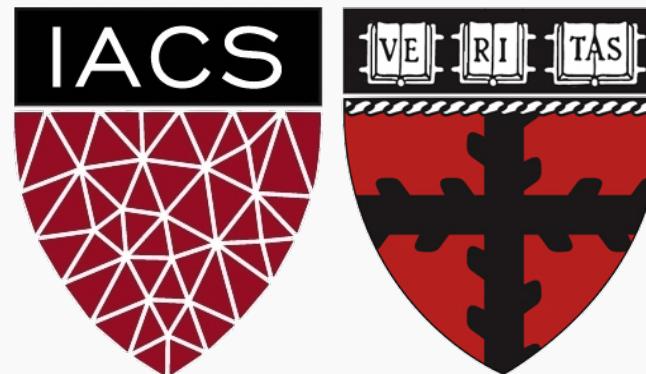


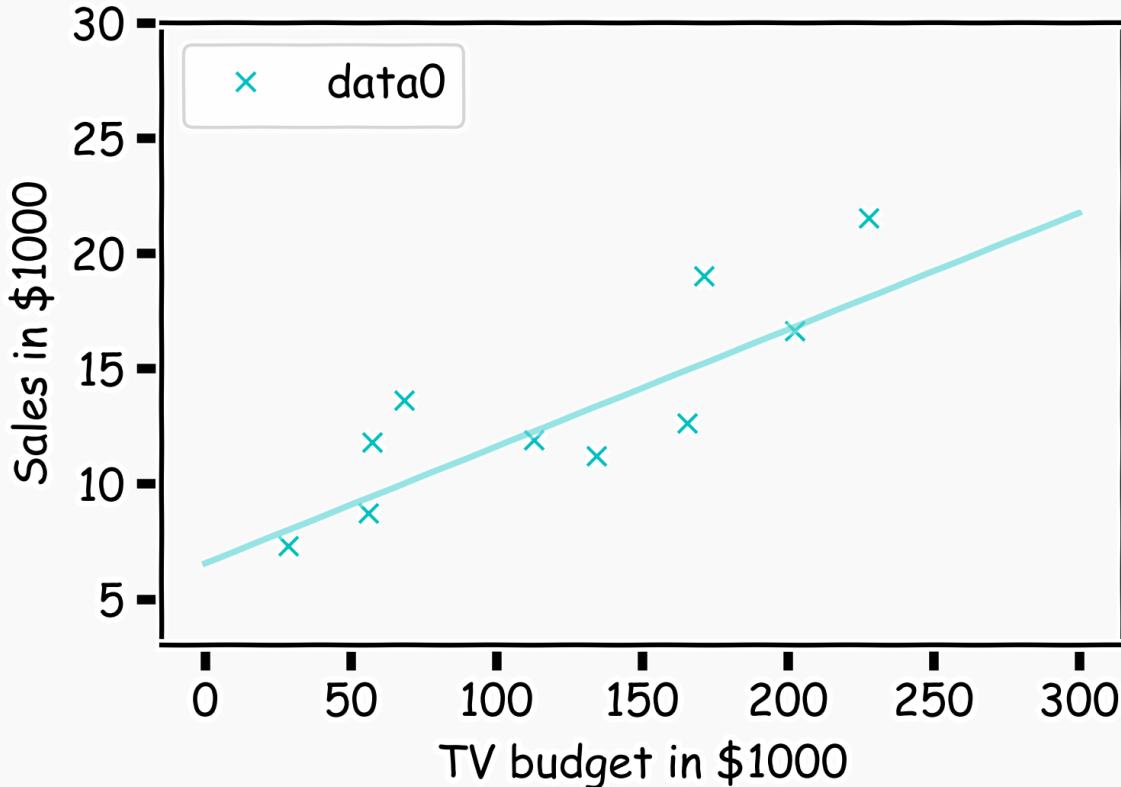
# Prediction Intervals

CS109A Introduction to Data Science  
Pavlos Protopapas, Kevin Rader and Chris Tanner



