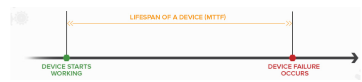
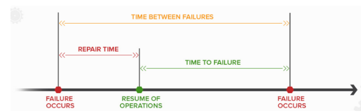
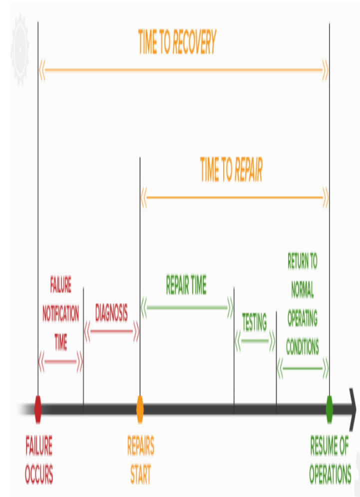
Life Analysis

Frequent Used Terms

acronym	full text	definition	illustration
MTTF	**Mean Time To Failure**	Mean Time To Failure is a very basic measure of reliability used for non-repairable systems. It represents the length of time that an item is expected to last in operation until it fails.	
MTBF	**Mean Time Between Failures**	The term MTBF is used for repairable systems, but it does not take into account units that are shut down for routine scheduled maintenance (re-calibration, servicing, lubrication) or routine preventive parts replacement. Rather, it captures failures that occur due to design conditions that make it necessary to take the unit out of operation before it can be repaired.	
MTTR	**Mean Time To Repair**	Mean Time To Repair (MTTR) refers to the amount of time required to repair a system and restore it to full functionality.	

Failure function up to time t

Cumulative distribution function: it tells the probability that the observed object is likely to fail up to time t.

P(T<=t) = F(t)

Survival function: it tells the probability that the observed object is likely to survive beyond time t.

P(T>t) = 1-F(t) = S(t)

The survival function has following properties:

$$S(0) = 1$$

$$\lim_{t \to \infty} S(t) = 0$$

$$\frac{\partial S}{\partial t} < 0$$

S(t) + F(t)=1

f(t) is defined as the corresponding probability density function of F(t).

Hazard function is defined as below:

$$\theta(t) = \frac{f(t)}{1 - F(t)} = \frac{f(t)}{S(t)}$$

The probability density function $f(t)$ summarizes the concentration of spell lengths (exit times) at each instant of time along the time axis. The hazard function summarizes the same concentration at each point of time, but conditions the expression on survival in the state up to that instant, and so can be thought of as summarizing the instantaneous *transition intensity*.

Integrated hazard function

$$\begin{aligned} H(t) &\equiv \int_0^t \theta(u)du \\ &= -\ln[S(t)]. \end{aligned}$$

The probability of exit within the interval is

$$\Pr(a_{j-1} < T \leq a_j) = F(a_j) - F(a_{j-1}) = S(a_{j-1}) - S(a_j)$$

Interval hazard rate, also known as the discrete hazard rate, is the probability of exit in the interval a[j-1]~a[j]

$$\begin{aligned} h(a_j) &= \Pr(a_{j-1} < T \leq a_j | T > a_{j-1}) \\ &= \frac{\Pr(a_{j-1} < T \leq a_j)}{\Pr(T > a_{j-1})} \\ &= \frac{S(a_{j-1}) - S(a_j)}{S(a_{j-1})} \\ &= 1 - \frac{S(a_j)}{S(a_{j-1})} \end{aligned}$$

The probability of survival until the end of interval j is the product of probabilities of not experiencing event in each of the intervals up to and including the current one.

$$
\begin{aligned}
S(j) \quad &\equiv \quad S_j = (1-h_1)(1-h_2).....(1-h_{j-1})(1-h_j) \\
&= \quad \prod_{k=1}^{j}(1-h_k)
\end{aligned}
$$

otherwise

$$
\begin{aligned}
F_j \quad &= \quad F(j) = 1 - S(j) \\
&= \quad 1 - \prod_{k=1}^{j}(1-h_k).
\end{aligned}
$$

