13 Integration and Differentiation

Solve all problems using numerical and scientific python packages and show the solutions in jupyter Notebook. References:

Chapra & Canale. (2010). Numerical Methods for Engineers, 6th edition. Part 6: Numerical Integration and Differentiation.

Kiusalaas. (2013). Numerical Methods in Engineering with Python 3. Third Edition. Ch 5. Numerical Differentiation, and Ch 6. Numerical Integration.

Johansson. (2015). Numerical Python: A Practical Techniques Approach for Industry. Ch 8. Integration.

1. Evalute the following integral:

$$\int_0^{\pi/2} (8 + 4\cos x)\mathrm{dx}$$

   (a) analytically

   (b) single application of the trapezoidal rule

   (c) multiple-application trapezoidal rule, with $n = 2$ and $n = 4$

   (d) single application of Simpson's 1/3 rule

   (e) multiple-application Simpson's rule, with $n = 5$

   For each of the numerical estimates determine the percent relative error based on the analytical results.

2. Evaluate the following integral:

$$\int_1^3 (1 - e^{-x})\mathrm{dx}$$

   (a) analytically

   (b) single application of the trapezoidal rule

   (c) multiple-application trapezoidal rule, with $n = 2$ and $n = 4$

   (d) single application of Simpson's 1/3 rule

   (e) multiple-application Simpson's 1/3 rule, with $n = 4$

   (f) multiple-application Simpson's rule, with $n = 5$.

   For each of the numerical estimates determine the percent relative true error based on the analytical solution.

3. Evaluate the following integral:

$$\int_{-2}^{4}(1 - x - 4x^3 + 2x^5)\mathrm{dx}$$

(a) analytically

(b) single application of the trapezoidal rule

(c) composite trapezoidal rule, with $n = 2$ and $n = 4$

(d) single application of Simpson's 1/3 rule

(e) Simpson's 3/8 rule

(f) *scipy.optimize.quad* function

For each of the numerical estimates determine the percent relative true error based on the analytical solution.

4. Integrate the following function analytically and using the trapezoidal rule, with $n = 1, 2, 3, ..., 20$:

$$\int_{1}^{2}(x + 1/x)^2\mathrm{dx}$$

Use the analytical solution to compute the true relative errors to evaluate the accuracy of the trapezoidal approximations. What do you observe?

5. Intergrate the following function both analytically and using Simpson's rules, with $n = 4, 5, ...$ When does the relative true error is less than 0.01

$$\int_{-3}^{5}(4x - 3)^3\mathrm{dx}$$