Filtered proportional controller

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***Abstract*—Previously, it was shown that P and PI control could be optimized for simple and consistent operation of the servo creel system. However, both control schemes have the shortcoming of high peak torque due to step velocity response. This new method looks to reduce the peak torque induced by changing the form of controller to a first order filter.**

I. Introduction

Proportional control on the servo creel system has been shown to be a very effective way to control dancer position through empirical testing. There are a few key strengths that are properties of the P controller and a couple weaknesses.

Pros:

1. Guaranteed zero overshoot if current saturation does not occur
2. Larger dancer displacements at larger speed reducing motor requirements
3. Simple implementation and prediction of response
4. Easy tuning based on dancer stroke and motor capabilities

Cons:

1. High torque peaks during step response
2. Tension variation with speed

The goal of this paper is to perform optimization around peak torque reduction during operation with minimal additions to the control scheme.

II. S-Domain Controller Solve

In order to generate a controller, the response of the input velocity to the output velocity was set to be a second order. This is due to the fact that second-order systems have an initial slope of zero, allowing torque to continuously change instead of jump. Recall in the previous controller design paper Equation (1) and (2) which define the characteristics of the plant.

Where
v_o is the feed velocity
v_i is the tow surface velocity leaving the spool
x_d is the dancer displacement from the initial position

The system dancer response can then be written in Laplace domain as shown in Equation (1).

$$x_d\ s = \frac{v_o\ s - v_i\ s}{2s} \quad (1)$$

$$G\ s = \frac{1}{2s} \quad (2)$$

The transfer function between v_0 and v_i is a function of the controller structure. Setting this to a second order allows the controller to be solved in the Laplace domain.

$$\frac{v_i}{v_o} = \frac{G}{CG+1} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (3)$$

Solving for C gives the following control structure

$$C\ s = \frac{\omega_n}{\zeta} \cdot \left(\frac{1}{\frac{1}{2\zeta\omega_n} s + 1} \right) \quad (4)$$

$$\tau = \frac{1}{2\zeta\omega_n} \quad (5)$$

Since we want the behavior to be responsive without overshoot, simply set $\zeta = 1$. Now in order to get long term performance similar to the proportional method the steady state gains are set based on proportional control scheme calculations.

Therefore $\frac{\omega_n}{\zeta} = K_p r_s$ in steady state.

Let $\zeta = 1$
Let $K_e = K_p r_s$

$$\omega_n = K_e \quad (6)$$

This means that the steady state displacement is completely determined by the quantity ω_n since the damping ratio is set to $\zeta = 1$.

III. Simulation

In order to see the percentage improvement, we compare the proportional controller with gain $K_e = 25rads^{-1}$ with the filtered controller corresponding to parameters $\omega_n = K_e$ and $\zeta = 1$. In Figure 1 a plot of the simulated dancer and torque response is shown.

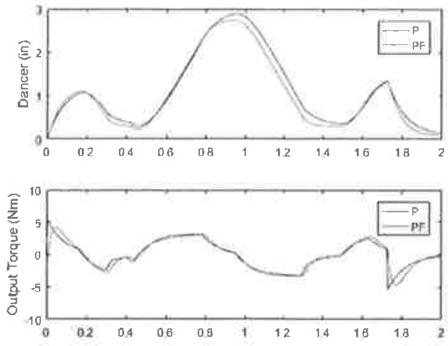

Figure 1 – Simulation of PF response vs. P response

Clearly, the torque ramps up to the maximum, and the maximum torque is also reduced when a spar simulation was conducted. In this case, the maximum torque was reduced by 22% with basically no significant effect on the dancer response as expected. Notably, the peak torques occur when a step velocity input occurs as expected.

IV. Overshoot-Peak Torque trade-off

When designing this type of system there is a need to understand the tradeoffs between certain parameters in the system. A filter is applied to the P controller in this case allowing the torque to ramp up slowly during a step velocity output. Intuitively, the larger the time constant of the filter, the slower the ramp and the lower the peak torque.

However, since the control structure has the damping ratio in the time constant expression in Equation (5), there are serious consequences of raising the time constant to reduce peak torque. The filter time constant is directly related to both the natural frequency and the damping ratio. Supposing that the steady-state controller gain is specified this gives the following expression for the time constant in terms of the damping ratio in Equation (8)

$$K_e = \frac{\omega_n}{\zeta} \tag{7}$$

$$\tau = \frac{1}{2K_e\zeta^2} \tag{8}$$

Therefore, the damping ratio can be expressed as proportional to the inverse square root of the time constant.

$$\zeta = \sqrt{\frac{1}{2K_e\tau}} \tag{9}$$

In Equation (9) the effect of larger time constants is the reduction of damping and therefore increase of overshoot. It is possible that some overshoot can be tolerated in the interest of torque reduction. In the interest of testing, we can easily calculate the damping ratio corresponding with a maximum overshoot we are willing to accept. From experience, roughly 2% is reasonable since there are oscillations and safety margins in the system.

$$\zeta = \sqrt{\ln 0.02^{2} / (\pi^2 + \ln 0.02^{2})} = 0.78$$

In this case $\omega_n = 19.5rads^{-1}$ is adjusted while $K_e = 25rads^{-1}$. Although only 2% overshoot results with this damping ratio, the peak torque is reduced by 30% compared to the original P control scheme and 10% compared to the critically damped PF scheme.

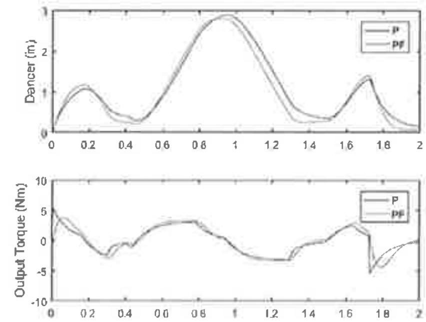

Figure 2 – Simulation of under-damped PF response vs. P response with $\zeta = 0.78$

It should be noted that even if the dancer was arbitrarily long, the peak torque can only be reduced to the peak torque caused by the acceleration ramp. This is due to the fact that in steady state, the accelerations will be matched. Thus, it is meaningless to decrease ζ below values of around 0.7 in any situation.

V. Implementation

In order to implement a filter on the controller, the sampling time T_s must be known. A bilinear approximation can easily be calculated to filter with any time constant. Therefore, the velocity command v_k can be written as function of previous values and the current error measurement in Equation (11).

$$s \approx \frac{2}{T_s}\left(\frac{z-1}{z+1}\right) \tag{10}$$

$$v_k = \frac{K_e(x_{e_k} + x_{e_{k-1}}) - \left(1 - \frac{2\tau}{T_s}\right) v_{k-1}}{\tau_s(1 + \frac{2\tau}{T})} \tag{11}$$

VI. Conclusion

It has been shown that smoothing the proportional controller can greatly affect the peak torque induced by the system. Reductions of up to 30% can be realized with minimal impact on the dancer response characteristics with a relatively simple implementation and the addition of one extra parameter. Testing must be done to see whether a controller based on a difference equation can be used to smooth torque in practice.