CS6220 Datamining Fall 2014 Homework 1, Wei Luo

Know Your Data

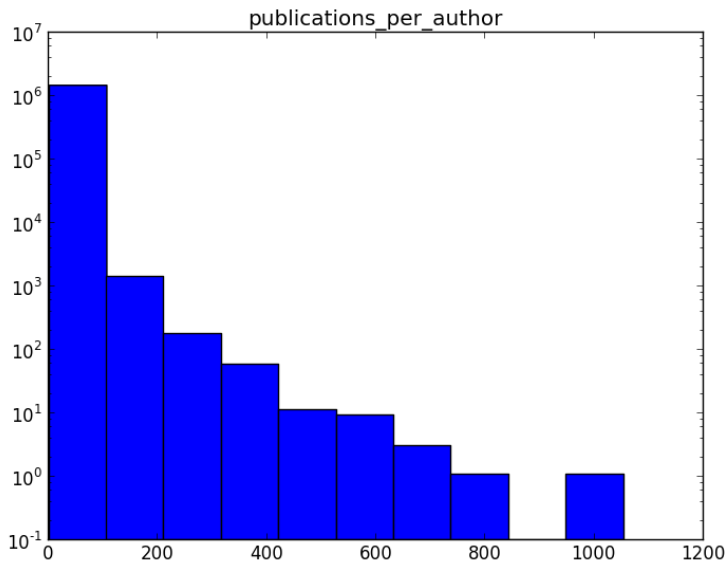
1. (1) The count of each item is:

Number of authors:	1484984
Number of publications:	1977248
Number of venues:	255686

(2) For publications per author, the statistical values are:

min	max	q1	q3	median
1	1054	1	2	1

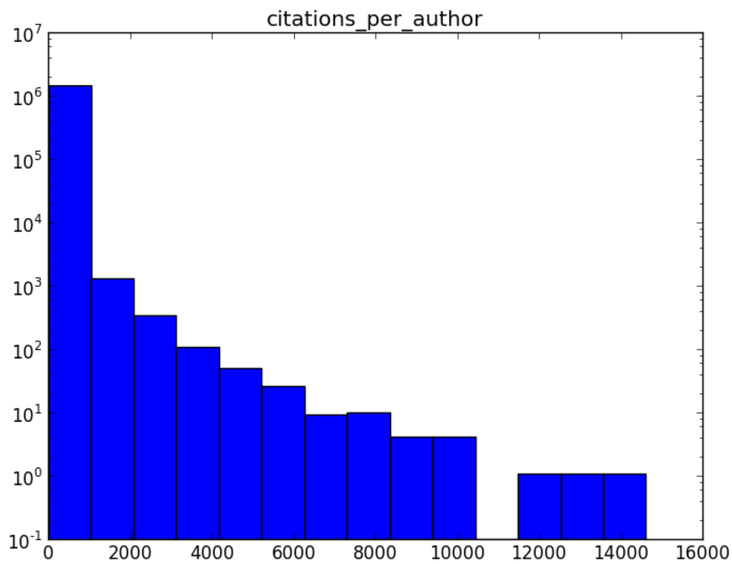
The histogram for number of publications per author is:



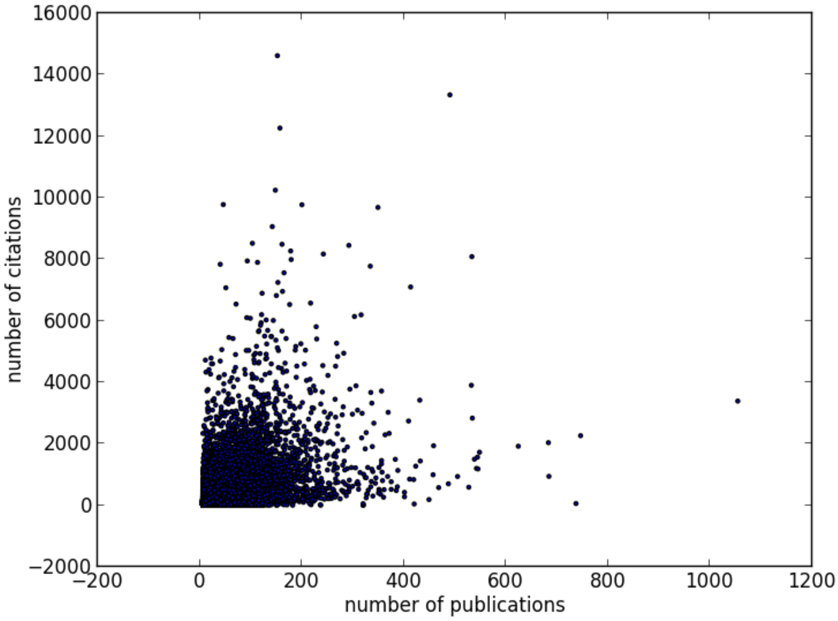
(3) For citations per author, the statistical values are:

