1. **(a.)**

We can compute the probability distribution as follows:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  | | --- | --- | | a | p(a) | | 0 | 0.6 | | 1 | 0.4 | | |  |  | | --- | --- | | b | p(b) | | 0 | 0.592 | | 1 | 0.408 | | |  |  |  | | --- | --- | --- | | a | b | p(a,b) | | 0 | 0 | 0.336 | | 0 | 1 | 0.264 | | 1 | 0 | 0.256 | | 1 | 1 | 0.144 | |

Therefore p(a)p(b) is:

|  |  |  |
| --- | --- | --- |
| a | b | p(a)p(b) |
| 0 | 0 | 0.3552 |
| 0 | 1 | 0.2448 |
| 1 | 0 | 0.2368 |
| 1 | 1 | 0.1632 |

Since p(a)p(b) p(a, b) , a and b are dependent.

|  |  |  |
| --- | --- | --- |
|  |  |  |
| |  |  | | --- | --- | | c | p(c) | | 0 |  | | 1 |  | | |  |  |  | | --- | --- | --- | | c | a | p(a|c) | | 0 | 0 | 0.5 | | 0 | 1 | 0.5 | | 1 | 0 | 0.6923 | | 1 | 1 | 0.3077 | | |  |  |  | | --- | --- | --- | | c | b | p(b|c) | | 0 | 0 | 0.8 | | 0 | 1 | 0.2 | | 1 | 0 | 0.4 | | 1 | 1 |  | |
|  |  |  |
| |  |  |  |  | | --- | --- | --- | --- | | c | a | b |  | | 0 | 0 | 0 | 0.4 | | 0 | 0 | 1 | 0.1 | | 0 | 1 | 0 | 0.4 | | 0 | 1 | 1 | 0.1 | | 1 | 0 | 0 | 0.2769 | | 1 | 0 | 1 | 0.4154 | | 1 | 1 | 0 | 0.1231 | | 1 | 1 | 1 | 0.1846 | | |  |  |  |  | | --- | --- | --- | --- | | c | a | b |  | | 0 | 0 | 0 | 0.4 | | 0 | 0 | 1 | 0.1 | | 0 | 1 | 0 | 0.4 | | 0 | 1 | 1 | 0.1 | | 1 | 0 | 0 | 0.2769 | | 1 | 0 | 1 | 0.4154 | | 1 | 1 | 0 | 0.1231 | | 1 | 1 | 1 | 0.1846 | |  |

Therefore p(a|c)p(b|c) = p(a,b|c), hence

**(b.)**

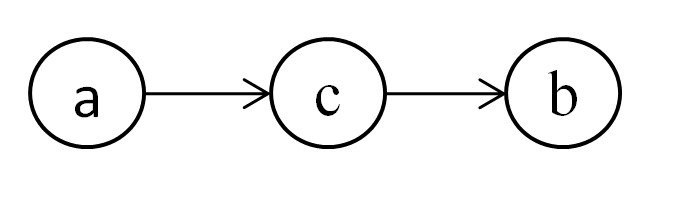
|  |
| --- |
|  |
| |  |  |  | | --- | --- | --- | | a | c | p(c|a) | | 0 | 0 | 0.4 | | 0 | 1 | 0.6 | | 1 | 0 | 0.6 | | 1 | 1 | 0.4 | |

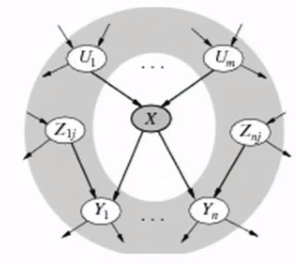
Then p(a)p(b|c)p(c|a) is

|  |  |  |  |
| --- | --- | --- | --- |
| a | b | c | p(a)p(b|c)p(c|a) |
| 0 | 0 | 0 | 0.192 |
| 0 | 0 | 1 | 0.144 |
| 0 | 1 | 0 | 0.048 |
| 0 | 1 | 1 | 0.216 |
| 1 | 0 | 0 | 0.192 |
| 1 | 0 | 1 | 0.064 |
| 1 | 1 | 0 | 0.048 |
| 1 | 1 | 1 | 0.096 |

Hence p(a)p(b|c)p(c|a) = p(a,b,c)

The corresponding Bayes network is:





