Description of Objective Functions

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***Abstract*— From discussion with AGS, the accuracy of the inverse algorithm to compute DAC commands from 3D coordinates is of critical importance. Thus, ensuring that the objective is created properly to result in good inverse performance and precision is necessary. However, calculation speed is also an important consideration as implementations may become impractical if this important element is neglected. The methods in this paper are all in different regions of the speed-precision spectrum.**

I. Introduction

The underlying problem of generating an objective that uses errors in DAC commands upon inversion of coordinates is the speed. The methods that are described all have different accuracy and speed characteristics when used as an objective and are used in different stages in the solving process.

II. Ray-Distance Method

This was the first objective used when building the projector model. The perpendicular distance between a modeled ray and a point was used to quantify the model performance. The DAC inputs would be used to generate the ray, and the point is measured so the distance can be computed.

The main advantage of this technique is that computing the resultant ray from an input-ray and two reflections are extremely fast. However, using the resultant model to generate DAC commands was not consistent. The root of the problem results from the fact that the ray position and orientation cannot be measured via the current procedure. Thus, simply because a mathematical ray is close to a point, does not mean the inverse function generates very accurate DAC commands

$$r = p \;\; DAC_1, DAC_2 \;\; + \lambda u \;\; DAC_1, DAC_2 \qquad (1)$$
$$\lambda \in (0, \infty)$$

III. Ray-Equation Method

In order to generate two DAC commands with XYZ coordinates, three equations with two unknowns must be solved. This is summarized by the vector equation in Eq. 2 which is equivalent to Eq. 3.

$$p + \|x - p\|u = x \qquad (2)$$

$$f = p + \|x - p\|u - x = \mathbf{0} \qquad (3)$$

Instead of varying DAC commands, model commands are varied in attempts to solve the equation. The computed vector norm of f during optimization measures how well the model fits the data. The number also represents the Euclidean distance between the model point and the measured point.

Thus, minimizing the sum of squared norms with respect to projector parameters results in better DAC agreement between the data and model.

$$\min \sum_i^n \|p_i + \|x_i - p_i\|u_i - x_i\|^2 \qquad (4)$$

IV. DAC Error Method

This is the most direct method of optimization with respect to an important objective. This objective uses the projector model parameters to invert positions to DAC commands and compares the sum of squared DAC errors. Using a linearization of the projector model with respect to the DAC commands the over determined system can be solved.

Thus we solve the vector equation in Eq. 5.

$$f_i = p_i + \|x_i - p_i\|u_i - x_i = 0 \qquad (5)$$

The solution is represented by Θ_i.

$$\Theta_i = \left(DAC_{1s_i}, DAC_{2s_i}\right) \qquad (6)$$

Let the DAC commands from the calibration data be represented by Γ_i.

$$\Gamma_i = \left(DAC_{1_i}, DAC_{2_i}\right) \qquad (7)$$

Thus, we select the parameters to solve Eq. 8.

$$\min \sum_i^n \|\Theta_i - \Gamma_i\|^2 \qquad (8)$$

V. Conclusion

The three objectives all have the inherent tradeoff between accuracy and speed. The Ray-Distance method is the fastest and the least accurate, while the DAC Error Method is the slowest and most accurate. In practice, it is helpful to start with the lower accuracy methods and quickly switch to the higher accuracy methods. This results in good error performance overall and reasonable solve time.