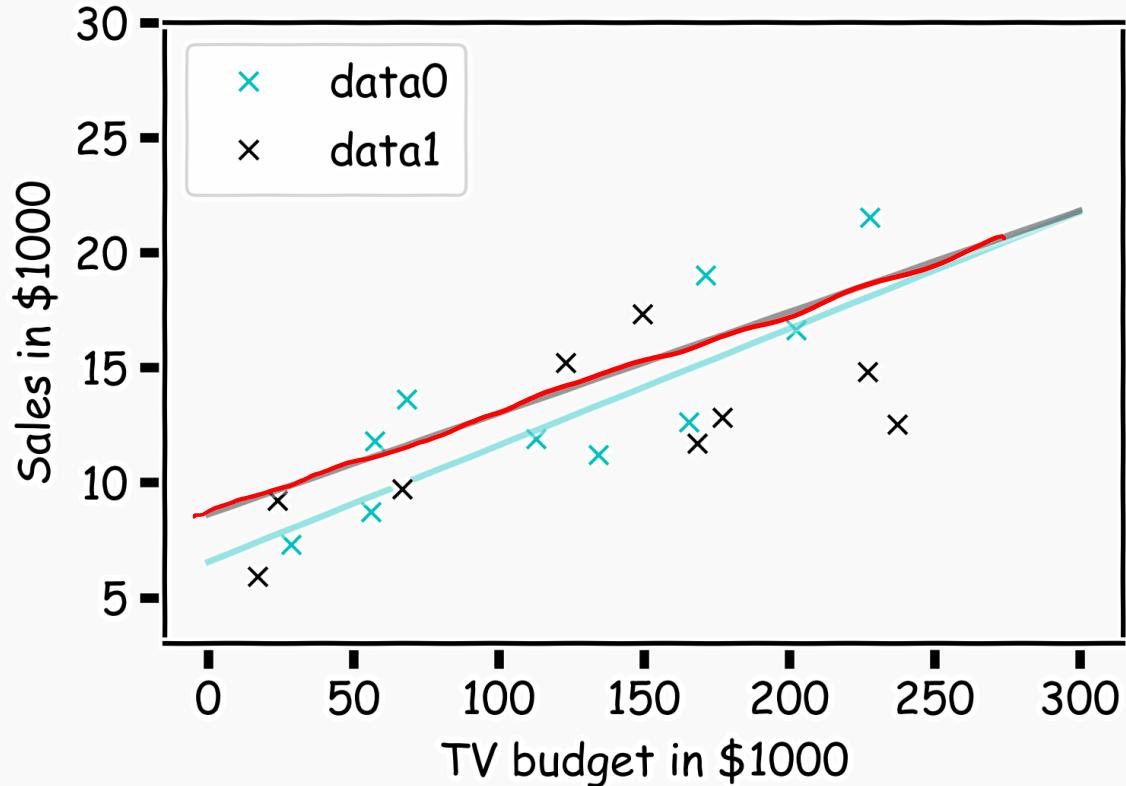
# How well do we know $\hat{f}$ ?

Our confidence in  $f$  is directly connected with our confidence in  $\beta$ s. For each bootstrap sample, we have one  $\beta$ , which we can use to determine the model,  $f(x) = X\beta$ .



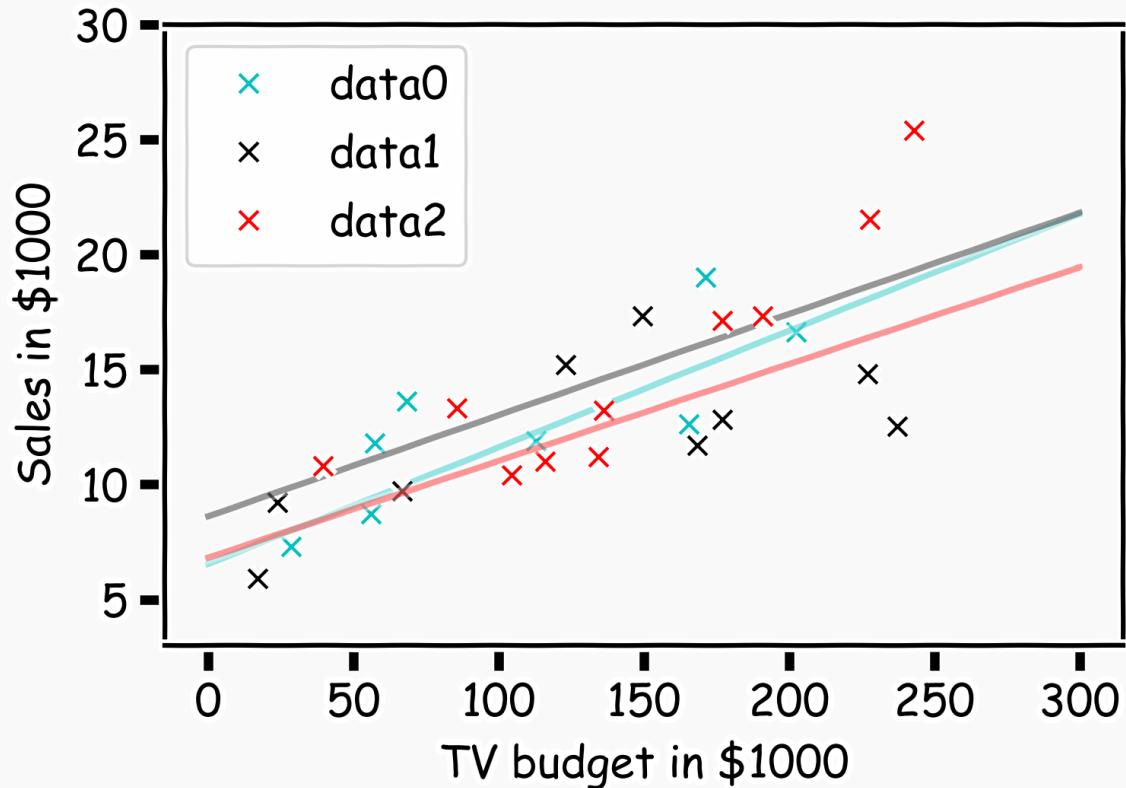
# How well do we know $\hat{f}$ ?

