Replication of Structural Estimations in Shapiro and Walker (2018, AER) *

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Abstract

This note covers the model used in Shapiro and Walker (2018): Why is pollution from US manufacturing declining? The roles of environmental regulation, productivity, and trade. *American Economic Review*, 108(12), 3814-3854. I summarize the equilibrium equations that are crucial for quantification and analyze the counterfactual. Several replications and extensions are conducted to illustrate how the model explains changes in pollution emissions. I show detailed derivations and include MATLAB code in the appendix.

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1 — Introduction to Shapiro and Walker (2018)

Asking what factors have driven the reduction of pollution emissions in the US, Shapiro and Walker (2018) develop a model to account for international trade, product composition, and environmental regulation. The model extends Melitz (2003) monopolistic competition by incorporating bilateral international trade and solves the counterfactual utilizing hat algebra. Shocks of international trade are characterized by iceberg trade costs and fixed costs of entering foreign markets; shocks of production composition are characterized by the expenditure share of each sector; shocks of environmental regulation are characterized by implicit pollution taxes.

The next section provides a sketch of the model. Section 3 presents the data and algorithm used for quantification and counterfactual analysis. Section 4 shows the replication results and some extensions. I also refer readers to Li and Zhu (2025), who also utilize the model under a Chinese context. The appendix includes detailed derivations and MATLAB code.

2 — Sketch of the Model

In the following, I list the basic elements comprising the model; detailed derivations can be found in the appendix.

2.1. Preference

A representative consumer in country d has a utility function with two layers. In the first layer, consumers source products within a sector according to a constant elasticity of substitution (CES) preference. In the second layer, consumers allocate a share $\beta_{d,s}$ of their total expenditure across sectors according to a Cobb–Douglas preference.

$$U_d = \prod_s \left[\left(\sum_o \int_{\omega \in \Omega} q_{od,s}(\omega)^{\frac{\sigma_s - 1}{\sigma_s}} \right)^{\frac{\sigma_s}{\sigma_s - 1}} \right]^{\beta_{d,s}}$$

We omit the variety index ω for simplicity. The common conclusions of CES preferences apply to within-sector product consumption.

$$q_{od,s} = \frac{p_{od,s}^{-\sigma_s}}{P_{d,s}^{1-\sigma_s}} E_{d,s}$$

where $P_{d,s}$ is the Dixit-Stigliz price index, and $E_{d,s}$ is the total expenditure in sector s:

$$P_{d,s} = \left[\sum_o \int_\Omega p_{od,s}^{(1-\sigma)} \right]^{\frac{1}{1-\sigma_s}}$$

Comment: $\beta_{d,s}$ is useful in the follow-up derivation. Common conclusion of C-D preference applies such that $E_d = \sum_s \beta_{d,s} \cdot E_{d,s}$ and $\sum_s \sigma_s = 1$.

2.2. Production

In SW(2018), the production technology follows Taylor and Copeland (2003), where labor is the only input, pollution is a by-product, and a firm can allocate some labor to pollution abatement. The market structure follows Melitz (2003) in a monopolistic competition framework with firm entry and exit depending on productivity drawn from a Pareto distribution.

2.2.1. Technology

Firm produces using labor $l_{od,s}$, and a proportion of $0 \geq a < 1$ of the employment is allocated to pollution abatement:

$$q_{od,s} = (1-a)\phi l_{od,s}$$

where ϕ is a productivity draw.

Pollution emissions $z_{od,s}$ follows certain form similar to production:

$$z_{od,s} = \nu(a)\phi l_{od,s}$$

where $\nu(a)$ means that the actually abatement is a function of abatement labor input. Following Taylor and Copeland (2003) to make $\nu(a)$ has economic meaning, let $\nu(a) = (1-a)^{\frac{1}{\alpha_s}}$, so that the production function can be written as a composite of labor and pollution inputs.

$$q_{od,s} = (z_{od,s})^{\alpha}_{s}(\phi l_{od,s})^{1-\alpha_s}$$

where α_s is interpreted as the elasticity of substitution between labor and pollution "input".

2.2.2. Market Structure

In the monopolistic competition, firms make entry decision before production. There is a mass of firm attempting entry $M^e_{o,s}$, they have to pay sunk cost $f^e_{o,s}$ to draw a productivity ϕ from Pareto distribution $G(\phi, b_{o,s})$,

$$G(\phi, b_{o,s}) = 1 - \left(\frac{\phi}{b_{o,s}}\right)^{-\theta_s}$$

where $b_{o,s}$ and θ_s represent the location and shape parameter of Pareto distribution.

Comment: $b_{o,s}$ is country-sector specific, and $-\theta_s$ is sector specific.

2.2.3. Profit Function

If a firm decides to produce and export, it pays wages w_o, pollution tax $t_{o,s}$, fixed costs of entry foreign/domestic market $f_{od,s}$, and faces iceberg trade costs $\tau_{od,s}$. Its profit function writes as

$$\pi_{o,s} = \sum_{d} \pi_{od,s} - w_o f^e_{o,s}$$

$$\pi_{od,s} = p_{od,s}q_{od,s} - w_o l_{od,s}\tau_{od,s} - t_{o,s}z_{od,s}\tau_{od,s} - w_d f_{od,s}$$

2.3. Equilibrium

The general equilibrium is characterized by consumer utility maximization, firms choosing optimal abatement shares to maximize profits upon entry, and labor-market clearing.

1. Firms choose an optimal abatement share that balances labor costs and pollution taxes given productivity draws.

$$1 - a = \left(\frac{w_o}{\phi t_{o,s}}\frac{\alpha_s}{1-\alpha_s}\right)^{\alpha_s}$$

2. In monopolistic competition, the expected profits of entrants must cover the sunk entry costs,

which defines a free-entry condition relating expected revenue and costs.

$$\frac{1-\alpha_s}{\theta_s}\frac{\sigma_s-1}{\sigma_s}R_{o,s} = w_o f^e_{o,s} m^e_{o,s}$$

3. Total labor supply L_d equals total labor demand used up in the following five purposes:
   - paying fixed costs of entry $L^e_{o,s} = f^e_{o,s} M^e_{o,s}$
   - production and pollution abatement $L^p_{o,s} = \sum_d M^e_{o,s} \text{Pr}(\phi > \phi^\star)\mathbf{E}[l_{od,s}\tau_{od,s}|\phi > \phi^\star]$, where $\phi^\star$ is the cutoff productivity satisfying zero-profit condition.
   - paying pollution tax $L^t_{o,s} = \frac{t_{o,s}}{w_o} Z_{o,s}$
   - paying foreign/domestic market entry cost $L^f_{o,s} = \sum_d M_{od,s} f_{od,s}$ where $M_{od,s}$ is the mass of successful entrants exporting to country d.
   - paying for net exports $L^{NX}_{o,s}$, which is a defined term according to the trade imbalance of $NX_d = R_d - E_d$

   such that

$$L_d = L^e_d + L^p_d + L^t_d + L^f_d + L^{NX}_d = \frac{R_d}{w_d}$$

Comment: For the ease of derivation and exposition, tax revenue is assumed be used for rent-seeking so that it requires labor resources.

