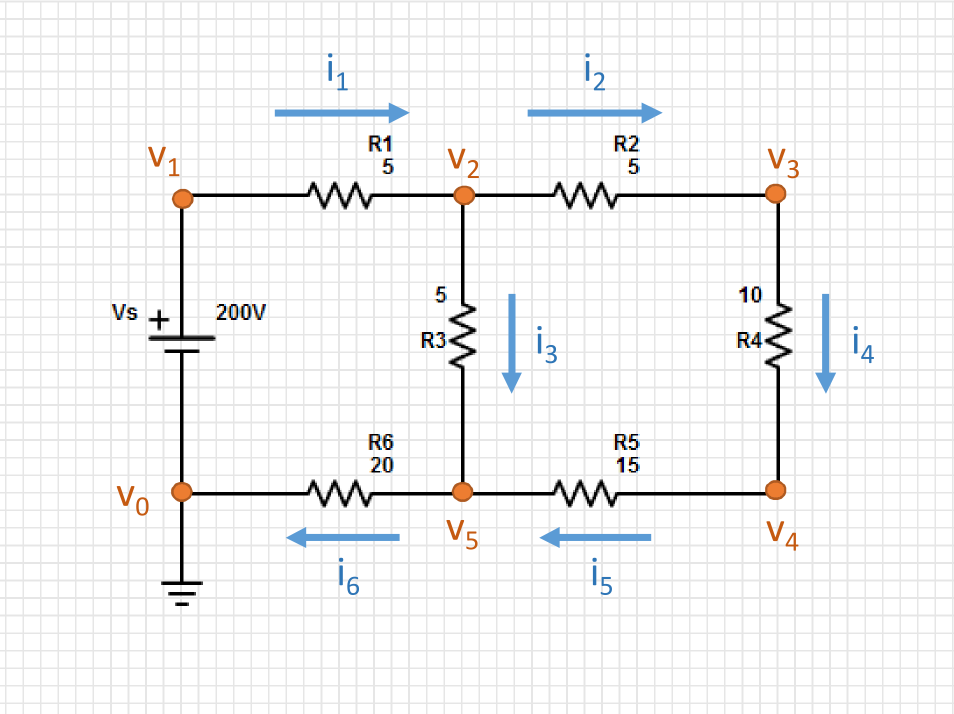


$$i_1 - i_2 - i_3 = 0$$
$$i_2 - i_4 = 0$$
$$i_4 - i_5 = 0$$
$$i_5 + i_3 - i_6 = 0$$
$$+v_s - r_1 i_1 - r_3 i_3 - r_6 i_6 = 0$$
$$+v_s - r_1 i_1 - r_2 i_2 - r_4 i_4 - r_5 i_5 - r_6 i_6 = 0$$

$$\begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 \\ -r_1 & 0 & -r_3 & 0 & 0 & -r_6 \\ -r_1 & -r_2 & 0 & -r_4 & -r_5 & -r_6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -v_s \\ -v_s \end{bmatrix}$$

Solving voltages

$$\Delta U = IR$$



$$v_0 = 0$$
$$v_0 - v_1 = -v_s$$
$$v_1 - v_2 = r_1 i_1$$
$$v_3 - v_2 = r_2 i_2$$
$$v_4 - v_3 = r_4 i_4$$
$$\mathrm{v}_5 - \mathrm{v}_4 = r_5 i_5$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} 0 \\ -v_s \\ r_1 i_1 \\ r_2 i_2 \\ r_4 i_4 \\ r_5 i_5 \end{bmatrix}$$

Application of Kirchhoffs Circuit Laws

The basic procedure for using **Kirchhoff's Circuit Laws** is as follows:

1. Label all voltages and resistances (V1, V2,… R1, R2, etc.)

2. Label each branch with a branch current. (I1, I2, I3 etc.)

3. Find Kirchhoff's first law equations for each node.

4. Find Kirchhoff's second law equations for each of the independent loops of the circuit.

5. Use Linear simultaneous equations as required to find the unknown currents.

Source: Kirchhoff's circuit laws. Electronics Tutorials.

Exercise

Solve currents and voltages given in the example circuit. Compare the results to given by electric circuit simulator.