Here we show two difference models predictions given the fitted coefficients.



# How well do we know $\hat{f}$ ?

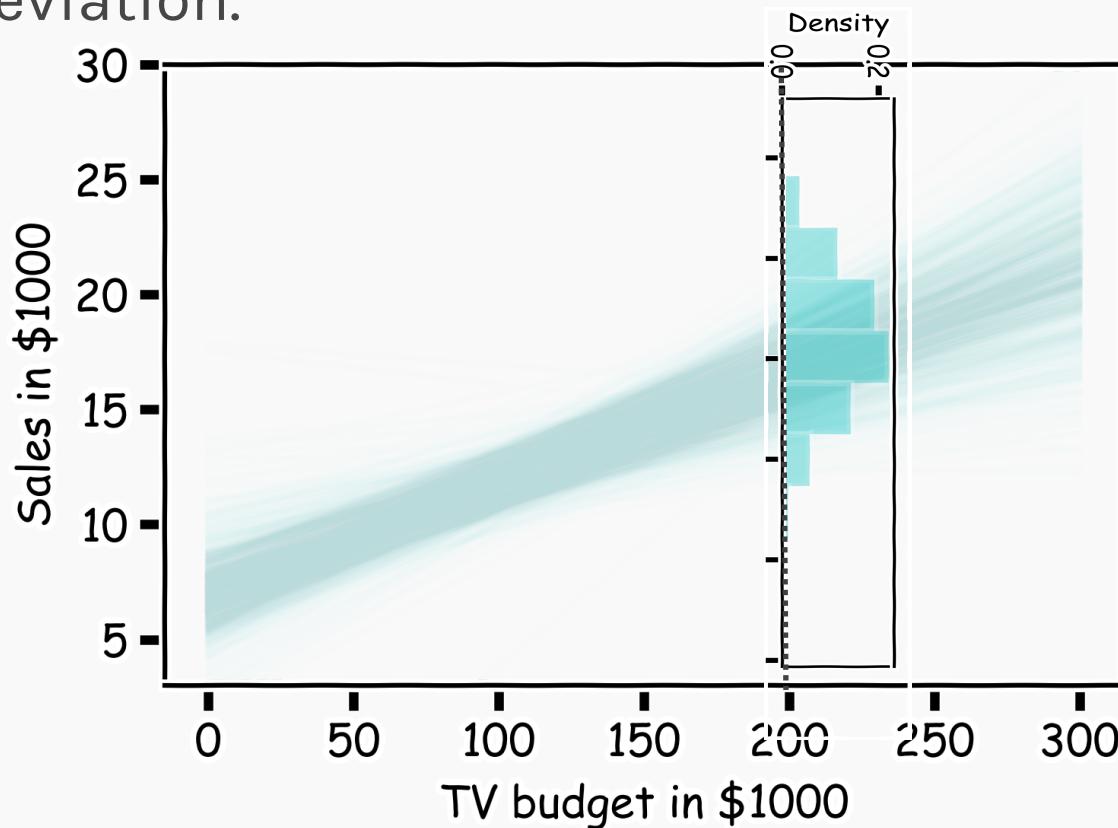
There is one such regression line for every bootstrapped sample.



# How well do we know $\hat{f}$ ?

Below we show all regression lines for a thousand of such bootstrapped samples.