The probability of exit within the j interval:

$$
\begin{aligned}
f(j) \quad &= \quad \Pr(a_{j-1} < T \leq a_j) \\
&= \quad S(j-1) - S(j) \\
&= \quad \frac{S(j)}{1-h_j} - S(j) \\
&= \quad \left(\frac{1}{1-h_j} - 1\right) S(j) \\
&= \quad \frac{h_j}{1-h_j} \prod_{k=1}^{j}(1-h_k)
\end{aligned}
$$

in further,

$$
f(j) = h_j S_{j-1} = \frac{h_j}{1-h_j} S_j
$$

Weibull model:

probability density function

$$
f(t) = \frac{\beta}{\eta}\left(\frac{t-\gamma}{\eta}\right)^{\beta-1} e^{-\left(\frac{t-\gamma}{\eta}\right)^{\beta}}
$$

Mean or MTTF

$$\overline{T} = \gamma + \eta \cdot \Gamma\left(\frac{1}{\beta} + 1\right)$$

The gamma function is defined as:

$$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} dx$$

Median:

$$\breve{T} = \gamma + \eta\left(\ln 2\right)^{\frac{1}{\beta}}$$

Mode:

$$\tilde{T} = \gamma + \eta\left(1 - \frac{1}{\beta}\right)^{\frac{1}{\beta}}$$

Standard deviation:

$$\sigma_T = \eta \cdot \sqrt{\Gamma\left(\frac{2}{\beta} + 1\right) - \Gamma\left(\frac{1}{\beta} + 1\right)^2}$$

Cumulative density function

$$F(t) = 1 - e^{-\left(\frac{t-\gamma}{\eta}\right)^{\beta}}$$

Survival function S(t) also known as reliability function:

$$R(t) = e^{-\left(\frac{t-\gamma}{\eta}\right)^{\beta}}$$

Conditional Reliability Function:

$$R(t|T) = \frac{R(T+t)}{R(T)} = \frac{e^{-\left(\frac{T+t-\gamma}{\eta}\right)^{\beta}}}{e^{-\left(\frac{T-\gamma}{\eta}\right)^{\beta}}}$$

The reliable life:

Given the reliability R, the reliable life of a unit is as below:

$$T_R = \gamma + \eta \cdot \{-\ln(R)\}^{\frac{1}{\beta}}$$

Failure Rate also known as hazard rate:

$$\lambda(t) = \frac{f(t)}{R(t)} = \frac{\beta}{\eta}\left(\frac{t-\gamma}{\eta}\right)^{\beta-1}$$

PdM

Based on all these mathematics, we can then answer the following PdM questions:

	Questions	Validity	Solution
1	Given reliability or survival objective probability, what is the most likely life length?		$T_R = \gamma + \eta \cdot \{-\ln(R)\}^{\frac{1}{\beta}}$
2	Given that the component survives at time t, what is the reliability for another time period delta t?		$R(t\|T) = \frac{R(T+t)}{R(T)} = \frac{e^{-\left(\frac{T+t-\gamma}{\eta}\right)^{\beta}}}{e^{-\left(\frac{T-\gamma}{\eta}\right)^{\beta}}}$

3	Given that a component survives to time t, what is the probability of the failure happening within next time unit?		$$\begin{aligned} h(a_j) &= \Pr(a_{j-1} < T \leq a_j \mid T > a_{j-1}) \\ &= \frac{\Pr(a_{j-1} < T \leq a_j)}{\Pr(T > a_{j-1})} \\ &= \frac{S(a_{j-1}) - S(a_j)}{S(a_{j-1})} \\ &= 1 - \frac{S(a_j)}{S(a_{j-1})} \end{aligned}$$ or 1-R(t|T)
4	How many components are most likely to fail within next time period?		N*H(t)
5	What is the failure probability for k components at next time unit?		b(n, p, k) here n is the total number of the components, p is the hazard probability
6	Given that k components are in stock, what is the probability of stock out?		sum of b(n, p, i) and i is from k to N
7	Given that k components are in stock, what is the mean time between stock-out?		solve H(t)=p and p is from above
8	What is the mean time of a component? Or, what is the life expectation? Or, what is the MTBF?		$$\overline{T} = \gamma + \eta \cdot \Gamma\left(\frac{1}{\beta} + 1\right)$$
9	What is the median time of component?		$$\breve{T} = \gamma + \eta\,(\ln 2)^{\frac{1}{\beta}}$$
10	What is the mode time of a component?		$$\tilde{T} = \gamma + \eta\left(1 - \frac{1}{\beta}\right)^{\frac{1}{\beta}}$$