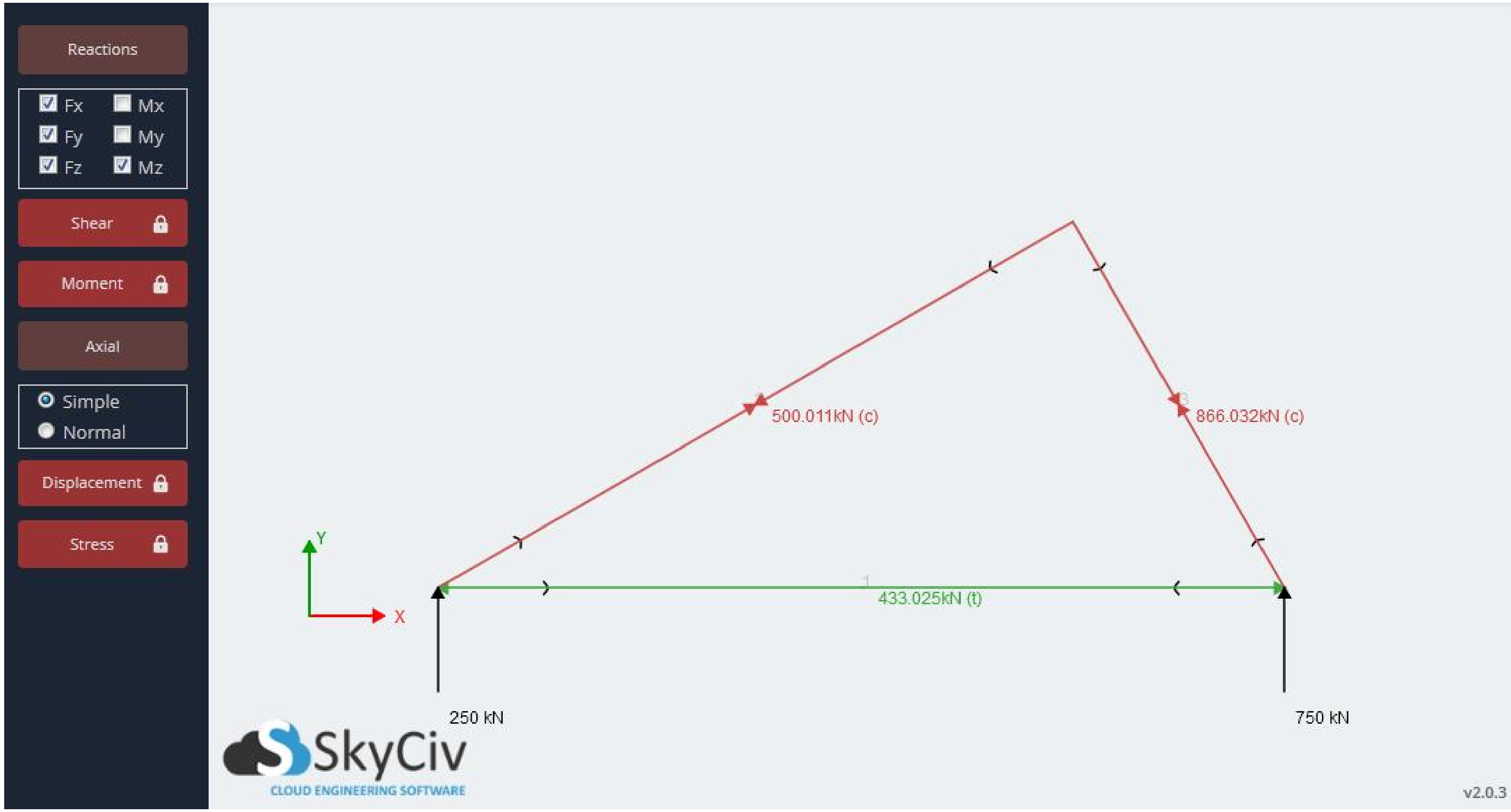
More info

Spice algorithm overview. eCircuit center.