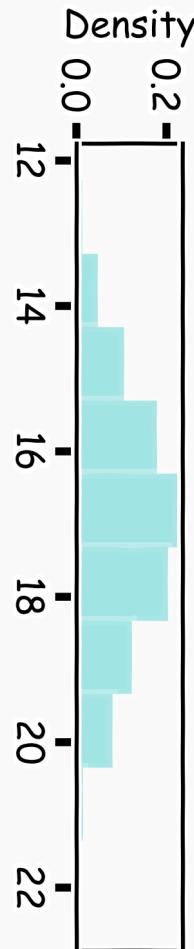
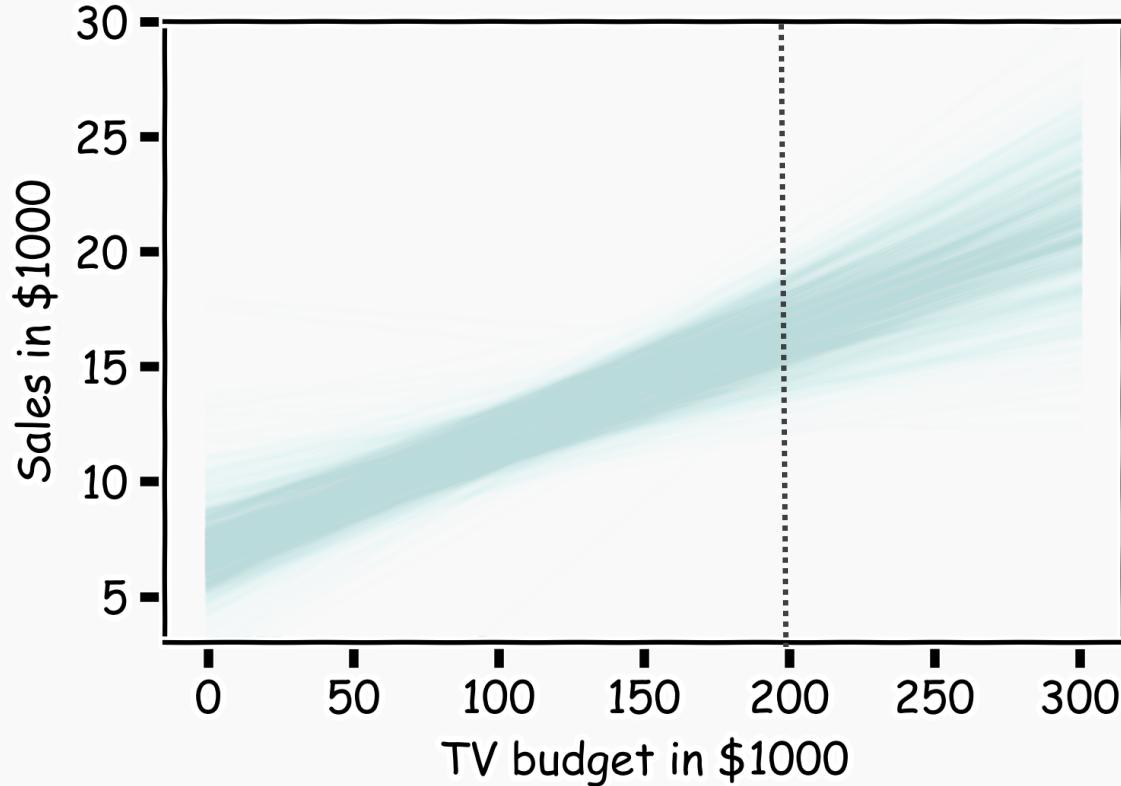
For a given  $x$ , we examine the distribution of  $\hat{f}$ , and determine the mean and standard deviation.