PdM GUI Design

subject	document	used tool	source of the tool
GUI	PdM.epgz	pencil	http://pencil.evolus.vn/
data tables	PdM.xlsx	excel	Microsoft excel

Model Performance Page

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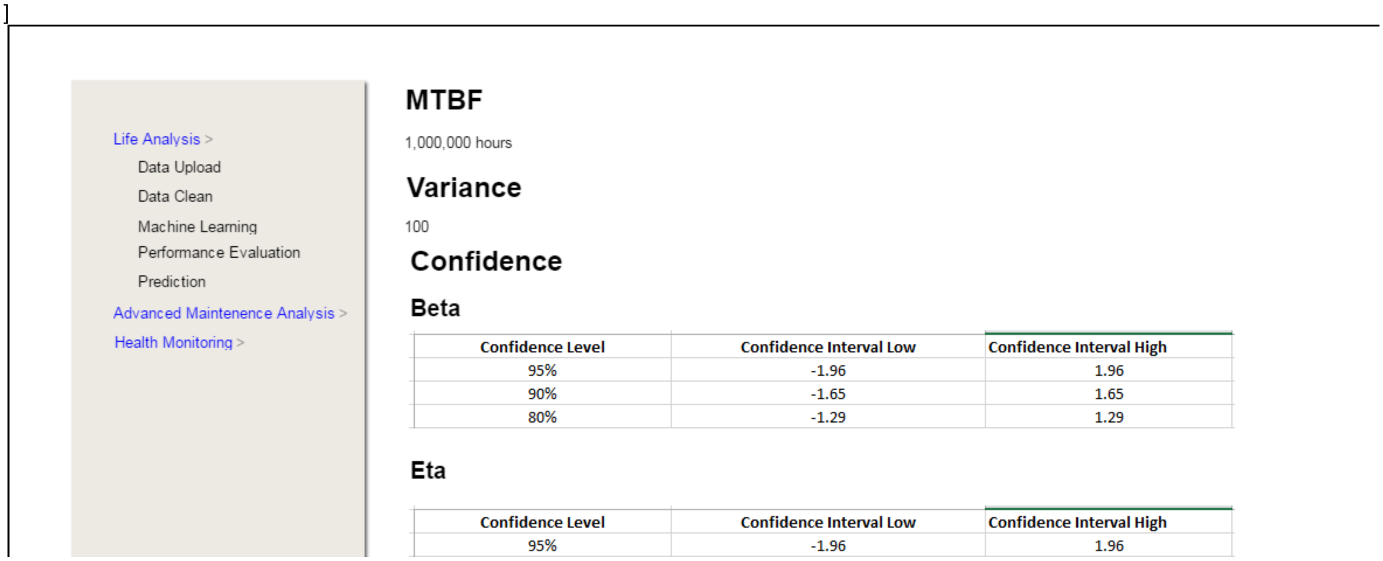
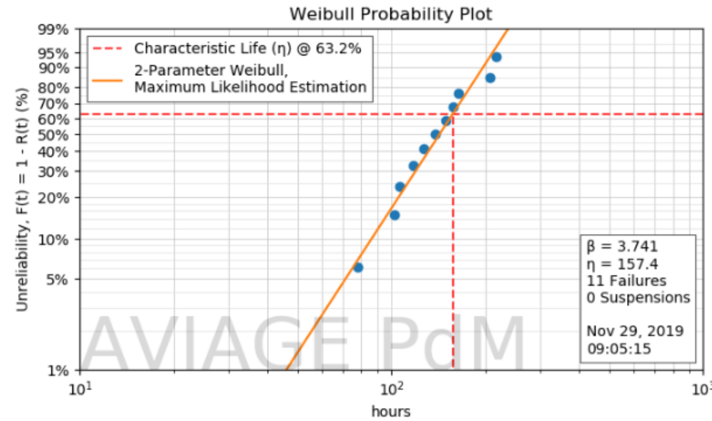
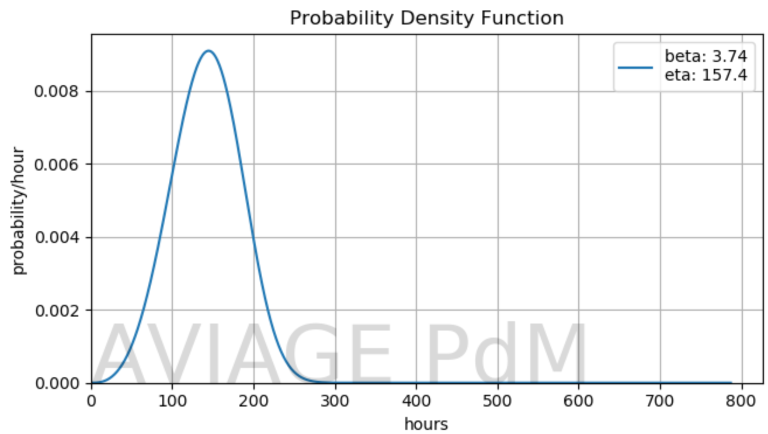
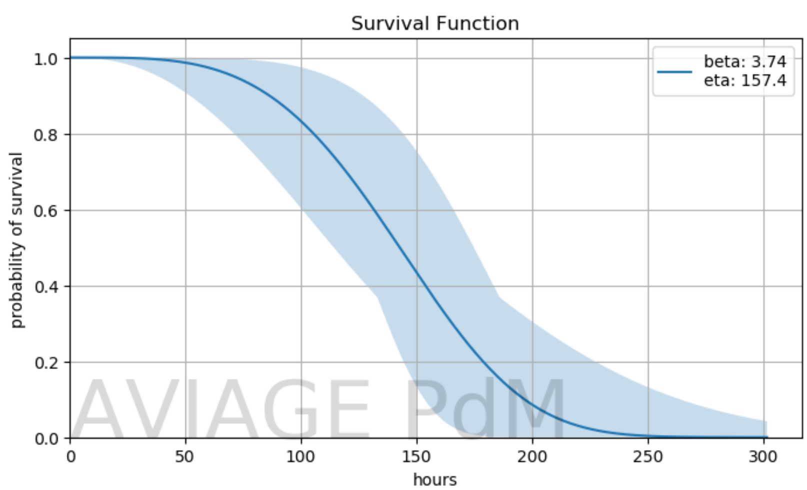


