

**21.1** Evaluate the following integral:

$$\int_0^{\pi/2} (8 + 4 \cos x) dx$$

(a) analytically; (b) single application of the trapezoidal rule; (c) multiple-application trapezoidal rule, with  $n = 2$  and 4; (d) single application of Simpson's 1/3 rule; (e) multiple-application

Simpson's 1/3 rule, with  $n = 4$ ; (f) single application of Simpson's 3/8 rule; and (g) multiple-application Simpson's rule, with  $n = 5$ . For each of the numerical estimates (b) through (g), determine the percent relative error based on (a).

**21.4** Integrate the following function analytically and using the trapezoidal rule, with  $n = 1, 2, 3$ , and 4:

$$\int_1^2 (x + 1/x)^2 dx$$

Use the analytical solution to compute true percent relative errors to evaluate the accuracy of the trapezoidal approximations.

**21.5** Integrate the following function both analytically and using Simpson's rules, with  $n = 4$  and 5. Discuss the results.

$$\int_{-3}^5 (4x - 3)^3 dx$$

**21.2** Evaluate the following integral:

$$\int_0^3 (1 - e^{-x}) dx$$

(a) analytically; (b) single application of the trapezoidal rule; (c) multiple-application trapezoidal rule, with  $n = 2$  and 4; (d) single application of Simpson's 1/3 rule; (e) multiple-application Simpson's 1/3 rule, with  $n = 4$ ; (f) single application of Simpson's 3/8 rule; and (g) multiple-application Simpson's rule, with  $n = 5$ . For each of the numerical estimates (b) through (g), determine the percent relative error based on (a).

**21.3** Evaluate the following integral:

$$\int_{-2}^4 (1 - x - 4x^3 + 2x^5) dx$$

(a) analytically; (b) single application of the trapezoidal rule; (c) composite trapezoidal rule, with  $n = 2$  and 4; (d) single application of Simpson's 1/3 rule; (e) Simpson's 3/8 rule; and (f) Boole's rule. For each of the numerical estimates (b) through (f) determine the percent relative error based on (a).