# How well do we know $\hat{f}$ ?

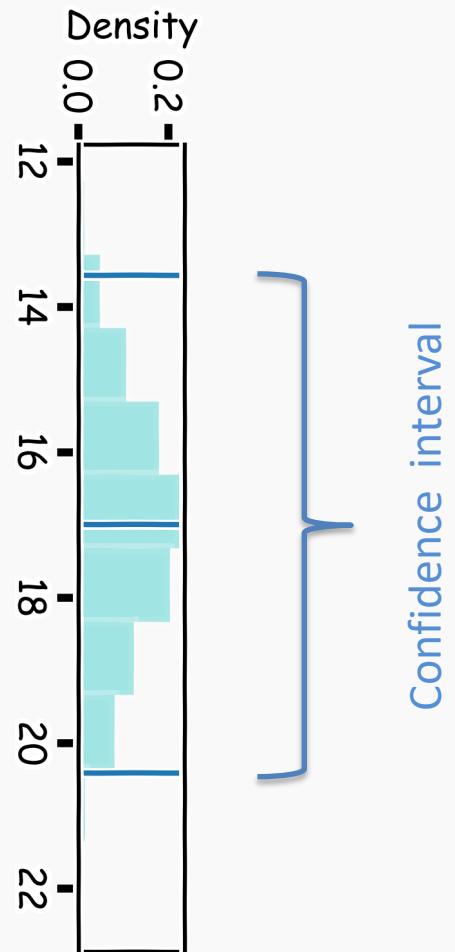
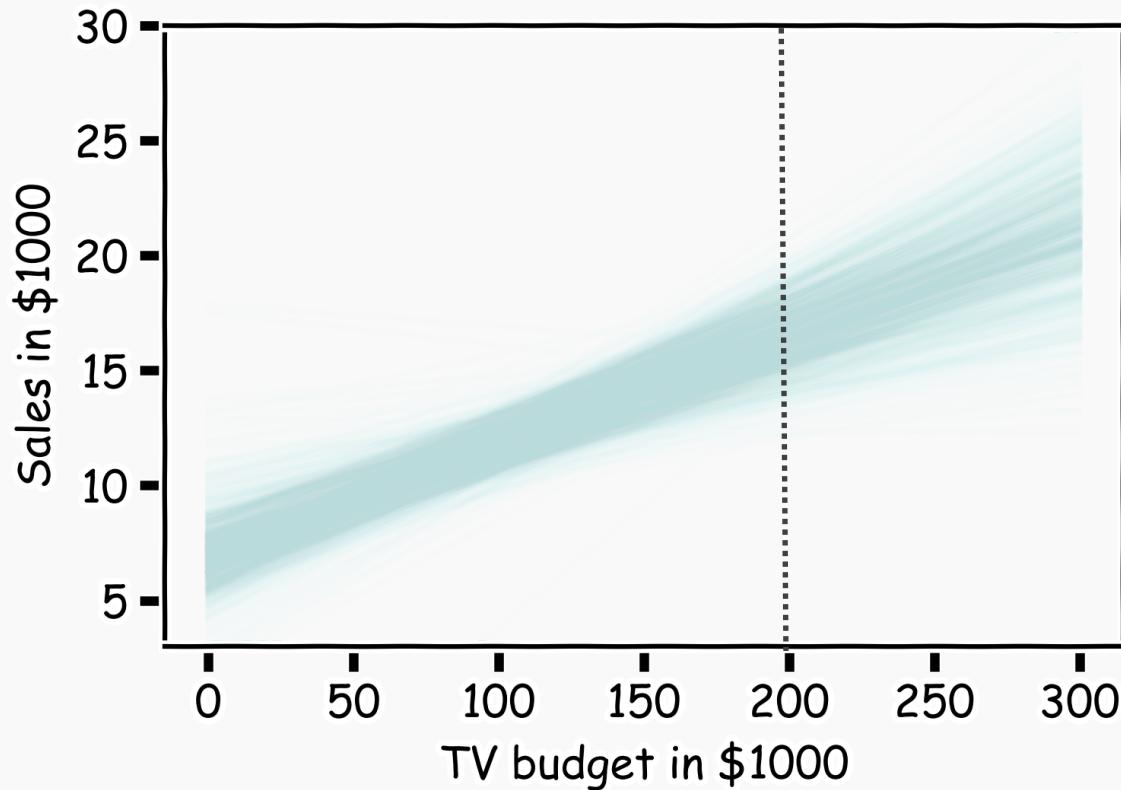
Below we show all regression lines for a thousand of such bootstrapped samples.

For a given  $x$ , we examine the distribution of  $\hat{f}$ , and determine the mean and standard deviation.



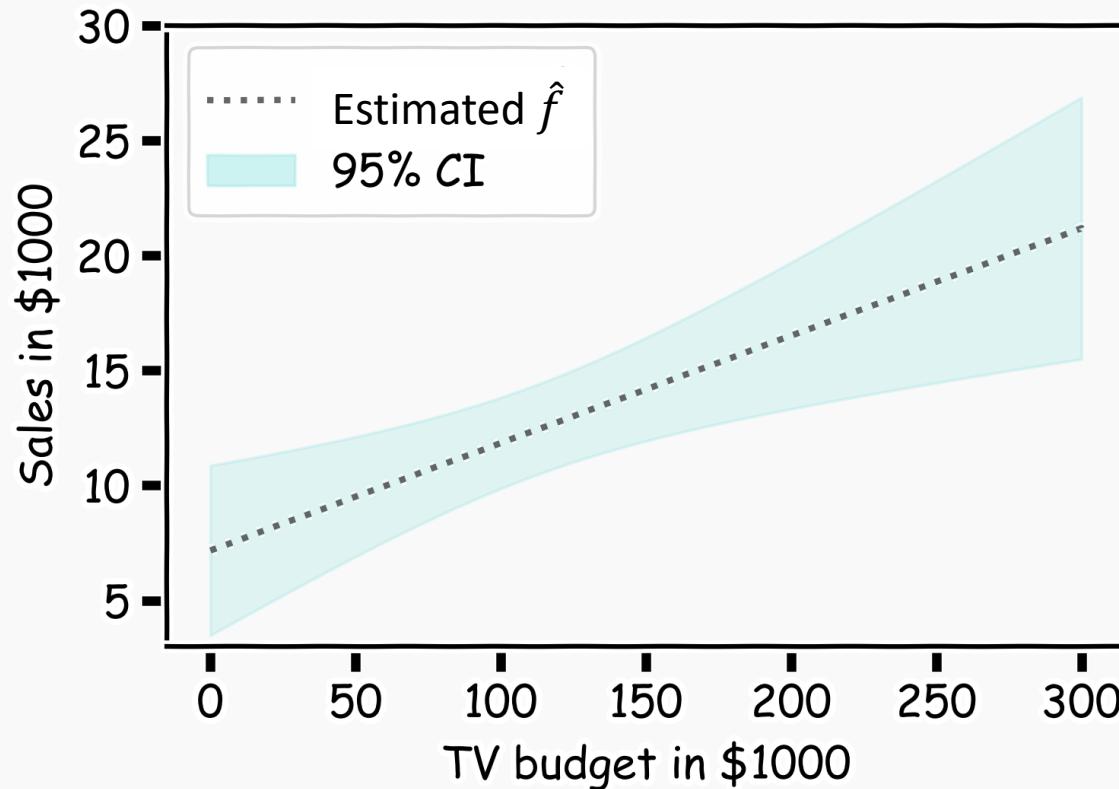
# How well do we know $\hat{f}$ ?

