11 Optimization

Solve all problems using numerical and scientific python packages and show the solutions in jupyter Notebook. References:

Chapra & Canale. (2010). Numerical Methods for Engineers, 6th edition. Part four: Optimization.

Kiusalaas. (2013). Numerical Methods in Engineering with Python 3. Third Edition. Ch. 10. Introduction to optimization.

Johansson. (2015). Numerical Python: A Practical Techniques Approach for Industry. Ch 6. Optimization.

1. Given
$$f(x) = -1.5x^6 - 2x^4 + 12x$$
   (a) Plot the function
   (b) Differentiate the function and then use a root location method to solve for the maximum $f(x)$ and the corresponding value of x.
   (c) Use *minimize* function from *scipy.optimize* to find the maximimum of the function.

2. Solve the problem 1 using the golden-section search. Use initial guesses of $x_l = 0.0$ and $x_h = 2.0$. Perform 30 iterations. How accurate the results is then?

3. Solve the problem 1 using parabolic interpolation method. Use initial guesses $x_0 = 0.0$, $x_1 = 1.0$ and $x_2 = 2.0$. Iterate until you reach the same accuracy as in previous problem. Did the parabolic interpolation find the solution with less or more iterations?

4. Solve the problem 1 using Newton-Raphson optimization method. Use initial guess of $x_0 = 2.0$. How many iterations are needed to achieve the same accuracy as in previous problems?

5. Employ the following methods to find the maximum of
$$f(x) = 4x - 1.8x^2 + 1.2x^3 - 0.3x^4$$
   (a) Golden-section method ($x_l = -2.0$, $x_u = 4.0$, $\epsilon_s = 0.0001$)
   (b) Parabolic interpolation ($x_0 = 1.74$, $x_1 = 2.0$, $x_2 = 2.5$, iterations $= 20$).
   (c) Newton-Raphson method ($x_0 = 3$, $\epsilon_s = 0.0001$).