



Assignment 2: logic and sets

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Answers

Sets

1. [10 marks]

a) [2 marks]

We have 3 activities; therefore, 8 total values will be needed:

- Students who joined a student club
- Students who ate at a cafe
- Students who went to a gym
- Students who joined a student club and ate at a cafe
- **Students who ate at a cafe and went to a gym** => This is the missing value.
- Students who went to the gym and joined a student club
- Students who joined a student club, ate at a cafe, and went to a gym
- Students who did not join a student club, eat at a cafe, or go to a gym

If we have x = the missing value, it can be calculated as:

$$465 = 101 + 220 - 68 - x + 208 - 126 + 32 + 159$$

$$\Leftrightarrow x = 101 + 220 - 68 + 208 - 126 + 32 + 159 - 465$$

$$\Leftrightarrow x = \mathbf{61}$$

b) [3 marks]

Values needed for the Venn diagram:

- Students who joined a student club and ate at a café but did not go to a gym
 $68 - 32 = \mathbf{36}$
- Students who ate at a café and went to the gym but did not join a student club
 $126 - 32 = \mathbf{94}$
- Students who went to the gym and joined a student club but did not eat at a café
 $61 - 32 = \mathbf{29}$
- Students who only joined a student club
 $220 - 36 - 32 - 94 = \mathbf{58}$
- Students who only ate at a café
 $208 - 32 - 36 - 29 = \mathbf{62}$
- Students who only went to a gym

$$208 - 32 - 29 - 94 = 53$$

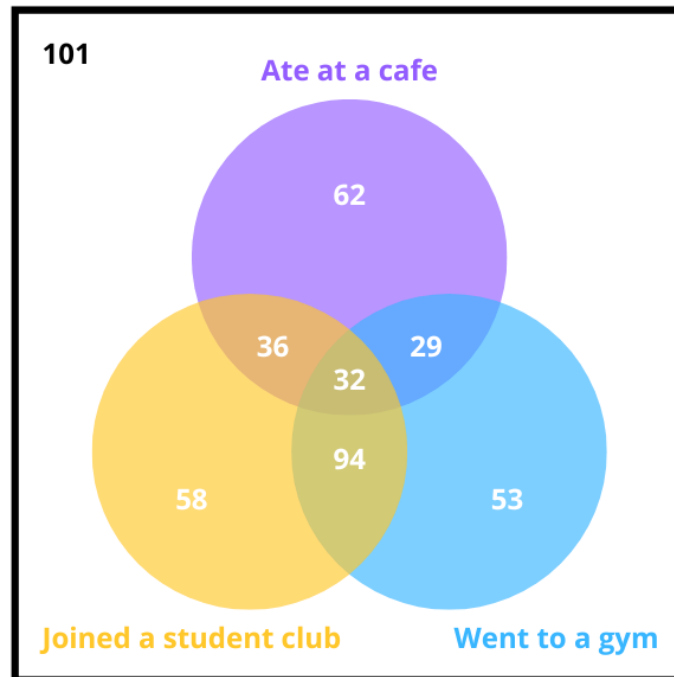


Figure 1: Venn diagram

c) [3 marks]

i. Did not go to a gym.

$$= \text{Total} - \text{Went to a gym} = 465 - 208 = \mathbf{257}$$

ii. Joined a student club but did not eat at a cafe.

$$= \text{Only joined a student club} + \text{Joined a student club and went to a gym but did not eat at a café}$$

$$= 58 + 94 = \mathbf{152}$$

iii. Did one only of joined a student club, eat at a cafe, or went to a gym.

$$= \text{Only did 1 of 3 things}$$

$$= 62 + 58 + 53 = \mathbf{173}$$

d) [2 marks]

Students who joined a student club = A

Students who ate at a café = B

Students who went to a gym = C

I. Did not go to a gym nor eat at a cafe.

$$(B \cap C)^c$$

ii. Joined a student club and ate at a cafe but did not go to a gym.

$$(A \cup B) \cap C^c$$

Logic

2. [3 + 3 = 6 marks]

a) Translate into English

- I. Huyen plays cricket, and either David or Adita plays esports.
- II. If David plays esports, then Adita does not play esports or Huyen plays cricket.
- III. Neither Huyen plays cricket nor does David play esports.

b) Translate into symbolic logic

- I. $d \rightarrow a$
- II. $\neg(a \wedge d)$
- III. $a \leftrightarrow (h \wedge d)$

3. [4 marks]

If we have:

$$\text{Width} > 10 = w$$

$$\text{Height} \leq 100 = h \Rightarrow \text{Height} > 100 = \neg h$$

$$(h \vee w) \wedge (\neg h \vee w) \wedge h$$

$$= (h \wedge \neg h) \vee w \wedge h \text{ (Distributive)}$$

$$= F \vee w \wedge h \text{ (Complement)}$$

$$= h \wedge w \text{ (Identity)}$$

Therefore, we can simplify the given statement to:

If width > 10 and height ≤ 100

Relations and functions

4. [4 marks]

a) Reflexive

$$(a, a), (b, b), (c, c), (d, d), (e, e)$$

b) Symmetric

$$(a, b) \rightarrow (b, a)$$

$(c, e) \rightarrow (e, c)$

$(d, e) \rightarrow (e, d)$

c) Transitive

$(c, e), (e, d)$, but no (c, d)

⇒ Not transitive

⇒ **Not equivalence**

5. $[2 + 3 + 1 = 6 \text{ marks}]$

a) We have $X: 2, Y: 3 \Rightarrow$ There are $Y^X = 3^2 = 9$ functions.