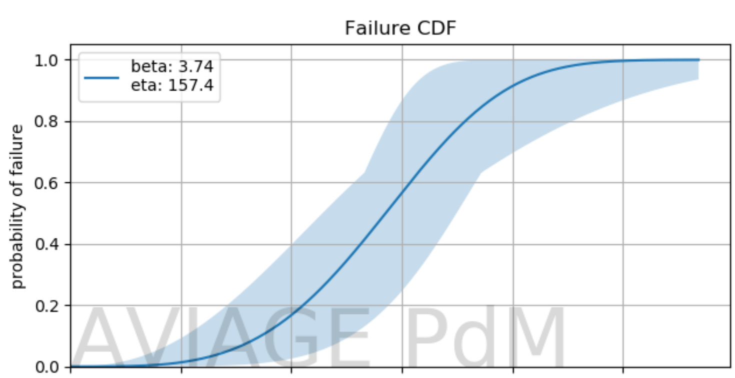
90%	-1.65	1.65
80%	-1.29	1.29

Graph

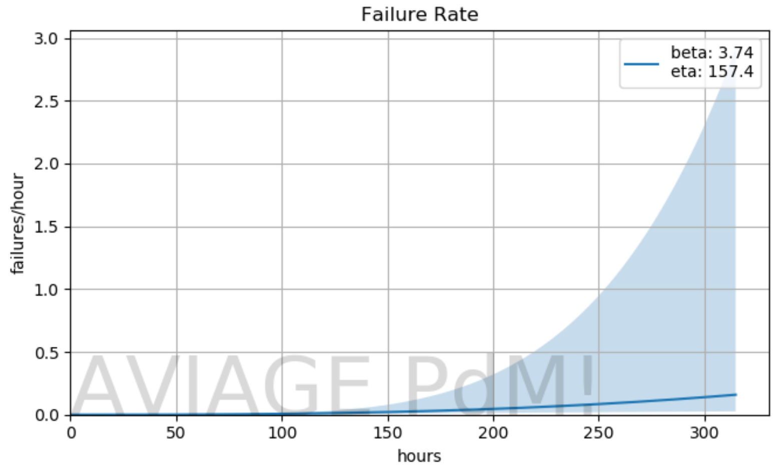









0
50
100
150
200
250
hours


Failure Rate
beta: 3.74
eta: 157.4
failures/hour
hours
AVIAGE PdM!

