4 Gaussian elimination

Exercises adapted from:

Chapra & Canale. 2010. Numerical Methods for Engineers, 6th edition. Ch 9. Gauss elimination.

Kiusalaas. (2013). Numerical Methods in Engineering with Python 3. Third Edition. Ch 2. Systems of Linear Algebraic Equations.

1. Solve the equation system

$$\begin{aligned} 3x_1 - 0.1x_2 - 0.2x_3 &= 7.85 \\ 0.1x_1 - 7x_2 - 0.3x_3 &= -19.3 \\ 0.3x_1 - 0.2x_2 - 10x_3 &= 71.4 \end{aligned}$$

   (a) by using the *solve* function found in *numpy.linalg* package

   (b) by using the matrix-vector *dot* product and inverse of matrix (*inv*)

2. Write a Python function called *gaussian* that implements the Gaussian elimination to solve a linear equation system presented in matrix form $Ax = b$. It should take two input parameters, the matrix A and a vector b, and return the values of the unknown variable x as results. Use the following code as a starting point:

```
def Gaussian(A, b):
        RowsA, ColsA = shape(A)
        Rowsb, Colsb = shape(b)
        x = zeros(Rowsb, Colsb)
        # Your code comes here
        # ....
        return x
```

   Use the equation system given in the previous problem to test and verify your code.

3. Improve your Gaussian elimination function implemented in previous problem by employing partial pivoting to avoid divisions by zeros. Test and check your code by solving the equation system

$$\begin{aligned} +2.0x_2 + 3.0x_3 &= 8.0 \\ 4.0x_1 + 6.0x_2 + 7.0x_3 &= -3.0 \\ 2.0x_1 + x_2 + 6.0x_3 &= 5.0 \end{aligned}$$

   Verify the solution by using the *solve* function or vector dot-product and inverse matrix calculation.

4. Given equation system

$$
\begin{aligned}
10.0x_1 + 2.0x_2 - x_3 &= 27.0 \\
-3.0x_1 - 6.0x_2 + 2.0x_3 &= -61.5 \\
x_1 + x_2 + 5.0x_3 &= -21.5
\end{aligned}
$$

   (a) Solve by Gaussian elimination
   (b) Substitute your solutions into the original equations in order to check answers.

5. Use Gaussian elimination to solve

$$
\begin{aligned}
4.0x_1 + x_2 - x_3 &= -2.0 \\
5.0x_1 + x_2 + 2.0x_3 &= 4.0 \\
6.0x_1 + x_2 + x_3 &= 6.0
\end{aligned}
$$

   Check your answers by substituting them into the original equation.

6. Solve the equations $Ax = b$ by Gauss elimination, where

$$
A = \begin{pmatrix}
0.0 & 0.0 & 2.0 & 1.0 & 2.0 \\
0.0 & 1.0 & 0.0 & 2.0 & -1.0 \\
1.0 & 2.0 & 0.0 & -2.0 & 0.0 \\
0.0 & 0.0 & 0.0 & -1.0 & 1.0 \\
0.0 & 1.0 & -1.0 & 1.0 & -1.0
\end{pmatrix}
$$

   and

$$
b = \begin{pmatrix} 1.0 \\ 1.0 \\ -4.0 \\ -2.0 \\ -1.0 \end{pmatrix}
$$

   Hint: You need to reorder the equations before solving.

Bonus exercises

7. Use Gauss-Jordan elimination method to solve

$$
\begin{aligned}
2x_1 + x_2 - x_3 &= 1 \\
5x_1 + 2x_2 + 2x_3 &= -4 \\
3x_1 + x_2 + x_3 &= 5
\end{aligned}
$$

   Do not employ pivoting. Check your answers by substituting them into the original equations.

8. Solve

$$\begin{aligned} x_1 + x_2 - x_3 &= -3 \\ 6x_1 + 2x_2 + 2x_3 &= 2 \\ -3x_1 + 4x_2 + x_3 &= 1 \end{aligned}$$

with

(a) naive Gauss elimination

(b) Gauss elimination with partial pivoting, and

(c) Gauss-Jordan without partial pivoting.

9. Use graphical method to solve

$$\begin{aligned} 2x_1 - 6x_2 &= -18 \\ -x_1 + 8x_2 &= 40 \end{aligned}$$

10. Given the equations

$$\begin{aligned} 0.5x_1 - x_2 &= -9.5 \\ 1.02x_1 - 2x_2 &= -18.8 \end{aligned}$$

(a) Solve graphically.

(b) Compute the determinant.

(c) On the basis of **(a)** and **(b)**, what would you expect regarding the system's condition?

(d) Solve by the elimination of unknowns.

(e) Solve again but with a_{11} modified slightly to 0.52. Interpret your results.

11. Given system of equations

$$\begin{aligned} -3.0x_2 + 7.0x_3 &= 2.0 \\ x_1 + 2.0x_2 - x_3 &= 3.0 \\ 5.0x_1 - 2.0x_2 &= 2.0 \end{aligned}$$

(a) Compute the determinant

(b) Use Cramer's rule to solve for the x's.

(c) Use Gauss elimination with partial pivoting to solve for x's.

Substitute your results back into the original equations in order to check your answers.