3 — Data, Equilibrium, and Algorithm for Estimation

3.1. Data

Main data used for structural estimation is country-sector-year trade data and pollution data in the US[1]. The data consists of two countries, US and the rest of the world (ROW), seventeen sectors, and nineteen years from 1990 to 2008. Bilateral trade data has been normalized to have a sum of one in each year to have an economic meaning.

The main data used for structural estimation are country–sector–year trade data and pollution data in the United States. The dataset consists of two countries (the US and the rest of the world), seventeen sectors, and nineteen years from 1990 to 2008. Bilateral trade data are normalized so

1. Since the focal country is US, and counterfactual pollution emissions are determined by changes in wages, the mass of firms, and pollution tax, we do not need pollution data in the rest of the world

that the total sum equals one in each year to maintain economic meaning[2]..

Denoting bilateral trade as $X_{od,s}$, where o represents origin, d represents destination, and s represents sector, we calculate several variables used for estimation.

Sector s's revenue R_{os} and a country total revenue R_o:

$$R_{o,s} = \sum_d X_{od,s}$$
$$R_o = \sum_s R_{os}$$

Sector s's expenditure E_{ds} and a country total expenditure E_d:

$$E_{d,s} = \sum_d X_{od,s}$$
$$E_d = \sum_s E_{ds}$$

Sector s's *net export* NX_{ds} and a country total expenditure NX_d:

$$NX_{d,s} = R_{d,s} - E_{d,s}$$
$$NX_d = R_d - E_d$$

The proportion of a sector's expenditure share in total expenditure $\beta_{d,s}$:

$$\beta_{d,s} = \frac{E_{ds}}{E_d}$$

The proportion of a sector's import share in total expenditure $\lambda_{od,s}$:

$$\lambda_{od,s} = \frac{X_{od,s}}{E_d}$$

2. Since the focal country is US, and counterfactual pollution emissions are determined by changes in wages, the mass of firms, and pollution tax, we do not need pollution data in the rest of the world

The proportion of a sector's export share in total revenue $\zeta_{od,s}$:

$$\zeta_{od,s} = \frac{X_{od,s}}{R_o}$$

The model's structural parameters $\{\alpha_s, \theta_s, \sigma_s\}$ are calibrated using US firm-level manufacturing data and are assumed constant over the nineteen years.

Total labor endowment L_o is assumed to one and no changes across all years.

Using *hat algebra* to denote changes,

$$\hat{x} \equiv \frac{x'}{x}$$

where x' is the counterfactual value.

Since we have nineteen years data, we assume x the base-year (1990) value, and in other years, x' are equilibrium values under all three shocks: foreign competition, US competition $\hat{\Gamma}_{od,s}$, environmental regulation $\hat{t}_{o,s}$, and production composition$\hat{\beta}_{d,s}$.

Using the hat algebra and the definition of wage $w_o \equiv \frac{R_o}{L_o}$, changes in wage $\hat{w}_o$ equals changes in a country's total revenue $\hat{R}_o$.

Using Equation (11) and Equation (15) in the main text of SW(2018), we can also recover two other variables, changes in the mass of firms attempting entry and changes in implicit pollution taxes from observed pollution emission changes.

$$\hat{M}^e_{o,s} = \frac{\hat{R}_{o,s}}{\hat{w}_o}$$

$$\hat{t}_{o,s} = \frac{\hat{M}_{o,s}\hat{w}_o}{\hat{Z}_{o,s}} \tag{1}$$

where $\hat{Z}_{o,s}$ is changes in pollution emissions observed in the data.

Comment: Historical value of $\{\hat{t}_{o,s}$ can be recovered using the second equality.

3.2. Equilibrium

Given exogenous parameters and model parameters, the equilibrium is characterized by consumer utility maximization, firm's profit maximization, and labor market clearing. Firm's profit maximization and labor market clearing in hat algebra form Equation (3) and Equation (2), respectively.

For simplicity, define two sector-level constants:

$$\Xi_s \equiv \frac{\theta_s - (\sigma_s - 1)(1 - \alpha_s)}{\sigma_s \theta_s}$$
$$\Theta_s \equiv \frac{(\sigma_s - 1)(\theta_s - \alpha_s + 1)}{\sigma_s \theta_s}$$

The equilibrium in counterfactual is that given the combination of changes or historical shocks $\{\hat{\Gamma}_{od,s}, \hat{t}_{o,s}, \hat{\beta}_{d,s}\}$, solve $\{\hat{w}_o, \hat{M}^e_{o,s}\}$ from the following system of non-linear equations [3]:

$$1 - \frac{1 - \sum_s \beta_{o,s}}{1 - \sum_s \Theta_s \hat{\beta}_{o,s}\beta_{o,s}} \cdot \frac{\sum_s \hat{M}^e_{o,s} R_{o,s} \Xi_s + \eta'_o}{\sum_s R_{o,s}\Xi_s + \eta_o} = 0 \tag{2}$$

$$\hat{w}_o - \sum_d \frac{\zeta_{od,s}\hat{w}_o^{-\theta_s}\hat{\Gamma}_{od,s}}{\sum_o \lambda_{od,s}\hat{w}_o^{-\theta_s}\hat{\Gamma}_{od,s}} \cdot \hat{\beta}_{d,s} \cdot \frac{\hat{w}_d R_d - \hat{NX}_d NX_d}{R_d - NX_d} = 0 \tag{3}$$