We determine the confidence interval of  $\hat{f}$  by selecting the region that contains 95% of the samples of  $\hat{f}(x) = X \hat{\beta}$ .



# How well do we know $\hat{f}$ ?

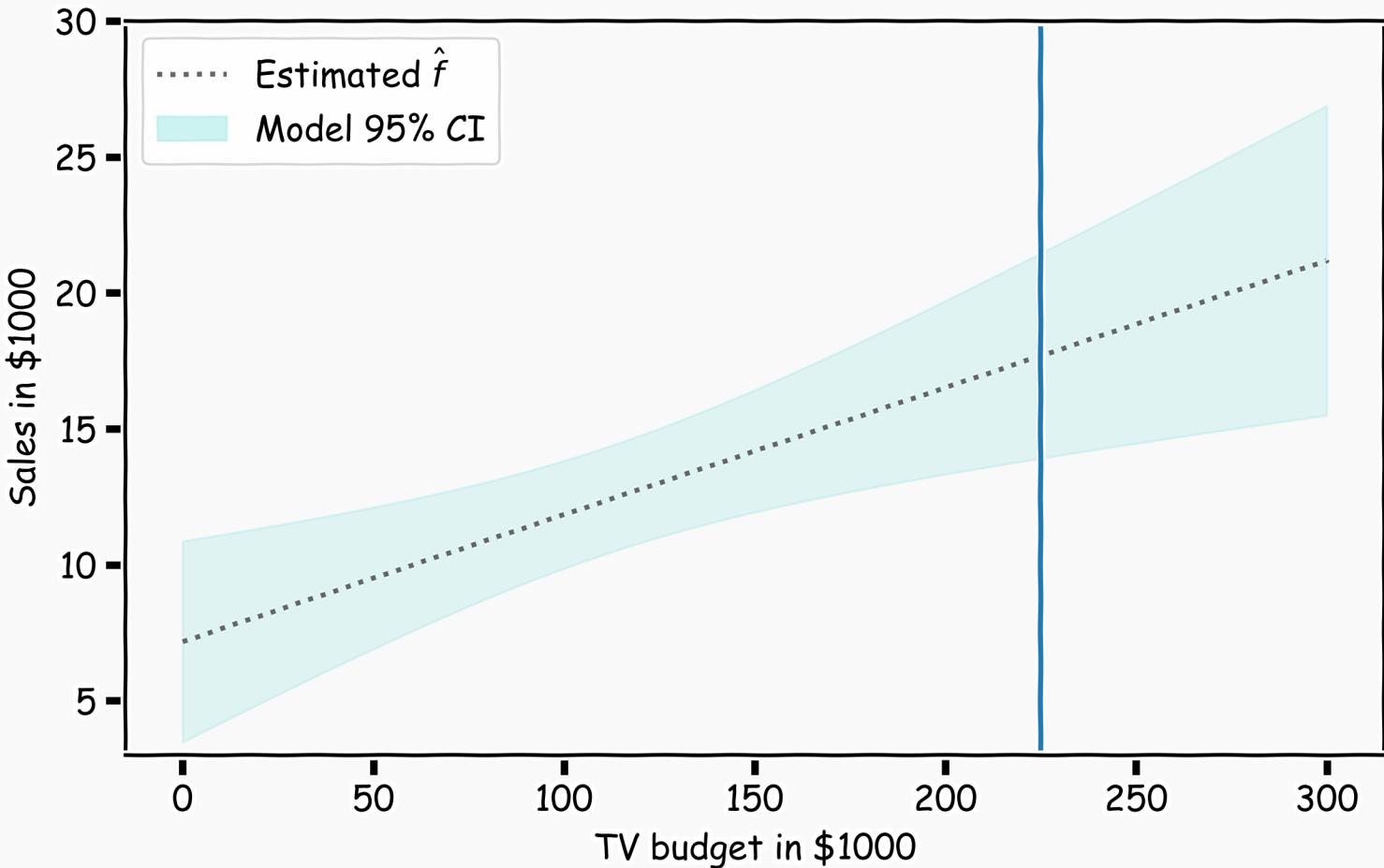
For every  $x$ , we calculate the mean of the models,  $\widehat{\mu}_f$  (shown with dotted line) and the 95% CI of those models (shaded area).