min	max	q1	q3	median
0	14618	0	4	0

The histogram for number of citations per author is:



(4) The scatter plot between the number of publications vs. the number of citations for authors who have more than 5 publications is:



Classification for Matrix Data

2. Decision Tree

From the problem we have features:

Color = {Yellow, Green}, Size = {Small, Large}, Shape = {Round, Irregular}

And label: Edible = $\{+, -\}$

Since each feature is binary, we don't have to consider it as a candidate again in the sub-trees once it is chosen for split.

And probabilities:

$P(g) = P(Color = Green)$ $\quad P(y) = P(Color = Yellow)$

$P(s) = P(Size = Small)$ $\quad P(l) = P(Size = Large)$

$P(r) = P(Shape = Round)$ $\quad P(i) = P(Shape = Irregular)$

$P(+) = P(Edible = +)$ $\quad P(-) = P(Edible = -)$

Before split:

$P(g) = \frac{3}{3+13} = \frac{3}{16}$ $\quad P(y) = \frac{13}{3+13} = \frac{13}{16}$ $\quad P(s) = \frac{8}{8+8} = \frac{1}{2}$ $\quad P(l) = \frac{8}{8+8} = \frac{1}{2}$

$P(r) = \frac{12}{12+4} = \frac{3}{4}$ $\quad P(i) = \frac{4}{12+4} = \frac{1}{4}$ $\quad P(+) = \frac{9}{9+7} = \frac{9}{16}$ $\quad P(-) = \frac{7}{9+7} = \frac{7}{16}$

$P(+|g) = \frac{1}{1+2} = \frac{1}{3}$ $\quad P(-|g) = \frac{2}{1+2} = \frac{2}{3}$ $\quad P(+|y) = \frac{8}{8+5} = \frac{8}{13}$ $\quad P(-|y) = \frac{5}{8+5} = \frac{5}{13}$

$P(+|s) = \frac{6}{6+2} = \frac{3}{4}$ $\quad P(-|s) = \frac{2}{6+2} = \frac{1}{4}$ $\quad P(+|l) = \frac{3}{3+5} = \frac{3}{8}$ $\quad P(-|l) = \frac{5}{3+5} = \frac{5}{8}$

$P(+|r) = \frac{6}{6+6} = \frac{1}{2}$ $\quad P(-|r) = \frac{6}{6+6} = \frac{1}{2}$ $\quad P(+|i) = \frac{3}{3+1} = \frac{3}{4}$ $\quad P(-|i) = \frac{1}{3+1} = \frac{1}{4}$

$H(Edible) = P(+)\log_2(\frac{1}{P(+)}) + P(-)\log_2(\frac{1}{P(-)}) = 0.9987$

$H(Edible|Color) = \sum_i P(Color_i) \sum_j P(Edible_j|Color_i) \log_2 \frac{1}{P(Edible_j|Color_i)} = 0.9532$

$H(Edible|Size) = \sum_i P(Size_i) \sum_j P(Edible_j|Size_i) \log_2 \frac{1}{P(Edible_j|Size_i)} = 0.8829$

$H(Edible|Shape) = \sum_i P(Shape_i) \sum_j P(Edible_j|Shape_i) \log_2 \frac{1}{P(Edible_j|Shape_i)} = 0.9528$

$IG(Color) = H(Edible) - H(Edible|Color) = 0.0455$

$IG(Size) = H(Edible) - H(Edible|Size) = 0.1158$

$IG(Shape) = H(Edible) - H(Edible|Shape) = 0.0459$

We choose feature Size for split at $root$ node since it has the most information gain.
Now for the left sub-tree, there are 8 data points, given that their Size is Small. (We are at $node_1$, the left child of $root$ node)

