# 10 Root finding - Part II

Solve all problems using numerical and scientific python packages and show the solutions in jupyter Notebook. References:

Chapra & Canale. (2010). Numerical Methods for Engineers, 6th edition. Part two: Roots of equations.

Kiusalaas. (2013). Numerical Methods in Engineering with Python 3. Third Edition. Ch 4. Roots of Equations.

Johansson. (2015). Numerical Python: A Practical Techniques Approach for Industry. Ch. 5. Equation Solving.

1. Determine the real root of $f(x) = -26 + 85x - 91x^2 + 44x^3 - 8x^4 + x^5$

   (a) Graphically

   (b) Using simple fixed-point iteration

   (c) Using Newton-Raphson method

   (d) Finding the roots of the polynomial

2. Determine the roots of $f(x) = -13 - 20x + 19x^2 - 3x^3$

   (a) Graphically

   (b) Using simple fixed-point iteration

   (c) Finding the roots of the polynomial

3. Determine the roots of $f(x) = -2x^6 - 1.6x^4 + 12x + 1$

   (a) Graphically

   (b) Using Newton-Raphson method

   (c) Finding the roots of the polynomial

4. A zero $x_0 = 1$ of $P_4(x) = x^4 - 3x^2 + 3x - 1$ is known. Verify that $x_0$ is indeed a zero, and then deflate the polynomial; that is find $P_3(x)$ so that $P_n(x) = P_{n-1}(x)(x - x_0)$. You can use `scipy.polynomial` module's polynomial algebra functions.

5. Study the example of using function `fsolve` from `scipy.optimize` module given in Johansson's (2015) book at pages 143-145. Similarly locate the two points where two circles $(x-2)^2+y^2 = 4$ and $x^2+(y-3) = 4$ interesect. Visualize also the convergence of different initial guesses to different solutions within a domain $x \in [-4, 6]$ and $y \in [-5, 7]$.

6. ~~The equations~~

$$\begin{aligned} \sin(x) + 3\cos(x) - 2 &= 0 \\ \cos(x) - 3\sin(y) &= 0 \end{aligned}$$

have a solution close the point $(1, 1)$. Refine the solution numerically. Can you represent the solution and the equations graphically?

7. Study the following simultaneous nonlinear equations

$$\begin{aligned} y &= -x^2 + x + 0.75 \\ y + 5xy &= x^2 \end{aligned}$$

(a) First manipulate the equations so that you can can draw them in x-y coordinates and graphically find where the roots approximately locate.

(b) Then find the roots using the fixed-point iteration and good initial guesses.

(c) Lastly find the roots using `fsolve` or `root` function.

8. Find a solution of

$$\begin{aligned} \sin(x) + y^2 + \ln(z) - 7 &= 0 \\ 3x + 2^y - z^3 + 1 &= 0 \\ x + y + z - 5 &= 0 \end{aligned}$$

Start with the point (1, 1, 1).