2 Approximations and round-off errors

Exercises adapted from: Chapra & Canale. 2010. Numerical Methods for Engineers, 6th edition. Ch 3. Approximations and round-off errors.

1. Why 0.1 + 0.1 + 0.1 == 0.3 is not true? How does the computer interpret internally the values of that statement? If you need to compare in your code two numerical expressions are they are equal or not, how would you do that? Write an example code.

2. Compose your own Python program based on the following pseudocode and use it to determine the smallest number that differs from 1.0 (e.g. computer's machine epsilon).

```
epsilon = 1.0
DO
    if (epsilon + 1.0 <= 1.0) EXIT
    epsilon = epsilon/2.0
END DO
epsilon = 2.0 * epsilon
```

3. Modify your code to calculate the smallest number x_{min}, that differs from zero.

4. How does the (epsilon) change, when you increase or decrease the study point (e.g. 1.0 or 0.0 in the previous exercise)? What is the relationship of the study point and epsilon? Use values that differ several decades.

5. The derivative of $f(x) = 1/(1 - 3x^2)$ is given by

$$f'(x) = \frac{6x}{(1 - 3x^2)^2}$$

   Do you expect to have difficulties evaluating this function at $x = 0.577$? How does the value of the derivative change when you increase the significant figures in your calculations. Try using from 3-digit to 16-digit arithmetics. Start with the following code:

```
from decimal import *
getcontext().prec = 3 % Define 3-digit precision
x = Decimal('0.577')
```

6. Determine the number of terms necessary to approximate $\cos(x)$ to 8 significant figures using the Maclaurin series approximation

$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + ...$$

Calculate the approximation using a value $x = 0.3\pi$. Write a program to determine your results. Note: $6! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6$ can be calculated in Python with *factorial(6)*, that you need to import from *scipy.misc* module.

7. Evaluate e^5 using two approaches

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + ...$$

and

$$e^{-x} = \frac{1}{e^x} = \frac{1}{1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + ...}$$

and compare with the true value of 6.737947×10^{-3}. Use 20 terms to evaluate each series and compute true and approximate relative errors as terms are added.

8. The infinite series

$$f(n) = \sum_{k=1}^{n} \frac{1}{k^4}$$

converges on a value of $f(n) = \pi^4/90$ as n approaches infinity. Write a program in *single* precision to calculate $f(n)$ for $n = 10,000$ by computing the sum from $k = 1$ to $10,000$. Then repeat the calculation but reverse the order – that is, from $k = 10,000$ to 1 using increments of -1. In each case, compute the true percent relative error. Explain the results.