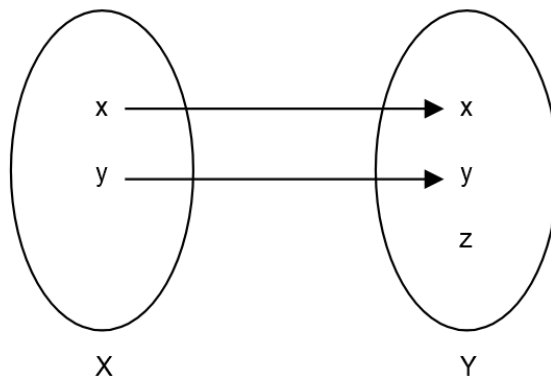


Figure 2: Number of functions

b) Number of injective functions: $3!/(3-2)! = 6$ functions

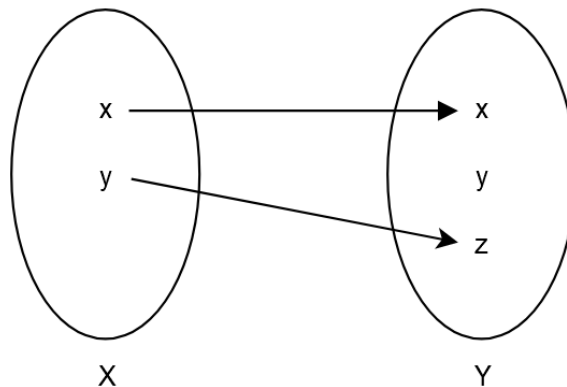


Figure 3: Injective example

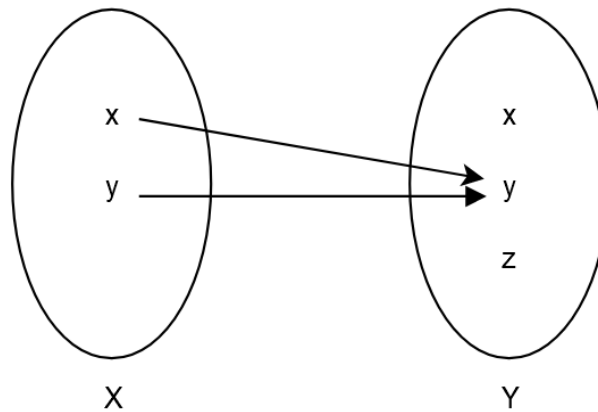


Figure 4: Example of not injective

- c) Domain X has 2 values, and co-domain Y has 3 values. For surjection, each value in co-domain Y must be used at least once. Given that 1 value in domain X can only be mapped with 1 value in co-domain Y, there will be no surjective function => **no bijective functions.**

Circuits

6. [2 + 2 = 4 marks]
- I. Boolean algebra notation
 - a. $(A + B') * B$
 - b. $(A + B) * (B + C) * (C + A)$
 - II. Using annotation for answer
 - a. $(1 + 0) * 1 = 1 * 1 = 1$
 - b. $(1 + 1) * (1 + 0) * (1 + 0) = 1 * 1 * 1 = 1 * 1 = 1$
7. [3 + 2 + 1 = 6 marks]
- a.

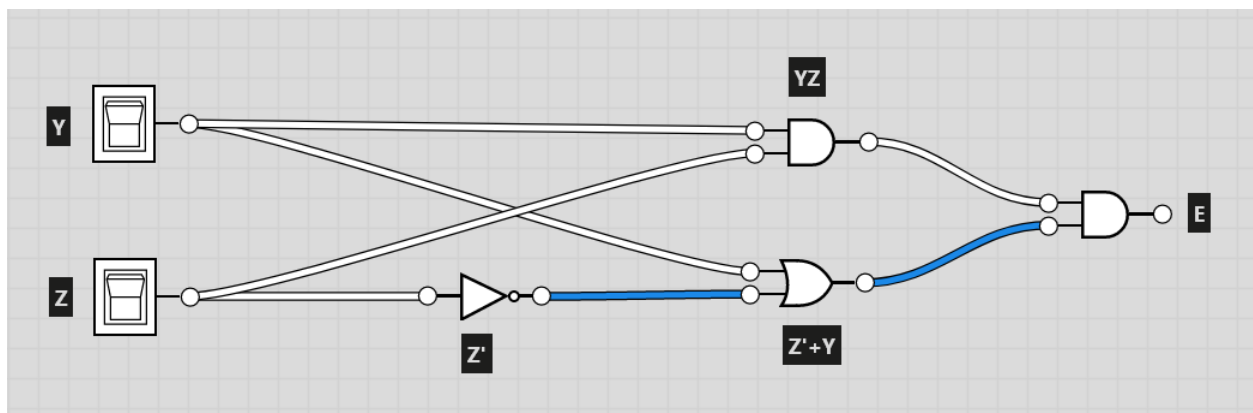


Figure 5: Question 7 circuit

- b. $E = (z' + y)yz$
 $= z'zy + yyz$ (Distributivity)
 $= 0y + yyz$ (Complement)
 $= 0 + yyz$ (Boundedness)
 $= 0 + yz$ (Idempotent)
 $= yz$ (Identity)
- c. $E = yz \Rightarrow \text{size} = 1, \text{depth} = 1$

For the original circuit, we have size = 4, depth = 3

In conclusion, the size of the circuit has reduced from 4 to 1 and the depth of the circuit has reduced from 3 to 1.