where

$$\hat{\Gamma}_{od,s} \equiv \left(\frac{1}{\hat{\beta}_{o,s}}\right)^{-\theta_s} (\hat{\tau}_{od,s})^{-\frac{\theta_s}{1-\alpha_s}} \left(\hat{f}_{od,s}\right)^{1-\frac{\theta_s}{(\sigma_s-1)(1-\alpha_s)}} \left(\hat{t}_{o,s}\right)^{-\frac{\alpha_s\theta_s}{1-\alpha_s}}$$
$$= \frac{\hat{\lambda}_{od,s}}{\hat{M}^e_{o,s}\hat{w}_o^{-\theta_s}} \left(\frac{\hat{\beta}_{d,s}}{\hat{w}_d} \frac{\hat{R}_d R_d - \hat{NX}_d NX_d}{R_d - NX_d}\right)^{1-\frac{\theta_s}{(\sigma_s-1)(1-\alpha_s)}}$$
$$\eta_o \equiv \sum_s (1 - \Xi_s)\beta_{o,s} NX_o - \Theta_s NX_{o,s}$$
$$\eta'_o \equiv \frac{1}{\hat{w}_o} \sum_s (1 - \Xi_s)\hat{\beta}_{o,s}\beta_{o,s}\hat{NX}_o NX_o - \Theta_s \hat{NX}_{o,s} NX_{o,s}$$

Comment: Historical value of $\{\hat{\Gamma}_{od,s}$ can be recovered using the second equality, as well as

3. These two equations are Equation (12) and (13) in the main text of SW(2018). $\hat{NX}_o$ and $\hat{NX}_{o,s}$ are assumed to one, suggesting no changes of shocks in net exports.

the following definition of foreign/US competition. Equation (2) forms a system of N (country) equations, and Equation (3) forms a system of $N \times J$ (country-sector) equations.

Additionally, SW(2018) defines shocks of foreign and US competition as follows. When the counterfactual is foreign competition,

$$\hat{\Gamma}_{od,s}^{\text{Foreign competition}} = \hat{\Gamma}_{od,s}, o \neq \text{US}$$
$$\hat{\Gamma}_{od,s}^{\text{Foreign competition}} = 1, o = \text{US}$$

When the counterfactual is US competition,

$$\hat{\Gamma}_{od,s}^{\text{US competition}} = 1, o \neq \text{US}$$
$$\hat{\Gamma}_{od,s}^{\text{US competition}} = \hat{\Gamma}_{od,s} \cdot \left(\hat{t}_{o,s}\right)^{\frac{\alpha_s \theta_s}{1-\alpha_s}}, o = \text{US}$$

When the counterfactual is US environmental regulation,

$$\hat{\Gamma}_{od,s}^{\text{Regulation}} = 1, o \neq \text{US}$$
$$\hat{\Gamma}_{od,s}^{\text{Regulation}} = \left(\hat{t}_{o,s}\right)^{-\frac{\alpha_s \theta_s}{1-\alpha_s}}, o = \text{US}$$

Note that the real shocks $\{\hat{\Gamma}_{od,s}, \hat{t}_{o,s}, \hat{\beta}_{d,s}\}$ relative to the base-year 1990 are observed in the data, and *counterfactual* means that *changing* only one shocks of $\{\hat{\Gamma}_{od,s}, \hat{t}_{o,s}, \hat{\beta}_{d,s}\}$ to the real data and setting the rest of them to one (i.e., the same relative to the base-year) to see how that shock affects the trending of $\{\hat{w}_o, \hat{M}_{o,s}^e\}$ and thus pollution emissions $\hat{Z}_{o,s}$ in years after 1990.

3.3. Algorithm

For N countries, each with J sectors, Equation (2) and Equation (3) have N unknown changes in wage $\hat{w}_o$ and $N \times J$ unknown changes in masses of firms $\hat{M}_{o,s}^e$. When solving the model, change in foreign wage is excluded as the numeria, which is solved by the normalization that the sum of two countries' revenue equals one in every year:

$$
\begin{aligned}
&\frac{R'_{\text{US}} + R'_{\text{Foreign}}}{R_{\text{US}} + R_{\text{Foreign}}} = 1 \\
\Rightarrow &\frac{\hat{R}_{\text{US}} R_{\text{US}}}{R_{\text{US}} + R_{\text{Foreign}}} + \frac{\hat{R}_{\text{Foreign}} R_{\text{Foreign}}}{R_{\text{US}} + R_{\text{Foreign}}} = 1 \\
\Rightarrow &\hat{w}_{\text{US}} \frac{R_{\text{US}}}{R_{\text{US}} + R_{\text{Foreign}}} + \hat{w}_{\text{Foreign}} \frac{R_{\text{Foreign}}}{R_{\text{US}} + R_{\text{Foreign}}} = 1
\end{aligned}
$$

The last equality utilizes $\frac{L'_d}{L_d} = \frac{\hat{R}_d}{\hat{w}_d} = 1$.

Algorithm 1 shows steps to calculate the counterfactual. SW(2018) exercises four counterfactual of $\{\hat{\Gamma}_{o,s}, \hat{t}_{o,s}, \hat{\beta}_{d,s}\}$:

1. Foreign competition: for $o =$ foreign countries, set $\hat{\Gamma}_{od,s}$ to $\hat{\Gamma}^{\text{Foreign competition}}_{od,s}$, set $\hat{\Gamma}_{od,s}$ for US and $\hat{\beta}_{d,s}$ to one.
2. US competition: for for $o =$ US, set $\hat{\Gamma}_{od,s}$ to $\hat{\Gamma}^{\text{US competition}}_{od,s}$, set $\hat{\Gamma}_{od,s}$ for foreign country and $\hat{\beta}_{d,s}$ to one.
3. Environmental regulation: for for $o =$ US, set $\hat{\Gamma}_{od,s}$ to $\hat{t}_{o,s}$, set $\hat{\Gamma}_{od,s}$ for foreign country to one, set $\hat{\beta}_{d,s}$ to one.
4. Product composition: set $\hat{\beta}_{d,s}$ to the historical real value, and set $\{\hat{\Gamma}_{od,s}\}$ to one.

