Quaternion solver verification

Owen Lu
Electroimpact Inc.
owenl@electroimpact.com

***Abstract*—In order to solve for the projector parameters, the extrinsic variables of the model must be consistently generated. If they cannot be consistently generated, then the model parameters will be compensated in order to adjust for the transforms. This creates a model that is only valid for the calibration data set. The main idea behind this concept is that if the projector is not moved relative to the tracker, the transform associated must stay constant no matter which data set is used.**

I. Data Collection

Using the methods described in "**Projector Calibration Procedure Verification**" a data set was generated with 3 different wall depths using a movable wall. This allowed the projector to stay in a constant position.

II. Hypothesis

During the experiment the projector was unmoved. Thus, the transform describing its orientation should be constant no matter which data set was used. It should be possible to use an optimization method to consistently generate quaternions that agree within a small error on each data set. This assumes that the nominal projector model is estimated to reasonable accuracy.

Suppose that q_i represents the quaternion that minimizes the error for each data set. Then since 3 data sets are present we expect that all quaternions are approximately equal.

Hypothesis 1:

$$q_1 \approx q_2 \approx q_3$$

The data used will be from the set transformed into projector coordinates via metrology, we expect that the solved quaternion has real part nearly 1.

Hypothesis 2:

$$\Re(q_{1,2,3}) \approx 1$$

This statement is equivalent to saying the rotation matrix is approximately the identity.

III. Solver Procedure

In order to estimate the orientation, the galvanometers must be calibrated. The method used was the method detailed in "**Explicit method to solving angle calibration**". This method uses a nominal model to solve the ray equations for angular configurations of both mirrors and performs a robust linear curve fit to map from DAC to mirror rotations.

After this is done, the quaternions are solved by minimizing the Ray Equation objective followed by the DAC Error described in "**Description of Objective Functions**".

To check whether galvanometer calibration has a substantial effect on the resultant quaternions, galvanometer calibrations were done on each data set, and the quaternion subsequently solved. Following are the steps clearly listed.

1. Galvanometer calibration using set 1
2. Quaternion solve on sets (1,2,3)
3. Repeat steps 1-2 using data sets 2 and 3 to calibrate galvanometers

This results in 9 quaternions total.

IV. Quaternion Results

The direct results of for the quaternions is given below.

Mean Value

$$mean\ q\ = [1, 0.0000158, 0.000322, -0.00081]$$

Standard Deviation

$$std\ q\ = 10^{-4} \cdot [0.00029, 0.87751, 0.60271, 0.433548]$$

There is agreement with Hypothesis 1 that all the quaternions are approximately equal as the standard deviation is on the order of 10^{-5}. The data also shows agreement with Hypothesis 2 since the quaternion is almost purely real.

V. Angular Deviance Results

Although the quaternions superficially look feasible, it is instructive to look at the relative angle between the quaternions and the real quaternion. Using the equation below, the geodesic angle between quaternions can be calculated.

$$\theta = \operatorname{acos}(2\ q \cdot q_0\ - 1)$$

$$q_0 = [1,0,0,0]$$

Mean angular deviance

$$mean\ \theta\ = 0.0715^\circ$$

Standard deviation of angular deviance

$$std\ \theta\ = 0.00263^\circ$$

The result shows that the quaternions are estimated within 3 millidegrees which is sufficient.