$P(+) = \frac{3}{4}$ $\quad P(-) = \frac{1}{4}$
$P(g) = \frac{1}{4}$ $\quad P(y) = \frac{3}{4}$ $\quad P(r) = \frac{3}{4}$ $\quad P(i) = \frac{1}{4}$
$P(+|g) = \frac{1}{2}$ $\quad P(-|g) = \frac{1}{2}$ $\quad P(+|y) = \frac{5}{6}$ $\quad P(-|y) = \frac{1}{6}$
$P(+|r) = \frac{2}{3}$ $\quad P(-|r) = \frac{1}{3}$ $\quad P(+|i) = 1$ $\quad P(-|i) = 0$

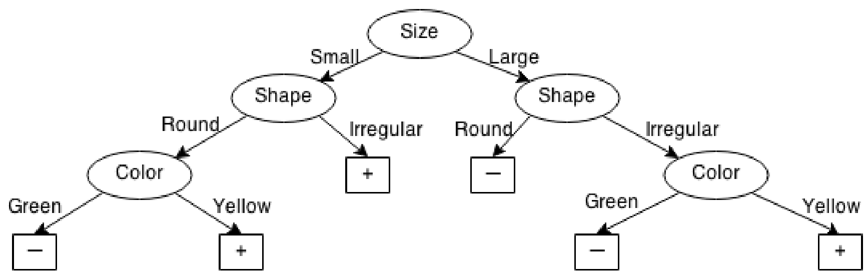
$H(Edible) = P(+)\log_2(\frac{1}{P(+)}) + P(-)\log_2(\frac{1}{P(-)}) = 0.8113$
$H(Edible|Color) = \sum_i P(Color_i)\sum_j P(Edible_j|Color_i)\log_2 \frac{1}{P(Edible_j|Color_i)} = 0.7375$
$H(Edible|Shape) = \sum_i P(Shape_i)\sum_j P(Edible_j|Shape_i)\log_2 \frac{1}{P(Edible_j|Shape_i)} = 0.6887$
$IG(Color) = H(Edible) - H(Edible|Color) = 0.0738$
$IG(Shape) = H(Edible) - H(Edible|Shape) = 0.1226$

We choose feature Shape for split at $node_1$ since it has more information gain.
Now we move one step further to the left sub-tree, there are 6 data points, given that their Size is Small and Shape is Round. (We are at $node_2$, the left child of $node_1$)
Since there is only one feature Color left as candidate, we use it for split. Since $P(-|g) = 1$ and $P(+|y) = \frac{3}{4}$, we make a leaf node of $-$ at the Green branch, a leaf node of $+$ at the Yellow branch.
We go back to the right child of $node_1$, there are 2 data points, given that their Size is Small and Shape is Irregular. Since all labels are $+$, we make a leaf node of $+$ here.
Now we go back to the right child node of $root$ node. There are 8 data points, given that their Size is Large. (We are at $node_3$, the right child of $root$ node)

$P(+) = \frac{3}{8}$ $\quad P(-) = \frac{5}{8}$
$P(g) = \frac{1}{8}$ $\quad P(y) = \frac{7}{8}$ $\quad P(r) = \frac{3}{4}$ $\quad P(i) = \frac{1}{4}$
$P(+|g) = 0$ $\quad P(-|g) = 1$ $\quad P(+|y) = \frac{3}{7}$ $\quad P(-|y) = \frac{4}{7}$
$P(+|r) = \frac{1}{3}$ $\quad P(-|r) = \frac{2}{3}$ $\quad P(+|i) = \frac{1}{2}$ $\quad P(-|i) = \frac{1}{2}$

$H(Edible) = P(+)\log_2(\frac{1}{P(+)}) + P(-)\log_2(\frac{1}{P(-)}) = 0.9544$
$H(Edible|Color) = \sum_i P(Color_i)\sum_j P(Edible_j|Color_i)\log_2 \frac{1}{P(Edible_j|Color_i)} = 0.7099$
$H(Edible|Shape) = \sum_i P(Shape_i)\sum_j P(Edible_j|Shape_i)\log_2 \frac{1}{P(Edible_j|Shape_i)} = 0.9387$
$IG(Color) = H(Edible) - H(Edible|Color) = 0.0157$
$IG(Shape) = H(Edible) - H(Edible|Shape) = 0.2445$