Comment: In all counterfactual, $\hat{NX}_{d,s}$ and $\hat{NX}_d$ are set to one. Equation (2 shows that $\hat{t}_{o,s}$ is actually a part of $\{\hat{\Gamma}_{od,s}, \}$,

Comment: As you will see in the MATLAB code, for each year, $\hat{\Gamma}_{od,s}$ is an $N \times J$ (i.e., 2×17) matrix. If the second row represents US value, then,the first counterfactual means that set the value in the first row to $\hat{\Gamma}^{\text{Foreign competition}}_{od,s}$, while set the second row to one.

4 — Results and Extensions

The main replication results are shown in Figure 1 including seven pollutants, which corresponds to SW(2018) Figure 5 (the first six pollutants) and Panel B of Figure 7 (CO_2) emissions. For the first six pollutants of CO, NX_x, $PM_{2.5}$, PM_{10}, SO_2, and VOC_s, environmental regulation explains most of the decline in pollution emissions.

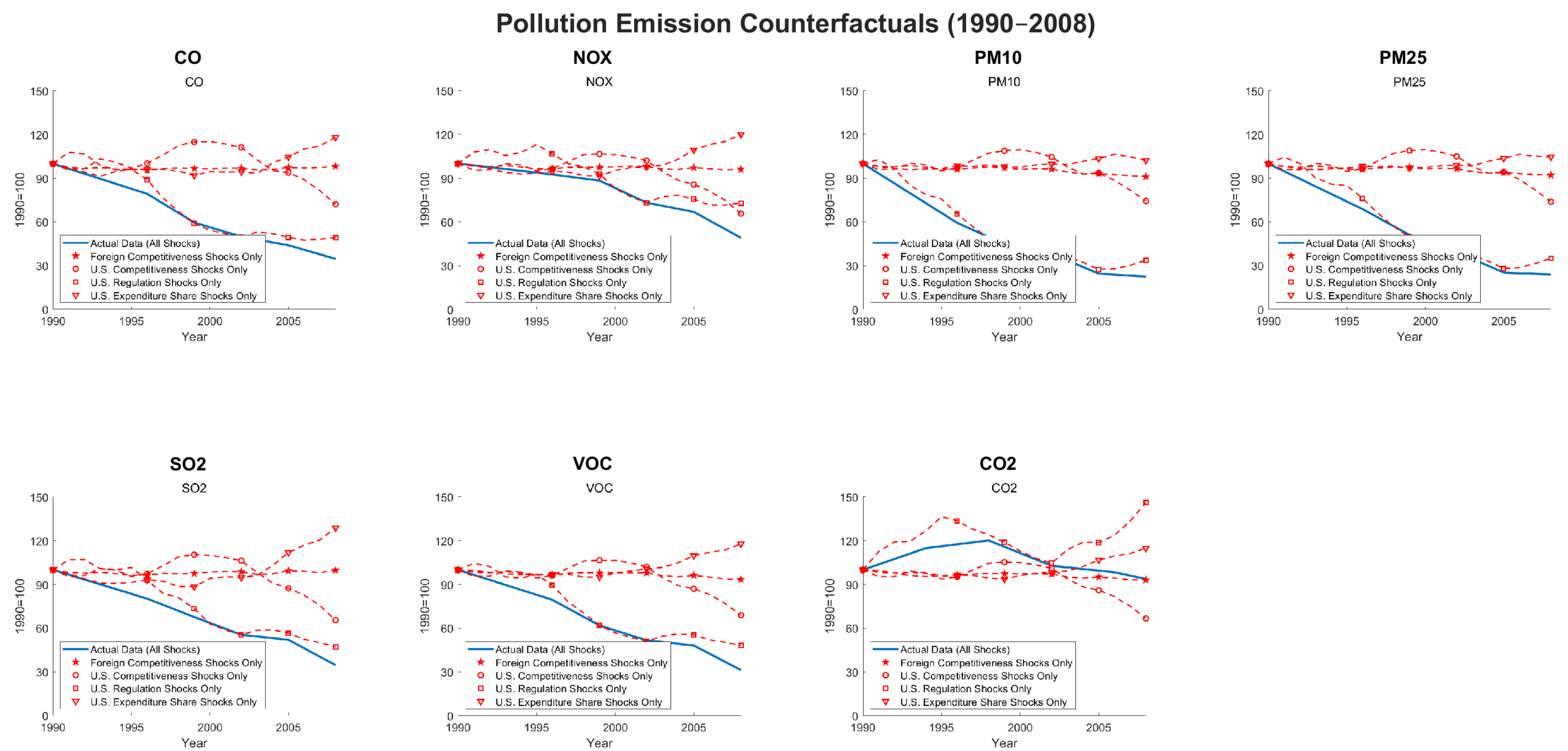

Figure 1 — Pollution Emission Counterfactual (1990–2008)

Notes. This figure replicates counterfactual Figure 5 in SW(2018) of seven pollutants. Value in 1990 has been normalized to 100, so that values in other years represent the relative change to 1990's.

Algorithm 1: Computing counterfactual pollution emissions

Data: Base-year production composition $\beta_{o,s}$, sectoral and country-level revenue $R_{o,s}, R_o$, sectoral and country-level net exports $NX_{o,s}, NX_o$, bilateral import share $\lambda_{od,s}$, bilateral export share $\zeta_{od,s}$. Structural parameters $\{\alpha_s, \theta_s, \sigma_s\}$.

Initialize shocks $\{\hat{\Gamma}_{od,s}, \hat{t}_{o,s}, \hat{\beta}_{d,s}\}$.

Initialize guesses of $\{\hat{w}_o, \hat{M}_{o,s}\}$.

Iterate $\{\hat{w}_o, \hat{M}_{o,s}\}$ until it satisfies both Equation (2) and Equation (3).

Calculate counterfactual pollution emissions $\hat{Z}_{o,s} = \frac{\hat{M}^e_{o,s}\hat{w}_o}{\hat{t}_{o,s}}$.

Result: Changes in wage, the mass of firms, and pollution emissions $\{\hat{w}_o, \hat{M}^e_{o,s}, \hat{Z}_{o,s}\}$

The second replication is the time trend of measurement of environmental regulation, "implicit pollution tax", $\hat{t}_{o,s}$. Figure 2 shows how the historical $\hat{t}_{o,s}$ for different pollutants, where $\hat{t}_{o,s}$ is calculated using Equation (1). Aligning with real trend of CO_2 emissions in Figure 1, implicit pollution tax for CO_2 emissions even has a decline in previous decades. However, pollution taxes for other pollutants have generally increased since 1990.

