一、defs

1.原函数 - Fx，f(x)

if $F'(x)=f(x)$，F(x)为f(x)的原函数

Notes:

①若f(x)有原函数，则一定有无数个原函数

$(x^3+C)'=3x^2$

② 任两个原函数差为常数

证设 $F'(x)=f(x)$，$G'(x)=f(x)$

$[F(x)-G(x)]'=0$

$\Rightarrow F(x)-G(x)=C$

（不定积分是找所有原函数的，不定积分从本质来说是集合）

2、不定积分 - $\int f(x)d(x)=F(x)+C$

找一个函数所有原函数，我们称为不定积分

Notes:

① $\frac{d}{d(x)}\int f(x)\,dx=f(x)$;

② $\int f'(x)\,dx=f(x)+C$

二、不定积分的工具

(一) 基本公式

$x>0$, $(\ln x)'=\frac{1}{x}$ $\int\frac{1}{x}dx=\ln x+C$

$x<0$ $[\ln(-x)]'=-\frac{1}{x}\cdot(-1)=\frac{1}{x}$

$\int\frac{1}{x}dx=\ln(-x)+C$

1. $\int k\,dx=kx+C$

2. $\int x^a dx=\begin{cases}\frac{1}{a+1}x^{a+1}+C, & a\neq -1\\ \ln|x|+C, & a=-1\end{cases}$

3. $\int a^x dx=\begin{cases}\frac{a^x}{\ln a}+C, & a\neq e\\ e^x+C, & a=e\end{cases}$

4. ① $\int\sin x\,dx=-\cos x+C$

② $\int\cos x\,dx=\sin x+C$

③ $\int\tan x\,dx=-\ln|\cos x|+C$

④ $\int\cot x\,dx=\ln|\sin x|+C$

⑤ $\int\sec x\,dx=\ln|\sec x+\tan x|+C$

⑥ $\int\csc x\,dx=\ln|\csc x-\cot x|+C$

⑦ $\int\sec^2 x\,dx=\tan x+C$

⑧ $\int \csc^2 x \, dx = -\cot x + C$

⑨ $\int \sec x \tan x \, dx = \sec x + C$

⑩ $\int \csc x \cot x \, dx = -\csc x + C$

5. 平方和、平方差

① $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$

② $\int \frac{1}{\sqrt{a^2-x^2}} dx =$

③ $\int \frac{1}{1+x^2} dx = \arctan x + C$

④ $\int \frac{1}{a^2+x^2} dx =$

⑤ $\int \frac{1}{\sqrt{x^2+a^2}}$

⑥ $\int \frac{1}{\sqrt{x^2-a^2}} dx$

⑦ $\int \frac{1}{x^2-a^2} dx =$

⑧ $\int \sqrt{a^2-x^2} \, dx =$

(二)、积分法

方法一 换元积分法 $\begin{cases} \text{第一类} \\ \text{第二类} \end{cases}$

1.第一类

例1. $\int \frac{1}{(3x+2)^2} dx \overset{\text{统一}}{=} \frac{1}{3}\int (3x+2)^{-2} d(3x+2)$

$\overset{3x+2=t}{=} \frac{1}{3}\int t^{-2} dt = \frac{1}{3} \times \frac{1}{(-1)} t^{-1} + C$

$= -\frac{1}{3t} + C = -\frac{1}{3(3x+2)} + C$

例2、$\int (1-\frac{1}{x^2}) e^{x+\frac{1}{x}} dx$

$= \int e^{x+\frac{1}{x}} d(x+\frac{1}{x}) = e^{x+\frac{1}{x}} + C$

例3、$\int \frac{1}{\sqrt{x(1+x)}} dx = 2\int \frac{dx}{2\sqrt{x}\cdot\sqrt{1+x}}$ $\quad (\sqrt{x})' = \frac{1}{2\sqrt{x}}$

$= 2\int \frac{d(\sqrt{x})}{\sqrt{1-(\sqrt{x})^2}} = 2\arcsin\sqrt{x} + C$

例4 $\int \frac{dx}{\sqrt{x}(1+x)}$

$$= 2\int \frac{d\sqrt{x}}{1+(\sqrt{x})^2} = 2\arctan\sqrt{x}+C$$

$$\int \tan x\,dx = -\int \frac{1}{\cos x}\,d(\cos x)$$

$$= -\ln|\cos x| + C;$$

$$\int \cot x\,dx = \int \frac{1}{\sin x}\,d(\sin x)$$

$$= \ln|\sin x| + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \int \frac{1}{\sqrt{1-(\frac{x}{a})^2}}\,d(\frac{x}{a})$$

$$= \arcsin\frac{x}{a} + C;$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\int \frac{1}{1+(\frac{x}{a})^2}\,d(\frac{x}{a})$$

$$= \frac{1}{a}\arctan\frac{x}{a} + C$$

Th1. $\int f[\varphi(x)]\cdot\varphi'(x)\,dx = \int f[\varphi(x)]\cdot d\varphi(x)$

$\underline{\underline{\varphi(x)=t}}$ $\int f(t)\,dt$

$F(t)+C = F[\varphi(x)]+C$

$$= F(t) + C = F(\varphi(x)) + C$$