LTSpice. Design simulation and device models. Linear Technology

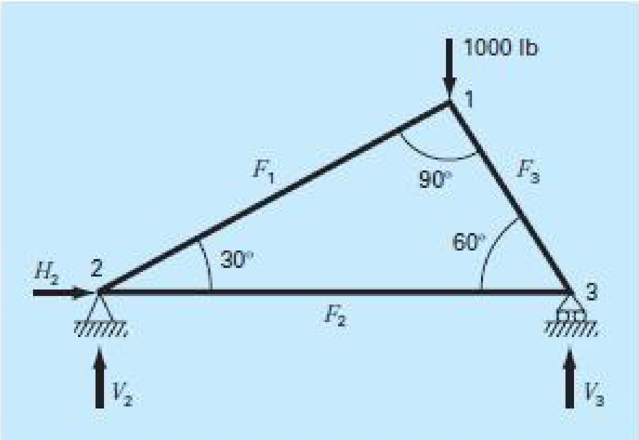
Nodal analysis

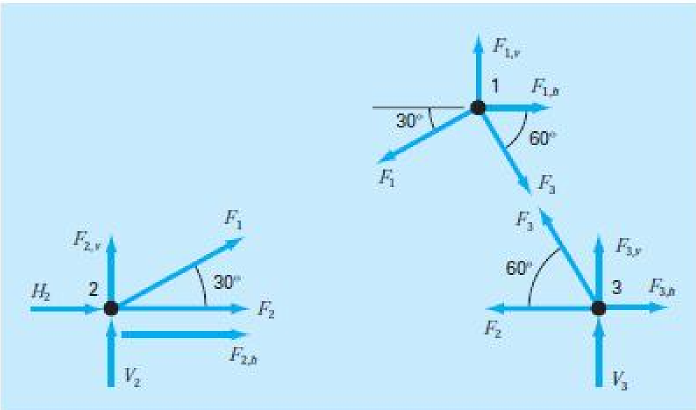
Mechanical struss analysis



https://skyciv.com/truss/v0/

Truss analysis $\sum_n F_{x,n} = 0,\ \sum_n F_{y,n} = 0$





Solution. This type of structure can be described as a system of coupled linear algebraic equations. Free-body force diagrams are shown for each node in Fig. 12.5. The sum of the forces in both horizontal and vertical directions must be zero at each node, because the system is at rest. Therefore, for node 1,

$$\Sigma F_H = 0 = -F_1 \cos 30° + F_3 \cos 60° + F_{1,h} \tag{12.3}$$
$$\Sigma F_V = 0 = -F_1 \sin 30° - F_3 \sin 60° + F_{1,v} \tag{12.4}$$

for node 2,

$$\Sigma F_H = 0 = F_2 + F_1 \cos 30° + F_{2,h} + H_2 \tag{12.5}$$
$$\Sigma F_V = 0 = F_1 \sin 30° + F_{2,v} + V_2 \tag{12.6}$$

for node 3,

$$\Sigma F_H = 0 = -F_2 - F_3 \cos 60° + F_{3,h}$$
$$\Sigma F_V = 0 = F_3 \sin 60° + F_{3,v} + V_3$$

Static analysis in matrix form

$$\Sigma F_H = 0 = -F_1 \cos 30^\circ + F_3 \cos 60^\circ + F_{1,h}$$
$$\Sigma F_V = 0 = -F_1 \sin 30^\circ - F_3 \sin 60^\circ + F_{1,v}$$

for node 2,

$$\Sigma F_H = 0 = F_2 + F_1 \cos 30^\circ + F_{2,h} + H_2$$
$$\Sigma F_V = 0 = F_1 \sin 30^\circ + F_{2,v} + V_2$$

for node 3,

$$\Sigma F_H = 0 = -F_2 - F_3 \cos 60^\circ + F_{3,h}$$
$$\Sigma F_V = 0 = F_3 \sin 60^\circ + F_{3,v} + V_3$$

$$\begin{bmatrix} -\cos 30^o & 0 & +\cos 60^o & 0 & 0 & 0 \\ -\sin 30^o & 0 & -\sin 60^o & 0 & 0 & 0 \\ \cos 30^o & \cos 0^o & 0 & 1 & 0 & 0 \\ \sin 30^o & \sin 90^o & 0 & 0 & 1 & 0 \\ 0 & -\cos 0^o & -\cos 60^o & 0 & 0 & 0 \\ 0 & -\sin 90^o & \sin 60^o & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ H_2 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -1000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Exercise

Solve the forces given in the truss example and compare them to the values given in truss analystator.

FEM applications

- Static heat transfer by conduction
- Static mechanical stress analysis
- Static electric current or field analysis

- VmWare Horizon Client -> Electronics HomeUsage
  - Comsol Multiphysics 5.1 (Classkit license)
    - Model Wizard, 2D
      - AC/DC —> Electric currents
      - Heat transfer —> Heat transfer in solids
      - Structural mechanics —> Solid mechanics
    - Preset studies —> Stationary

Exercise

Find a simple static heat transfer, mechanical stress analysis or electrostatic FEM model or application and study how the results were solved.