Optimization method to solving angle calibration functions and projector geometry

Owen Lu
Electroimpact Inc.
owenl@electroimpact.com

***Abstract*— Using nominal projector dimensions and solving for mirror angles, a preliminary curve fit can be created to map both DAC angles to real angles. This allows optimization to begin near a feasible region. Ideally, all parameters that describe the geometry of the projector could be solved on in order to get the real dimensions of the projector. However, measurement error and redundancy in the effects of parameters on the objective function may prevent this from being possible. This document describes the main parameters that will be solved on to generate projector structures to be used for ray angle calculations.**

I. Introduction

In order to account for errors in construction a more general model is necessary to account for variations. Fortunately although the explicit equations to generate the output ray are very long expressions, the core of the problem is simple. There are two mirrors, described as planes in space, with a ray input that is reflected off both mirrors. It is known that any deviation from the mirror normal from the ideal orientation can greatly affect the ray output. Thus, the optimization method is used to solve on 8 scalar values. The first 4 describe the linear calibration constants to convert DAC to angles, the last 4 describe the initial configuration of the normal. All other dimensions that describe the projector are assumed to be nominal

II. Unit vector generation

To generate unit vectors spherical coordinates are used. This allows the optimization to be done without constraints on vector length.

$$u_x = \sin(\phi)\cos(\theta) \tag{1}$$

$$u_y = \sin(\phi)\sin(\theta) \tag{2}$$

$$u_z = \cos(\phi) \tag{3}$$

Therefore, two angles can describe any unit vector in 3 space.

III. Ray-plane intersection

Laser ray r is defined by a point and a vector for some interval of λ. For some interval $\lambda \in (a, b)$ the finite line describes the light ray in space.

$$r\colon \bar{x}_0 + \lambda\bar{v} \tag{4}$$

The mirror plane is defined also with a point and vector by the below equation.

$$m\colon (\bar{x} - \bar{p}_0)\cdot\bar{n} = 0 \tag{5}$$

The intersection between the line and the plane $\bar{y}$ is then given by (7).

$$s = \frac{\bar{p}_0\cdot\bar{n} - \bar{x}_0\cdot\bar{n}}{\bar{v}\cdot\bar{n}} \tag{6}$$

$$\bar{x}' = \bar{x}_0 + s\bar{v} \tag{7}$$

IV. Ray reflection

The final equation to generate the ray output from a mirror is calculated using the ray orientation and the normal vector. The calculation is to reflect $\bar{v}$ across $\bar{n}$ shown in (8).

$$\bar{v}' = \bar{v} - 2(\bar{v}\cdot\bar{n})\bar{n} \tag{8}$$

V. Serial ray generation

Using a ray and a plane, the output ray can be generated. This process can repeated from multiple mirrors. In this case there are only two mirrors defined by $\bar{n}_1, \bar{p}_1, \bar{n}_2, \bar{p}_2$. If the ray input is defined by $\bar{x}_0, \bar{v}_0$ then the following calculations will generate the output of the projector.

The output of m_1, r_1 is defined by the ray-plane intersection and ray reflection equations.

$$\bar{x}_1 = \bar{x}'(\bar{x}_0, \bar{v}_0, \bar{p}_1, \bar{n}_1) \tag{9}$$

$$\bar{v}_1 = \bar{v}'(\bar{v}_0, \bar{n}_1) \tag{10}$$

$$r_1\colon \bar{x}_1 + \lambda\bar{v}_1 \tag{11}$$

Applying the same equations to r_1 using m_2 generates r_2 in (14).

$$\bar{x}_2 = \bar{x}'(\bar{x}_1, \bar{v}_1, \bar{p}_2, \bar{n}_2) \tag{12}$$

$$\bar{v}_2 = \bar{v}'(\bar{v}_1, \bar{n}_2) \tag{13}$$

$$r_2\colon \bar{x}_2 + \lambda\bar{v}_2 \tag{14}$$

VI. Objective Formulation

The objective used is the average perpendicular distance from the point in space to the corresponding ray generated by the project with corresponding angle pairs.

The perpendicular distance between $\bar{x}_i$ and line r_i is given by (15).

$$d = norm\left((\bar{x}_i - \bar{x}_0) - \left((\bar{x}_i - \bar{x}_0)\cdot\bar{v}\right)\bar{v}\right) \tag{15}$$

The objective is then the average of d over all data points. Currently, the first optimization algorithm takes into account only 8 variables describing the projector. Until dimensions can be verified, these will be the only ones used.

$$M = \frac{1}{n}\sum_{i=1}^{n} d(k_1, b_1, k_2, b_2, \phi_1, \theta_1, \phi_2, \theta_2) \tag{16}$$

VII. Initial conditions

Using an initial conditions generated from the “Explicit method to solving angle calibration functions to dual mirror galvonometers" the seed values for k_1, b_1, k_2, b_2