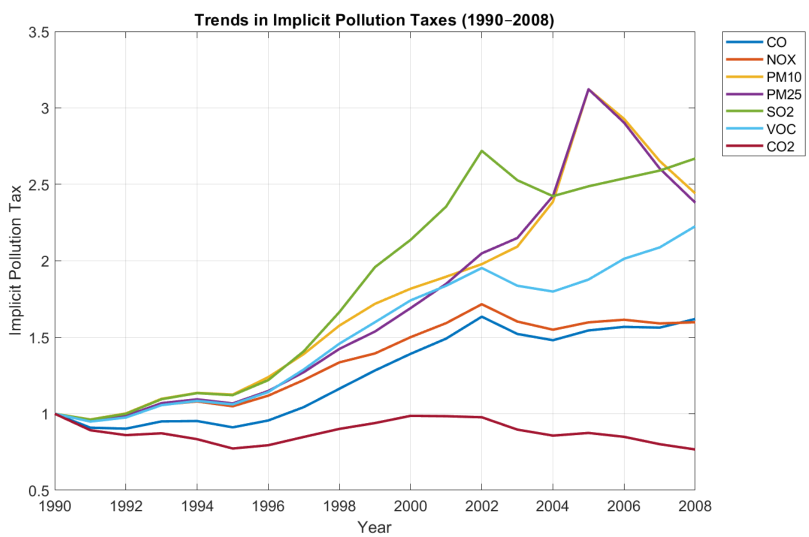

Figure 2 — Implicit Pollution Taxes (1990–2008)

Notes. This figure replicates Figure 4 in SW(2018) of implicit pollution taxes of seven pollutants. Value in 1990 has been normalized to one, so that values in other years represent the relative change to 1990's.

The third replication is the counterfactual of US international trade revenue (i.e, export) share in the world. Figure 3 shows that changes in environmental regulation (rectangle) and product composition (triangle) cannot explain the falling share of US export. Rather, both foreign and US competition account for most of the decline.

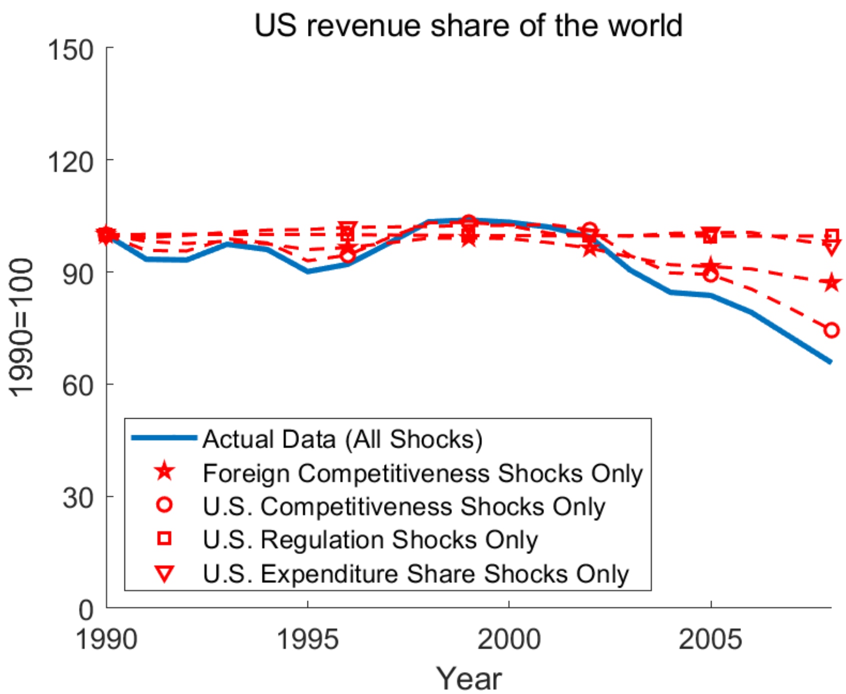

Figure 3 — US Export Share Counterfactual (1990–2008)

Notes. This figure shows how US export share changes under four different shocks. Value in 1990 has been normalized to 100, so that values in other years represent the relative change to 1990's.

The last replication extends the third replication by plotting changes in $\Gamma_{o,s}$ to explain why foreign and US competition explain falling US exports. Figure 4 shows continuously increasing foreign competitiveness relative to both foreign countries and the US, while Figure 5 shows that US competitiveness increased until around 2000 and then began to decline. While several factors contribute to these competitiveness changes — such as productivity, iceberg trade costs, fixed entry costs to foreign markets, and pollution taxes — the model does not separately identify the individual contributions of each component.

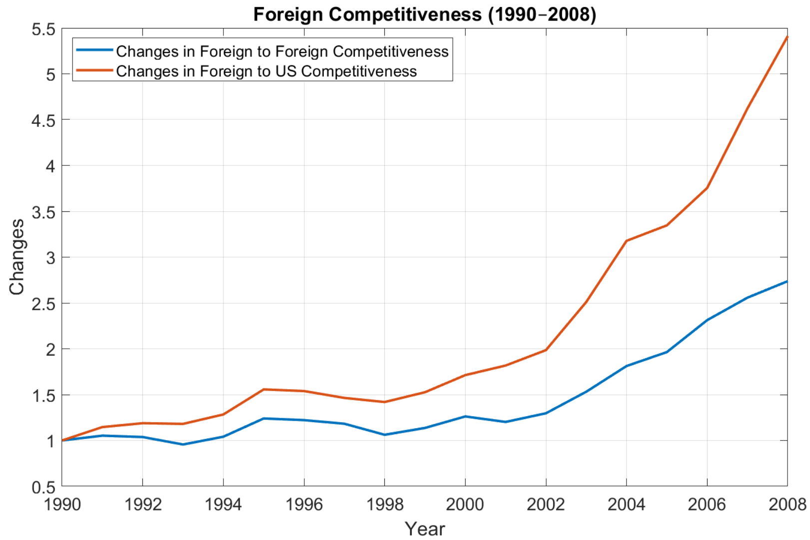

Figure 4 — Changes in Foreign Competitiveness (1990–2008)

Notes. This figure shows time trend of foreign competitiveness $\Gamma_o^{\text{Foreign Competitiveness}}$. Value in 1990 has been normalized to one, so that values in other years represent the relative change to 1990's.

