信息与计算科学 1601 骆天奇 2016254060407

Answer 1

answer 1.1

由 *Bayes* 公式得

$$P(H_i|E_1) = \frac{P(H_i)P(E_1|H_i)}{\sum_{j=1}^{n} P(H_j)P(E_1|H_j)} \tag{1}$$

其中

$$\begin{aligned}&\sum_{i=1}^{n} P(H_i)P(E_1|H_i)\\ =&(0.4 \times 0.5) + (0.3 \times 0.3) + (0.3 \times 0.5)\\ =&0.44\end{aligned}$$

$$\begin{aligned}P(H_1|E_1) =&\frac{P(H_1)P(E_1|H_1)}{\sum_{j=1}^{3} P(H_j)P(E_1|H_j)}\\ =&\frac{0.4 \times 0.5}{0.44}\\ =&\frac{5}{11} > P(H_1)\end{aligned} \tag{2}$$

$$\begin{aligned}P(H_2|E_1) =&\frac{P(H_2)P(E_1|H_2)}{\sum_{j=1}^{3} P(H_j)P(E_1|H_j)}\\ =&\frac{0.3 \times 0.3}{0.44}\\ =&\frac{9}{44} < P(H_2)\end{aligned} \tag{3}$$

$$\begin{aligned}P(H_3|E_1) =&\frac{P(H_3)P(E_1|H_3)}{\sum_{j=1}^{3} P(H_j)P(E_1|H_j)}\\ =&\frac{0.3 \times 0.5}{0.44}\\ =&\frac{15}{44} > P(H_3)\end{aligned} \tag{4}$$

可见由于证据 E_1 的出现，H_1, H_3 成立的可能性有所增加，而 H_2 成立的可能性有所下降。

answer 1.2

由 *Bayes* 公式得

$$P(H_i|E_1E_2 \ldots E_m) = \frac{P(H_i)\prod_{j=1}^{m} P(E_j|H_i)}{\sum_{j=1}^{n}\left(P(H_j)\prod_{k=1}^{m} P(E_k|H_j)\right)} \tag{5}$$

其中

$$\begin{aligned}&\sum_{i=1}^{n}\left(P(H_i)\prod_{j=1}^{m} P(E_j|H_i)\right)\\ =&\big(0.4 \times (0.5 \times 0.7)\big)+\\ &\big(0.3 \times (0.3 \times 0.9)\big)+\\ &\big(0.7 \times (0.5 \times 0.1)\big)\\ =&0.256\end{aligned}$$

$$\begin{aligned}P(H_1|E_1E_2) =&\frac{P(E_1|H_1)P(E_2|H_1)P(H_1)}{\sum_{i=1}^{3} P(E_1|H_i)P(E_2|H_i)P(H_i)}\\ =&\frac{0.4 \times (0.5 \times 0.7)}{0.256}\\ \approx&0.54 > P(H_1)\end{aligned} \tag{6}$$

$$\begin{aligned}P(H_2|E_1E_2) =&\frac{P(E_1|H_2)P(E_2|H_2)P(H_2)}{\sum_{i=1}^{3} P(E_1|H_i)P(E_2|H_i)P(H_i)}\\ =&\frac{0.3 \times (0.3 \times 0.9)}{0.256}\\ \approx&0.31 > P(H_2)\end{aligned} \tag{7}$$

$$\begin{aligned}P(H_3|E_1E_2) =&\frac{P(E_1|H_3)P(E_2|H_3)P(H_3)}{\sum_{i=1}^{3} P(E_1|H_i)P(E_2|H_i)P(H_i)}\\ =&\frac{0.7 \times (0.5 \times 0.1)}{0.256}\\ \approx&0.13 < P(H_3)\end{aligned} \tag{8}$$

可见由于证据 E_1, H_2 的出现，H_1 成立的可能性有所增加，H_2 成立的可能性略有增加,H_3 成立的可能性有所下降。