# Confidence in predicting $\hat{y}$

Even if we knew  $f(x)$  –the response value cannot be predicted perfectly because of the random error in the model (irreducible error).

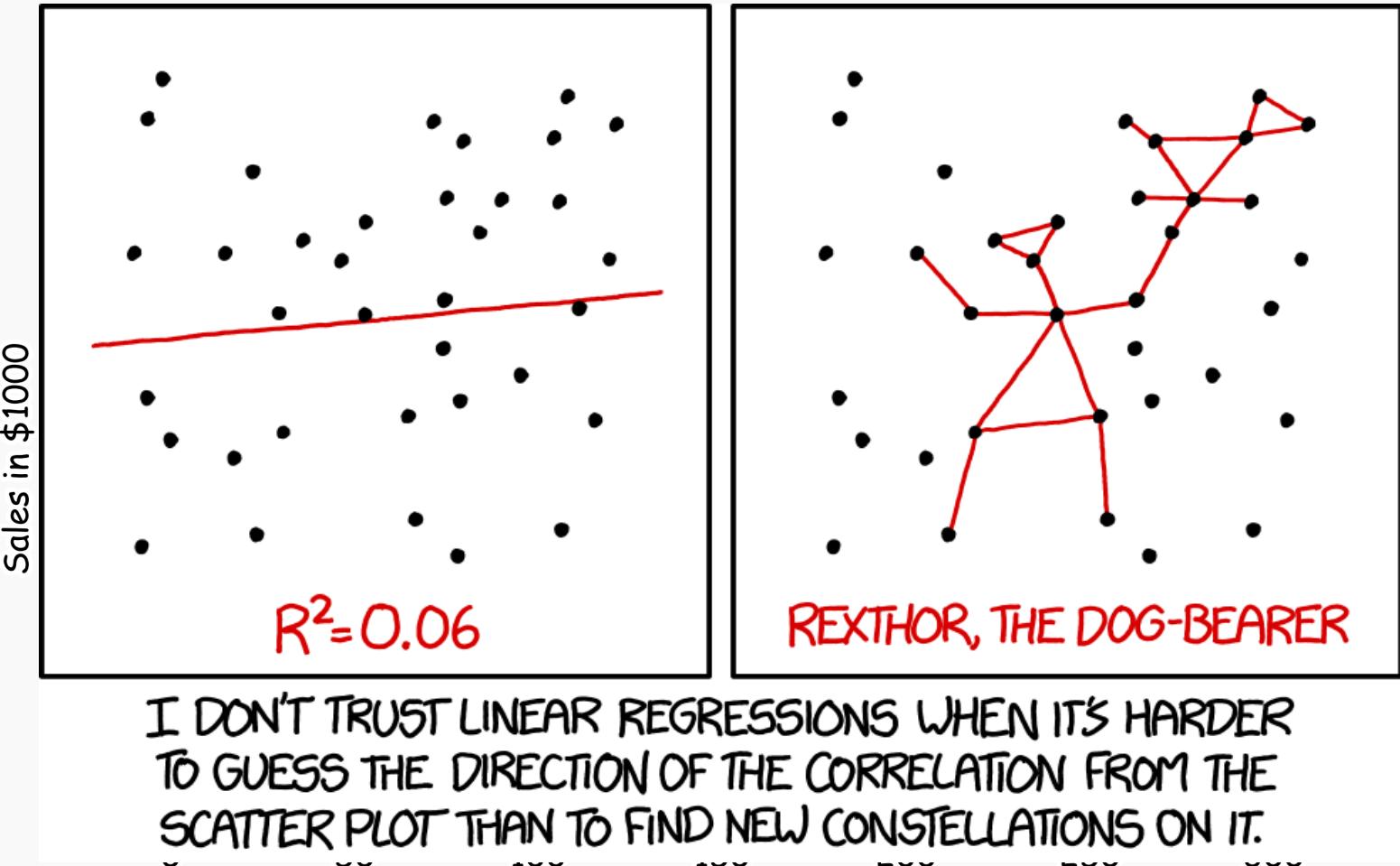
How much will  $Y$  vary from  $\hat{Y}$ ? We use **prediction intervals** to answer this question.