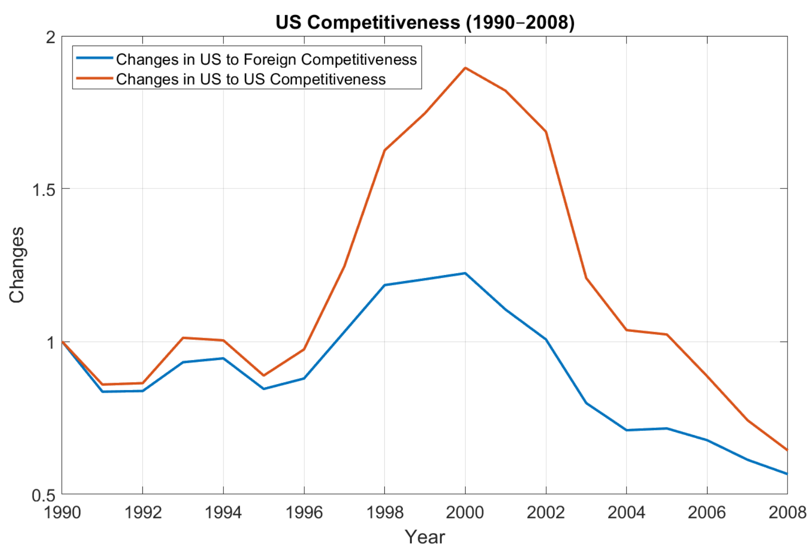

Figure 5 — Changes in US Competitiveness (1990–2008)

Notes. This figure shows time trend of US competitiveness $\Gamma_o^{\text{US Competitiveness}}$. Value in 1990 has been normalized to one, so that values in other years represent the relative change to 1990's.

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A — Appendix for Derivations

Notations

In SW(2018), subscript o denotes origin country, d denotes destination country, and s denotes sector.

Table A1 — Codebook for SW2018

SW2018	Description
	Structural parameters
α_s	pollution elasticity
$b_{o,s}$	location parameter of Pareto distribution
θ_s	shape parameter of Pareto distribution
σ_s	elasticity of substitution
	Exogenous characteristics
L_d	labor endowment normalized to one
$f^e_{o,s}$	fixed costs of drawing a productivity
$f_{od,s}$	fixed costs of entering a market d
	Endogenous characteristics and scalars
ϕ	Productivity draw
$l_{od,s}$	Labor demand
$z_{od,s}$	Pollution emissions
a	Share of labor allocated to pollution abatement
$q_{od,s}$	Output
$p_{od,s}$	Price
$c_{o,s}$	Variable cost
$w_{o,s}$	Wage
$t_{o,s}$	Pollution tax

Continued on next page

Table A1 (continued from previous page)

SW2018	Description
$\tau_{od,s}$	Iceberg trade cost
$\pi_{od,s}$	Profit
$X_{od,s}$	Bilateral trade share
$E_{d,s}$	Expenditure of sector s in country d
E_d	Expenditure of country d
$R_{o,s}$	Revenue of sector s in country o
R_o	Revenue of country o
$NX_{d,s}$	Net exports of sector s in country d
NX_d	Net exports of country o
$\beta_{d,s}$	Cobb-Douglas expenditure share of sector s in country d
$P_{d,s}$	Price index of sector s in country d
$Z_{o,s}$	Pollution emissions of sector s in country d
$M^e_{o,s}$	Mass of firms attempting to entry
$M_{o,s}$	Mass of firms successfully entry
$\phi^\star_{od,s}$	Cutoff productivity making zero profits
$\lambda_{od,s}$	Bilateral trade share from county d to country o
$\zeta_{od,s}$	Bilateral trade share from county o to country d

Comment: ω and ϕ can both represent a firm. Since the equilibrium and the counterfactual actually utilize the sector-level aggregates, I have omitted these two notations.

A.1. Derivation of preference

Consumer demand in country d is defined as

$$U_d = \prod_s \left[\left(\sum_o \int_{\omega \in \Omega} q_{od,s}(\omega)^{\frac{\sigma_s - 1}{\sigma_s}} \right)^{\frac{\sigma_s}{\sigma_s - 1}} \right]^{\beta_{d,s}} \tag{A.1}$$

Within each sector, the common conclusions of CES preference applies:

$$q_{od,s} = \frac{p_{od,s}^{-\sigma_s}}{P_{d,s}^{1-\sigma_s}} E_{d,s} \tag{A.2}$$

where $P_{d,s}$ is the Dixit-Stigliz price index, and $E_{d,s}$ is the total expenditure in sector s:

$$P_{d,s} = \left[\sum_o \int_\Omega p_{od,s}^{(1-\sigma)} \right]^{\frac{1}{1-\sigma_s}} \tag{A.3}$$

A.2. Derivation of production

Firm produces using labor $l_{od,s}$, and a proportion of $0 \geq a < 1$ of the employment is allocated to pollution abatement:

$$q_{od,s} = (1-a)\phi l_{od,s} \tag{A.4}$$

where ϕ is a productivity draw.

Under the assumption of pollution emissions

$$z_{od,s} = (1-a)^{\frac{1}{\alpha_s}} \phi l_{od,s} \tag{A.5}$$

We can calculate

$$1 - a = \left(\frac{z_{od,s}}{\phi l_{od,s}} \right)^{\alpha_s} \tag{A.6}$$

Substitute in Equation (A.4) gives

$$q_{od,s} = (z_{od,s})^{\alpha}_{s}(\phi l_{od,s})^{1-\alpha_s} \tag{A.7}$$

A.2.1. Optimal share of labor allocation a

To derive the optimal share of labor allocated to pollution abatement a, we turn to firm's profit maximization problem:

$$\max_{a,l_{od,s}} \pi_{od,s} = p_{od,s} q_{od,s} - w_o l_{od,s} \tau_{od,s} - t_{o,s} z_{od,s} \tau_{od,s} - w_d f_{od,s} \tag{A.8}$$

$$= p_{od,s}(1-a)\phi l_{od,s} - w_o l_{od,s} \tau_{od,s} - t_{o,s} z_{od,s} = (1-a)^{\frac{1}{\alpha_s}} \phi l_{od,s} \tau_{od,s} - w_d f_{od,s} \tag{A.9}$$

The second equality follows by substituting in Equation (A.4) and Equation (A.5).

