

Answer 1

answer 1.1

由 Bayes 公式得

$$P(H_i|E_1) = \frac{P(H_i)P(E_1|H_i)}{\sum_{j=1}^n P(H_j)P(E_1|H_j)} \quad (1)$$

其中

$$\begin{aligned} & \sum_{i=1}^n P(H_i)P(E_1|H_i) \\ &= \sum_{i=1}^3 P(H_i)P(E_1|H_i) \\ &= (0.4 \times 0.5) + (0.3 \times 0.3) + (0.3 \times 0.5) \\ &= 0.44 \end{aligned}$$

$$\begin{aligned} P(H_1|E_1) &= \frac{P(H_1)P(E_1|H_1)}{\sum_{j=1}^3 P(H_j)P(E_1|H_j)} \\ &= \frac{0.4 \times 0.5}{0.44} \\ &\approx 0.45 > P(H_1) \end{aligned} \quad (2)$$

$$\begin{aligned} P(H_2|E_1) &= \frac{P(H_2)P(E_1|H_2)}{\sum_{j=1}^3 P(H_j)P(E_1|H_j)} \\ &= \frac{0.3 \times 0.3}{0.44} \\ &\approx 0.20 < P(H_2) \end{aligned} \quad (3)$$

$$\begin{aligned} P(H_3|E_1) &= \frac{P(H_3)P(E_1|H_3)}{\sum_{j=1}^3 P(H_j)P(E_1|H_j)} \\ &= \frac{0.3 \times 0.5}{0.44} \\ &\approx 0.34 > P(H_3) \end{aligned} \quad (4)$$

可见由于证据 E_1 的出现, H_1, H_3 成立的可能性有所增加, 而 H_2 成立的可能性有所下降。

answer 1.2

由 Bayes 公式得

$$P(H_i|E_1 E_2 \dots E_m) = \frac{P(H_i) \prod_{j=1}^m P(E_j|H_i)}{\sum_{j=1}^n (P(H_j) \prod_{k=1}^m P(E_k|H_j))} \quad (5)$$

$$\begin{aligned} & \sum_{i=1}^n (P(H_i) \prod_{j=1}^m P(E_j|H_i)) \\ &= \sum_{i=1}^3 P(H_i)P(E_1|H_i)P(E_2|H_i) \\ &= (0.4 \times (0.5 \times 0.7)) + \\ & \quad (0.3 \times (0.3 \times 0.9)) + \\ & \quad (0.7 \times (0.5 \times 0.1)) \\ &= 0.256 \end{aligned}$$

$$\begin{aligned} P(H_1|E_1 E_2) &= \frac{P(E_1|H_1)P(E_2|H_1)P(H_1)}{\sum_{i=1}^3 P(H_i)P(E_1|H_i)P(E_2|H_i)} \\ &= \frac{0.4 \times (0.5 \times 0.7)}{0.256} \\ &\approx 0.54 > P(H_1) \end{aligned} \quad (6)$$

$$\begin{aligned} P(H_2|E_1 E_2) &= \frac{P(E_1|H_2)P(E_2|H_2)P(H_2)}{\sum_{i=1}^3 P(H_i)P(E_1|H_i)P(E_2|H_i)} \\ &= \frac{0.3 \times (0.3 \times 0.9)}{0.256} \\ &\approx 0.31 > P(H_2) \end{aligned} \quad (7)$$

$$\begin{aligned} P(H_3|E_1 E_2) &= \frac{P(E_1|H_3)P(E_2|H_3)P(H_3)}{\sum_{i=1}^3 P(H_i)P(E_1|H_i)P(E_2|H_i)} \\ &= \frac{0.7 \times (0.5 \times 0.1)}{0.256} \\ &\approx 0.13 < P(H_3) \end{aligned} \quad (8)$$

可见由于证据 E_1, E_2 的出现, H_1 成立的可能性有所增加, H_2 成立的可能性略有增加, H_3 成立的可能性有所下降。