**2.**

Consider Figure 14.4 (b) on the textbook:

The grey part is the Markov Blanket. We need to consider all possible paths from the center node X to all possible nodes outside the blanket. There are three kinds of such paths. First, consider U1…Um, they are parents of X. For any node Ui among U1…Un, the link between X and Ui has its tail on Ui. In this case, the path between X and outer nodes on the parent could be either tail-to-tail or head-to-tail. Once the parent is given, X is independent with all such nodes. Second, consider Y1…Yn, they are children of node X. For any node Yi among Y1…Yn, the link between X and Yi has its head on Yi. In this case, the path between X and outer nodes which are children of any Yi is head-to-tail on Yi. Once Yi is given, X is independent with all such nodes. The last kind of path comes from Z1j…Znj, they share children with X. The path will be head-to-head on the shared children Y1…Yn, thus given Yi cannot ensure independence of X and nodes on such paths. However, such path will be either tail-to-tail or head-to-tail on Zij (a node among Z1j…Znj), then, once Zij and Yi is given, X will be independent with outer nodes in such path. So, conditioned on all the nodes in the Markov Blanket, X will be independent with all the remaining variables in the graph.

**3. (a.)**

Independent pairs of variables are:

**(b.)**

Given a third variables, conditionally independent pairs of variables are:

**4. (a.)**

p(a,b,c) = p(a)p(b|a)p(c|b)

**(b.)**

Therefore a and c are dependent.

**(c.)**

Therefore p(a|b)p(c|b) = p(a,c|b) , a and c are conditionally independent given b.

**5. (a.)**

p(a,b,c) = p(a)p(b|a,c)p(c)

**(b.)**

Therefore a and c are independent.

**(c.)**

Therefore a and c are not conditionally independent given b.

**6. (a.)**

From the given net work, we know that

Therefore (ii) **P**(J|G)=**P**(J|G,I) and (iii) **P**(M|G,B,I)=**P**(M|G,B,I,J) are asserted.

**(b.)**

From the given Bayes net, we know that

**P**(B,I,M,G,J) = **P**(J|G)**P**(G|B,I,M)**P**(I|B,M)**P**(B)**P**(M)

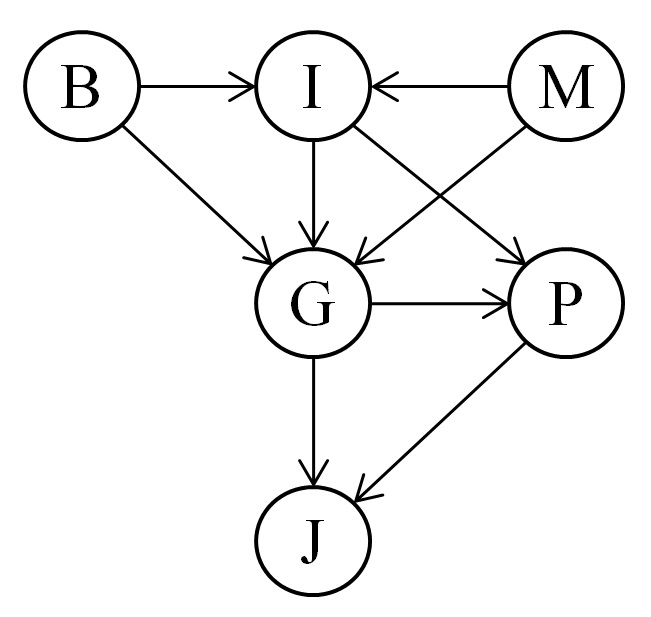
Therefore:

**(c.)**

**P**(J|B,I,M) = , and the problem can be described as P(J|b,i,m)

Therefore, the probability is

**(d.)**

****In this Bayes net, one will not be FoundGuilty if he/she is not Indicted regardless of whether he/she BrokeElectionLaw, regardless of whether he/she PoliticallyMotivatedProsecutor.

So G is context-specifically independent of B and M given I = false.

**(e.)**

Three links added to the net.

I to P : Inducted could result in PresidentPardon

G to P : FoundGuilty could result in PresidentPardon

P to J : PresidentPardon could lead to not Jailed

**7. (a.)**

Therefore the combination is in detailed balance with

**(b.)**

Therefore:

Therefore, q also has as its stationary distribution.