6 Applications

Exercises adapted from:

Kiusalaas. (2013). Numerical Methods in Engineering with Python 3. Third Edition.

1. Four blocks of different masses M_i are connected by ropes of negligble mass. Three of the blocks lie on a inclined plane, the coefficients of friction between the blocks and the plane being μ_i. The equations of motion for the blocks can be shown to be:

$$\begin{aligned}
T_1 + m_1 a &= m_1 g(\sin\theta - \mu_1 \cos\theta) \\
-T_1 + T_2 + m_2 a &= m_2 g(\sin\theta - \mu_2 \cos\theta) \\
-T_2 + T_3 + m_3 a &= m_3 g(\sin\theta - \mu_3 \cos\theta) \\
-T_3 + m_4 a &= m_4 g(\sin\theta - \mu_4 \cos\theta)
\end{aligned}$$

   where T_i denotes the tensile forces in the ropes and a is the acceleration of the system. (a) Determine a and T_i, when $\theta = 45^o$ and $g = 9.81\ m/s^2$, $m = [10.0,\ 4.0,\ 5.0,\ 6.0]\ kg$, and $\mu = [0.25,\ 0.30,\ 0.20]$. (b) What the angle should be in order that the system is in balance? Try a couple of different values for angle and find out what are the values for a and T_i. Based on these values make a graph (x-axis = angle = θ, y-axis = acceleration = a) and based on the graph estimate the angle giving the acceleration $a = 0.0\ m/s^2$.

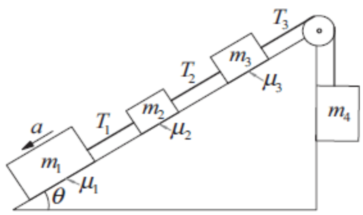

Figure 1: Four blocks of different masses

2. The edges of the square plate are kept at the temperatures shown. Assuming steady-state heat conduction, the differentatial equation governing the temperature T in the interior is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

   If the equation is approximated by finite differences using the mesh shown, we obtains the following algebraic equations for temperatures

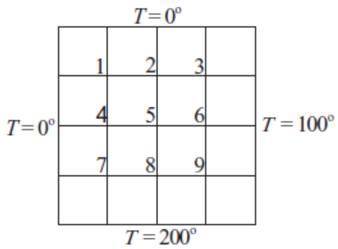

Figure 2: Square plate temperatures

at the mesh points

$$\begin{bmatrix} -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & -4 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -4 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -4 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 100 \\ 0 \\ 0 \\ 100 \\ 200 \\ 200 \\ 300 \end{bmatrix}$$

Solve these equations.

3. The equilibrium equations of the blocks in the spring-block system are

$$\begin{aligned} 3(x_2 - x_1) - 2x_1 &= -80 \\ 3(x_3 - x_2) - 3(x_2 - x_1) &= 0 \\ 3(x_4 - x_3) - 3(x_3 - x_2) &= 0 \\ 3(x_5 - x_4) - 3(x_4 - x_3) &= 60 \\ -2x_5 - 3(x_5 - x_4) &= 0 \end{aligned}$$

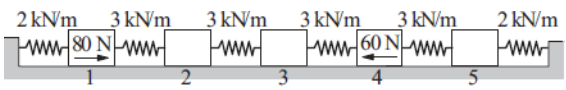

Figure 3: Spring-block system

where x_i are the horizontal displacements of the blocks measured in mm. Solve these equations with Gauss-Seidel method without relaxation. Start with $x_i = 0$ and iterate until four-figure accuracy after the decimal point is achieved. Also print out the number of iterations required. Then try different relaxation parameter values to find the best iteration speed.