Poisson distribution

Probability, Statistics and Discrete Mathematics, Spring 2017

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What is the Poisson distribution?

A discrete probability distribution that expresses the probability of a given number of independent events occurring in a fixed interval of time and/or space.

Properties:

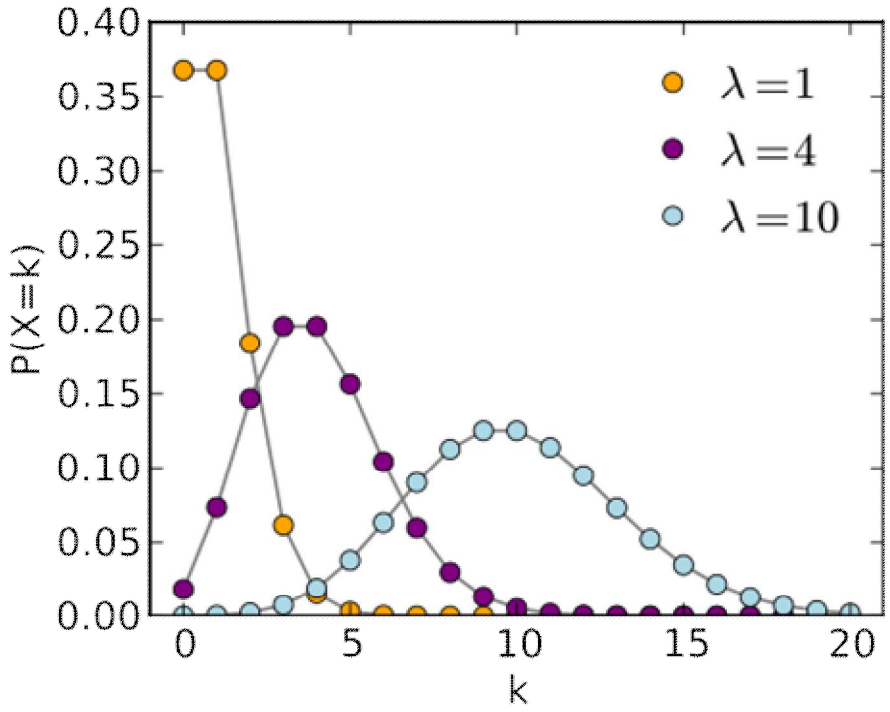
Mean = λ (the expected number of occurrences)

Variance = λ

Propability density function P(k) = $\frac{\lambda^k e^{-\lambda}}{k!}$

The Poisson distribution can also be used for the number of events in other specified intervals such as distance, area or volume.

Source: Wikipedia, Poisson distribution



Examples

The Poisson distribution may be useful to model events such as

- The amount of mail received each day
- The number of meteors greater than 1 meter diameter that strike earth in a year
- The number of patients arriving in an emergency room between 10 and 11 pm

What is image noise?

Image noise is random variation of brightness or color information in images, and is usually an aspect of electronic noise. It can be produced by the sensor and circuitry of a scanner or digital camera.

Source: Wikipedia, image noise

Noise clearly visible in an image from a digital camera

Shot noise

Photon noise simulation. Number of photons per pixel increases from left to right and from upper row to bottom row.

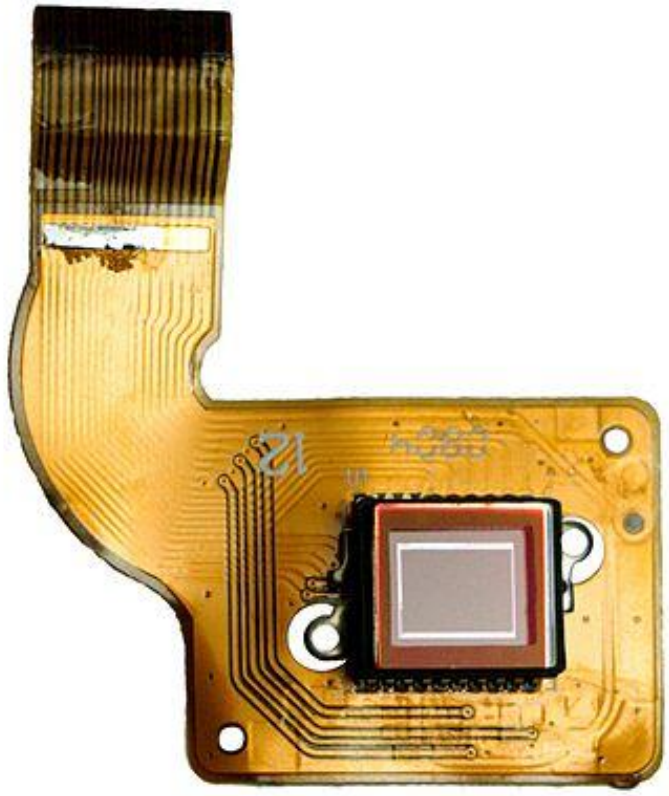
Shot noise or Poisson noise is a type of electronic noise which can be modeled by a Poisson process.

In electronics shot noise originates from the discrete nature of electric charge.

Shot noise also occurs in photon counting in optical devices, where shot noise is associated with the particle nature of light.

Source: Wikipedia, Shot noise

Charged-Coupled Device (CCD) image sensor



A CCD image sensor on a flexible circuit board

Each cell of a CCD image sensor is an analog device.

When light strikes the chip it is held as a small electrical charge in each photo sensor.

The charges in the line of pixels nearest to the (one or more) output amplifiers are amplified and output, then each line of pixels shifts its charges one line closer to the amplifier(s), filling the empty line closest to the amplifiers(s).

This process is then repeated until all the lines of pixels have had their charge amplified and output.

Source: Wikipedia, image sensor

Signal-to-Noise ratio (SNR)

Signal-to-noise ratio (abbreviated SNR or S/N) is a measure used in science and engineering that compares the level of a desired signal to the level of background noise. It is defined as **the ratio of signal power to the noise power**, often expressed in decibels. A ratio higher than 1:1 (greater than 0 dB) indicates more signal than noise.

$$\mathrm{SNR} = \frac{P_{\mathrm{signal}}}{P_{\mathrm{noise}}}$$

$$\mathrm{SNR}_{\mathrm{dB}} = 10 \log_{10} \left(\frac{P_{\mathrm{signal}}}{P_{\mathrm{noise}}} \right)$$

SNR for shot noise

Since the standard deviation of shot noise is equal to the square root of the average number of events *N* (Poisson distribution property), the signal-to-noise ratio (SNR) is given

$$\mathrm{SNR} = \frac{N}{\sqrt{N}} = \sqrt{N}$$

Lab exercises

- Facebook messages per day
- Image scanner shot noise simulation
- Two-color image pattern simulation
- Camera ISO noise test
- Own experiments