Taking partial derivation of $\pi_{od,s}$ with respect to a and $l_{od,s}$:

$$\frac{\partial pi}{\partial a} = -p\phi l + t\phi l \frac{1}{\alpha}(1-a)^{\frac{1}{\alpha}-1}\tau = 0 \Rightarrow p = \frac{t\tau}{\alpha}(1-a)^{\frac{1}{\alpha}-1} \tag{A.10}$$

$$\frac{\partial pi}{\partial l} = p(1-a)\phi - w\tau - t(1-a)^{\frac{1}{\alpha}}\phi tau = 0 \Rightarrow p = \frac{w\tau + t(1-a)^{\frac{1}{\alpha}}\phi\tau}{(1-a)\phi} \tag{A.11}$$

Comment: I omit the subscripts above for the ease of exposition when possible.

Equaling the above two expressions for p gives the optimal share of labor inputs for production $1-a$:

$$1 - a = \left(\frac{w_o}{\phi t_{o,s}} \frac{\alpha_s}{1-\alpha_s}\right)^{\alpha_s} \tag{A.12}$$

A.2.2. Pricing

Under monopolistic competition, firm's pricing is a constant markup to marginal costs

$$p_{od,s} = \frac{\sigma_s}{\sigma_s - 1} MC \tag{A.13}$$

where MC refers to marginal costs.

To derive MC, we begin with total costs TC:

$$TC = wl\tau + tz\tau + wf \tag{A.14}$$

$$= wl\tau + t\left[\frac{q}{(\phi l)^{1-\alpha}}\right]^{\frac{1}{\alpha}}\tau + wf \tag{A.15}$$

where the second equality follows by substituting in Equation (A.7).

Comment: Sunk entry costs f^e do not influence firms' pricing decision.

Taking partial derivation with respect to l to derive the optimal labor inputs:

$$\frac{\partial TC}{\partial l} = 0 \Rightarrow l = \left[\frac{w}{tq^{\frac{1}{\alpha}}\frac{1-\alpha}{\alpha}\phi^{\frac{\alpha-1}{\alpha}}}\right]^{-\alpha} \tag{A.16}$$

Substitute the above optimal l into TC to derive TC as a function of q

$$TC = w^{1-\alpha}t^{\alpha}\left(\frac{1-\alpha}{\alpha}\right)^{\alpha}\phi^{\alpha-1}\tau q \tag{A.17}$$

$$+ w^{1-\alpha}t^{\alpha}\left(\frac{1-\alpha}{\alpha}\right)^{\alpha-1}\phi^{\alpha-1}\tau q \tag{A.18}$$

$$+ wf \tag{A.19}$$

Taking partial derivation of TC w.r.t q derives MC:

$$MC = w_o^{1-\alpha_s}t_{o,s}^{\alpha_s}\left(\frac{1-\alpha_s}{\alpha_s}\right)^{\alpha_s-1}\frac{1}{\alpha_s}\phi^{\alpha_s-1}\tau_{od,s} \tag{A.20}$$

Define

$$c_{o,s} \equiv \left(\frac{t_{o,s}}{\alpha_s}\right)^{\alpha_s}\left(\frac{w_o}{1-\alpha_s}\right)^{1-\alpha_s} \tag{A.21}$$

we now writes firms pricing as

$$p_{od,s} = \frac{\sigma_s}{\sigma_s - 1}\frac{c_{o,s}\tau_{od,s}}{\phi^{1-\alpha_s}} \tag{A.22}$$

A.2.3. Cutoff productivity $\phi^{\star}_{od,s}$; Zero-profit condition (ZPC)

There exists a cutoff productivity $\phi^{\star}_{od,s}$ making firms earn zero profits after entry, such that productivity draw lower than $\phi^{\star}_{od,s}$ will not enter the market.

Cutoff productivity $\phi^{\star}_{od,s}$ satisfies

$$\pi_{od,s}(\phi^{\star}_{od,s}) = 0 \tag{A.23}$$

$$\begin{aligned}
\pi_{od,s} &= pq - TC && \text{(A.24)}\\
&= pq - \left[MC \cdot \frac{1-\alpha}{\alpha}\alpha q + MC \cdot \alpha q\right] - wf && \text{(A.25)}\\
&= pq - \left(pq\frac{\sigma - 1}{\sigma} + wf\right) && \text{(A.26)}\\
&= \frac{pq}{\sigma} - wf && \text{(A.27)}\\
&= \frac{r_{od,s}}{\sigma_s} - w_d f_{od,s} && \text{(A.28)}\\
& && \text{(A.29)}
\end{aligned}$$

where the third equality utilizes the pricing rule $p = \frac{\sigma}{\sigma-1}MC$.

Let $\pi_{od,s} = 0$ and substitute in Equation (A.2)

$$\frac{p_{od,s}^{1-\sigma_s}}{P_{d,s}^{1-\sigma_s}} E_{d,s} \frac{1}{\sigma_s} = w_d f_{od,s} \tag{A.30}$$

substitute in Equation A.22 solves $\phi^{\star}_{od,s}$

$$\phi^{\star}_{od,s} = \left[\frac{\sigma_s}{\sigma_s - 1}\frac{c_{o,s}\tau_{od,s}}{P_{d,s}}\left(\frac{\sigma_s w_d f_{od,s}}{E_{d,s}}\right)^{\frac{1}{\sigma_s - 1}}\right]^{\frac{1}{1-\alpha_s}} \tag{A.31}$$

A.2.4. Free-entry condition

Free-entry condition says that the expected profits of firms attempting to entry have to at least cover the sunk fixed costs of entry. Applying total probability theorem:

$$\sum_d \Pr(\phi > \phi^{\star}_{od,s})\mathbf{E}[\phi_{od,s}|\phi > \phi^{\star od,s}] = w_o f^e_{o,s} \tag{A.32}$$

We first list several characteristics of Pareto distribution, solve for the conditional density distribution, then calculate $\mathbf{E}[\phi_{od,s}|\phi > \phi^{\star}_{od,s}]$.