Life Analysis >
- Data Upload
- Data Clean
- Machine Learning
- Performance Evaluation
- Prediction

Advance Maintainance Analysi >

Health Monitoring >

Reliability Prediction

* Given reliability objective, what is the most likely life length for the component

Reliability	Expected Running Time
99.999999%	1,000 hours
99.999900%	10,000 hours
99.99%	100,000 hours

Stock Out Prediction

* Given the number of components in stock, what is the stock out probability and what is the mean time between stock out

Quantity	Stock Out Probability %	Stock Out Mean Time
1	10	1 years
2	5	5 years
3	1	8 years
4	0.1	12 years
5	0.01	20 years
6	0.0001	25 years
7	0.00001	30 years

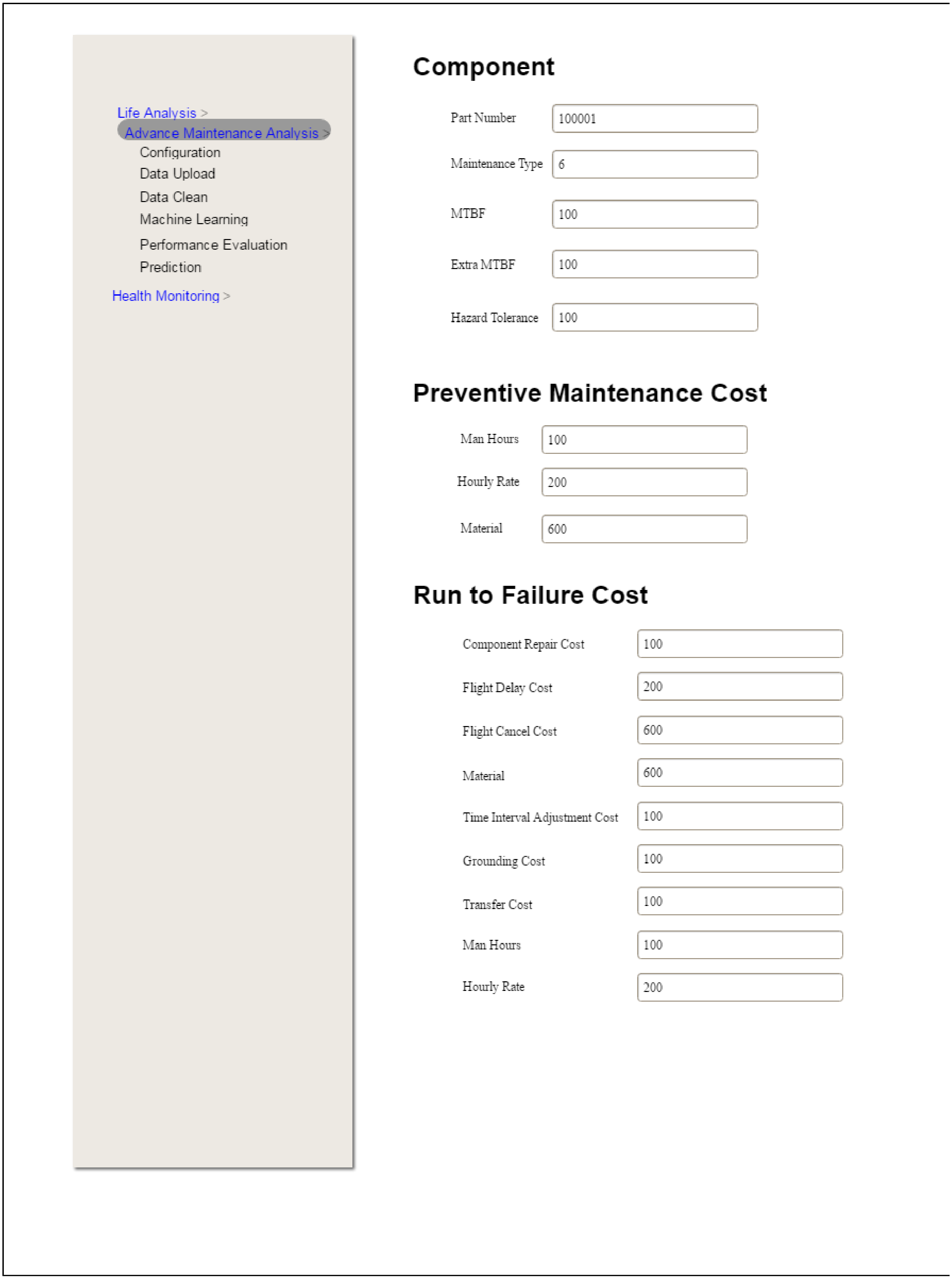
Realtime Reliability Prediction

* Given the active hours of a specific part, what is the reliability sitautation

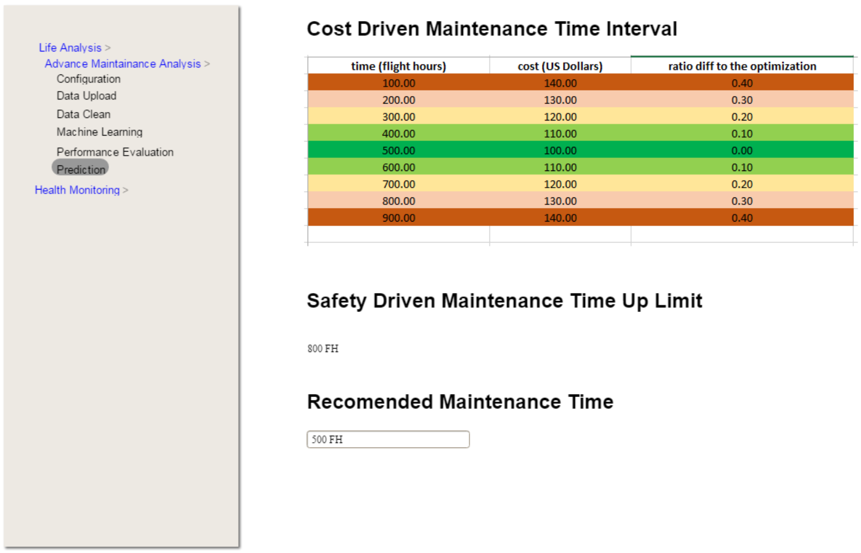
Part Number	Active Hours	Reliability for Next [100] hours	Reliability for Next [200] hours
100001	1000	99%	98%
100002	1000	99%	98%
100003	1000	99%	98%
100004	1000	99%	98%
100005	1000	99%	98%
100006	1000	99%	98%