# Confidence in predicting $\hat{y}$

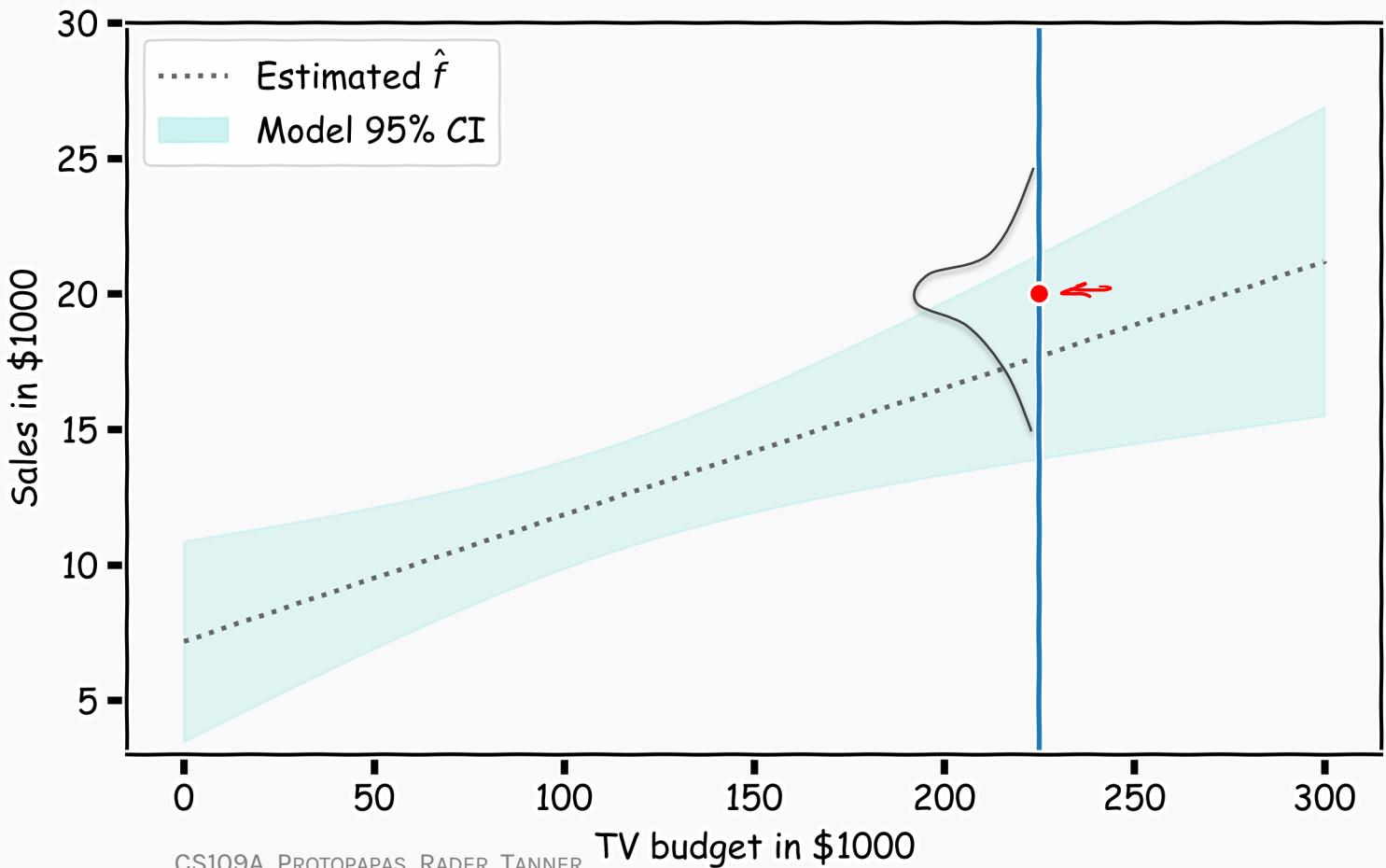
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# Confidence in predicting $\hat{y}$

- for a given  $x$ , we have a distribution of models  $f(x)$
- for each of these  $f(x)$ , the prediction for  $y \sim N(f(x), \sigma_\epsilon)$



# Confidence in predicting $\hat{y}$

- for a given  $x$ , we have a distribution of models  $f(x)$
- for each of these  $f(x)$ , the prediction for  $y \sim N(f(x), \sigma_\epsilon)$
- The prediction confidence intervals are then ...