We choose feature Shape for split at $node_3$ since it has more information gain.
Now we move one step further to the left sub-tree, there are 6 data points, given that their Size is Large and Shape is Round. All the Color for this level is Yellow, so we stop splitting. And since $P(-) = \frac{3}{4}$, we make a leaf node of $-$ here.
Now we go back and see the right child of $node_3$, there are 2 data points, given that their Size is Large and Shape is Irregular. (We are at $node_4$, the right child of $node_3$)
Since there is only one feature Color left as candidate, we use it for split. Since $P(-|g) = 1$ and $P(+|y) = 1$, we make a leaf node of $-$ at the Green branch, a leaf node of $+$ at the Yellow branch.
Then we finished building the decision tree. And it looks like



3. Naïve Bayes

By looking in to three features {secret, sports, dollar}, the message table can be abstracted as:

secret	sports	dollar	label
0	0	1	spam
1	0	0	spam
1	0	0	spam
0	0	0	non-spam
1	1	0	non-spam
0	1	0	non-spam
0	0	0	non-spam

Then by counting, we can calculate MLEs as:

$\theta_{spam} = P(C_{spam}) = 3/7$

$\theta_{secret|spam} = P(secret = 1|C_{spam}) = 2/3$

$\theta_{secret|non-spam} = P(secret = 1|C_{non-spam}) = 1/4$

$\theta_{sports|non-spam} = P(sports = 1|C_{non-spam}) = 2/4$

$\theta_{dollar|spam} = P(dollar = 1|C_{spam}) = 1/3$

4. Support Vector Machine

(1) The support vectors are data points {7,18,20}:

$(0.53, 0.77)$, $(2.05, -0.62)$, $(1.63, -0.91)$

(2) $\mathbf{w} = [w_0, w_1, w_2]$

$w_0 + 0.53w_1 + 0.77w_2 = 1$

$w_0 + 2.05w_1 - 0.62w_2 = -1$

$w_0 + 1.63w_1 - 0.91w_2 = -1$

Solve the equations, we get $w_0 = 0.6687$, $w_1 = -0.5661$,$w_2 = 0.8198$

So $\mathbf{w} = [0.6687, -0.5661, 0.8198]$

(3) For calculating bais b:

$x[\alpha_k \neq 0] = [(0.53, 0.77), (2.05, -0.62), (1.63, -0.91)]$

$w' = [-0.5661, 0.8198]$,$y = [1, -1, -1]$, $N_k = 3$

$b = \sum_{k:\alpha_k \neq 0}(y_k - w'x_k)/N_k = 0.6687$

(4) The learned decision boundary function is:

$f(x) = f((x_1, x_2)) = 0.6687 - 0.5661x_1 + 0.8198x_2$

(5) For data point $x = (-1, 2)$:

$f(x) = 0.6687 - 0.5661x_1 + 0.8198x_2 = 2.8744 > 1$

So the predicted label for x is 1.

5. Mutual Information and Information Gain

Consider Y as the class label, and X as the attribute to predict Y, we have:

$H(Y) = \sum_y p(y) \log \frac{1}{p(y)}$

$H(Y|X) = \sum_x p(x) \sum_y p(y|x) \log \frac{1}{p(y|x)}$

$IG(X) = H(Y) - H(Y|X) = \sum_y p(y) \log \frac{1}{p(y)} - \sum_x p(x) \sum_y p(y|x) \log \frac{1}{p(y|x)}$

Since $p(y) = \sum_x p(x, y)$, $p(y|x) = p(x, y)/p(x)$, we have:

$$\begin{aligned} IG(X) &= \sum_y \sum_x p(x, y) \log \frac{1}{p(y)} - \sum_x p(x) \sum_y p(x, y)/p(x) \log \frac{1}{p(x,y)/p(x)} \\ &= \sum_x \sum_y p(x, y)(\log \frac{1}{p(y)} - \log \frac{1}{p(x,y)/p(x))}) \\ &= \sum_x \sum_y p(x, y)(\log \frac{p(x,y)}{p(x)p(y)}) \end{aligned}$$

It is the same as mutual information, $I(X; Y)$