Make the percent number have six digital accuracy.

Advanced Maintenance Analysis Configuration Page


Life Analysis >
Advance Maintenance Analysis >
Configuration
Data Upload
Data Clean
Machine Learning
Performance Evaluation
Prediction
Health Monitoring >
Component
Part Number
100001
Maintenance Type
6
MTBF
100
Extra MTBF
100
Hazard Tolerance
100
Preventive Maintenance Cost
Man Hours
100
Hourly Rate
200
Material
600
Run to Failure Cost
Component Repair Cost
100
Flight Delay Cost
200
Flight Cancel Cost
600
Material
600
Time Interval Adjustment Cost
100
Grounding Cost
100
Transfer Cost
100
Man Hours
100
Hourly Rate
200

Maintenance Timing Recommendation Page



Questions and Solutions

	question	solution	source
1	How to detect a reasonable statistical model from the given data?	mean square error	
2	How to evaluate the estimated model's performance?	QQ-plot	QQ_plot.pdf
3	Goodness-of-Fit	Kolmogorov-Smirnov	https://www.itl.nist.gov/div898/handbook/eda/section3/eda35g.htm

Reference

	subject	source
1	survival analysis	Survival Analysis.pdf
2	weibull distribution	http://reliawiki.org/index.php/The_Weibull_Distribution