Ray equation solver

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***Abstract*— In order to generate DAC commands for the projector three equations are solved for two unknowns. In this implementation, Newton's method is used.**

I. Linearization

The projector model generates the point on the second mirror from which the ray exits the projector as well as the direction of the ray.

This gives rise to three equations when solved generate a theoretical intersection of the ray and the point. Below it is represented by a vector equation.

$$f(x_{dac}, y_{dac}) = x + \|x_i - x\|u - x_i = 0 \tag{1}$$

$$f = [f_1 \quad f_2 \quad f_3]^T \tag{2}$$

Both x and u are both functions of the projector geometry and the DAC angles. We wish to find the DAC commands that best satisfies Equation 1.

Linearizing with respect to the DAC angles allows us to calculate the solution very quickly. In practice, numerically differentiating using a forward difference with a step of 1 DAC count is sufficient. In short, we wish to find the derivative matrix $\boldsymbol{D}$ to calculate the solution of Equation 4.

$$\theta_{dac} = \left[x_{dac_0}, y_{dac_0}\right]^T \tag{3}$$

$$f(\theta_{dac_0}) + \boldsymbol{D}(\theta_{dac_0})(\theta_{dac_1} - \theta_{dac_0}) = 0 \tag{4}$$

$$\boldsymbol{D} = \begin{bmatrix} \dfrac{\partial f_1}{\partial x_{dac}} & \dfrac{\partial f_1}{\partial y_{dac}} \\ \dfrac{\partial f_2}{\partial x_{dac}} & \dfrac{\partial f_2}{\partial y_{dac}} \\ \dfrac{\partial f_2}{\partial x_{dac}} & \dfrac{\partial f_3}{\partial y_{dac}} \end{bmatrix} \tag{5}$$

II. Numerical calculation

To calculate the derivative matrix with respect to x_{dac}, y_{dac} we simplify evaluate the function f three times.

Function evaluations:

$$f(x_{dac_0}, y_{dac_0})$$
$$f(x_{dac_0} + 1, y_{dac_0})$$
$$f(x_{dac_0}, y_{dac_0} + 1)$$

$$D_x = f(x_{dac_0} + 1, y_{dac_0}) - f(x_{dac_0}, y_{dac_0}) \tag{6}$$

$$D_y = f(x_{dac_0}, y_{dac_0} + 1) - f(x_{dac_0}, y_{dac_0}) \tag{7}$$

$$\boldsymbol{D}(\theta_{dac_0}) \approx [D_x \quad D_y] \tag{8}$$

Then we simply solve the overdetermined system for the increment.

Let

$$\boldsymbol{D}(\theta_{dac_0}) \approx [D_x \quad D_y] \tag{9}$$

$$d\theta = (\theta_{dac_1} - \theta_{dac_0})$$

Solve

$$\boldsymbol{D}(\theta_{dac_0}) \approx [D_x \quad D_y] \tag{10}$$

$$\boldsymbol{D}(\theta_{dac_0})d\theta = -f(\theta_{dac_0})$$

Iterate

$$\boldsymbol{D}(\theta_{dac_0}) \approx [D_x \quad D_y] \tag{11}$$

$$\theta_1 = \theta_0 + d\theta$$

In practice, this takes 1 iteration to converge for the nominal projector model parameters within 1 DAC count. However, 2 or 3 iterations is recommended.