The cumulative density function (CDF) and probability density function (PDF) of Pareto distribution are as follows:

$$G(\phi, b_{o,s}) = 1 - \left(\frac{\phi}{b_{o,s}}\right)^{-\theta_s} \tag{A.33}$$

$$g(\phi, b_{o,s}) = \theta_s b_{o,s}^{\theta_s} \phi^{-\theta_s - 1} \tag{A.34}$$

To derive conditional PDF $g(\phi|\phi > \phi^{\star}_{od,s})$, we first derive conditional CDF:

$$G(\phi|\phi > \phi^{\star}_{od,s}) = \Pr\left[\Phi \leq \phi|\Phi > \phi^{\star}\right] = \frac{\Pr(\phi^{\star} < \Phi \leq \phi)}{\Pr(\Phi > \phi^{\star})} \tag{A.35}$$

$$= \frac{G(\phi) - G(\phi^{\star})}{1 - G(\phi^{\star})} \tag{A.36}$$

The first equality utilizes conditional probability

$$\Pr(B|A) = \frac{\Pr(AB)}{\Pr(A)}$$

Taking differentiation solves conditional CDF:

$$g(\phi|\phi > \phi^{\star}_{od,s}) = \frac{g(\phi)}{1 - G(\phi^{\star})} = \theta_s \frac{\left(\phi^{\star}_{od,s}\right)^{\theta_s}}{\phi^{\theta_s + 1}} \tag{A.37}$$

A contrast of $g(\phi)$ and $g(\phi|\phi > \phi^{\star}_{od,s})$ is plotted in Figure A1.

We now turn to derive conditional expected profits $\mathbf{E}[\phi_{od,s}|\phi > \phi^{\star}_{od,s}]$:

$$\mathbf{E}[\phi_{od,s}|\phi > \phi^\star_{od,s}] = \int_{\phi^\star_{od,s}}^{\infty} \pi_{od,s} g(\phi|\phi > \phi^\star_{od,s}) d\phi \tag{A.38}$$

$$= \int_{\phi^\star_{od,s}}^{\infty} \left(\frac{r}{\sigma} - w_d f_{od,s}\right) g(\phi|\phi > \phi^\star_{od,s}) d\phi \tag{A.39}$$

$$= \int_{\phi^\star_{od,s}}^{\infty} \frac{r}{\sigma} g(\phi|\phi > \phi^\star_{od,s}) d\phi - \int_{\phi^\star_{od,s}}^{\infty} w_d f_{od,s} g(\phi|\phi > \phi^\star_{od,s}) d\phi \tag{A.40}$$

$$= \int_{\phi^\star_{od,s}}^{\infty} \frac{r}{\sigma} g(\phi|\phi > \phi^\star_{od,s}) d\phi - w_d f_{od,s} \tag{A.41}$$

$$= \int_{\phi^\star_{od,s}}^{\infty} \frac{pq}{\sigma} g(\phi|\phi > \phi^\star_{od,s}) d\phi - w_d f_{od,s} \tag{A.42}$$

$$= \int_{\phi^\star_{od,s}}^{\infty} p^{1-\sigma} P^{\sigma-1} E_{d,s} \cdot \frac{1}{\sigma} \cdot g(\phi|\phi > \phi^\star_{od,s}) d\phi - w_d f_{od,s} \tag{A.43}$$

$$= \int_{\phi^\star_{od,s}}^{\infty} p^{1-\sigma} \frac{w_d f_{od,s} \sigma}{p(\phi^\star)^{1-\sigma}} \cdot \frac{1}{\sigma} \cdot g(\phi|\phi > \phi^\star_{od,s}) d\phi - w_d f_{od,s} \tag{A.44}$$

$$= \int_{\phi^\star_{od,s}}^{\infty} \left[\frac{\phi^{\alpha-1}}{(\phi^\star)^{\alpha-1}}\right]^{1-\sigma} w_d f_{od,s} g(\phi|\phi > \phi^\star_{od,s}) d\phi - w_d f_{od,s} \tag{A.45}$$

$$= \int_{\phi^\star_{od,s}}^{\infty} \phi^{(\alpha-1)(1-\sigma)} (\phi^\star)^{-(\alpha-1)(1-\sigma)} w_d f_{od,s} \cdot \theta_s \left(\phi^\star_{od,s}\right)^{\theta_s} \phi^{-\theta_s - 1} - w_d f_{od,s} \tag{A.46}$$

$$= w_d f_{od,s} \theta \left(\phi^\star\right)^{\theta - (\alpha-1)(1-\sigma)} \cdot \int_{\phi^\star_{od,s}}^{\infty} \phi^{(\alpha-1)(1-\sigma)-\theta-1} d\phi - w_d f_{od,s} \tag{A.47}$$

$$= w_d f_{od,s} \theta \frac{1}{\theta - (\alpha-1)(1-\sigma)} - w_d f_{od,s} \tag{A.48}$$

$$= w_d f_{od,s} \frac{(\sigma-1)(1-\alpha)}{\theta - (\sigma-1)(\alpha-1)} \tag{A.49}$$

The third equality follows by $\int_{\phi^\star_{od,s}}^{\infty} g(\phi|\phi > \phi^\star_{od,s}) d\phi = 1$ (See Figure A1). The sixth equality follows by substituting in Equation (A.30), and the seventh equality follows by substituting in Equation (A.22) and note that $\phi^\star$ is a constant.

Finally, we can derive the free entry condition. Substitute $\mathbf{E}[\phi_{od,s}|\phi > \phi^\star_{od,s}]$ into Equation(A.32) and rewrite $\Pr(\phi > \phi^\star) = [1 - G(\phi^\star)]$

$$\sum_d [1 - G(\phi^\star)] \cdot w_d f_{od,s} \frac{(\sigma-1)(1-\alpha)}{\theta - (\sigma-1)(\alpha-1)} = w_o f^e_{o,s} \tag{A.50}$$

$$\Rightarrow \sum_d \left(\frac{b_{o,s}}{\phi^\star_{od,s}}\right)^{\theta_s} \cdot w_d f_{od,s} \frac{(\sigma-1)(1-\alpha)}{\theta - (\sigma-1)(\alpha-1)} = w_o f^e_{o,s} \tag{A.51}$$

$$\Rightarrow f^e_{o,s} \frac{\theta - (\sigma-1)(\alpha-1)}{(\sigma-1)(1-\alpha)} = \sum_d \left(\frac{b_{o,s}}{\phi^\star_{od,s}}\right)^{\theta_s} \frac{w_d}{w_o} f_{od,s} \tag{A.52}$$

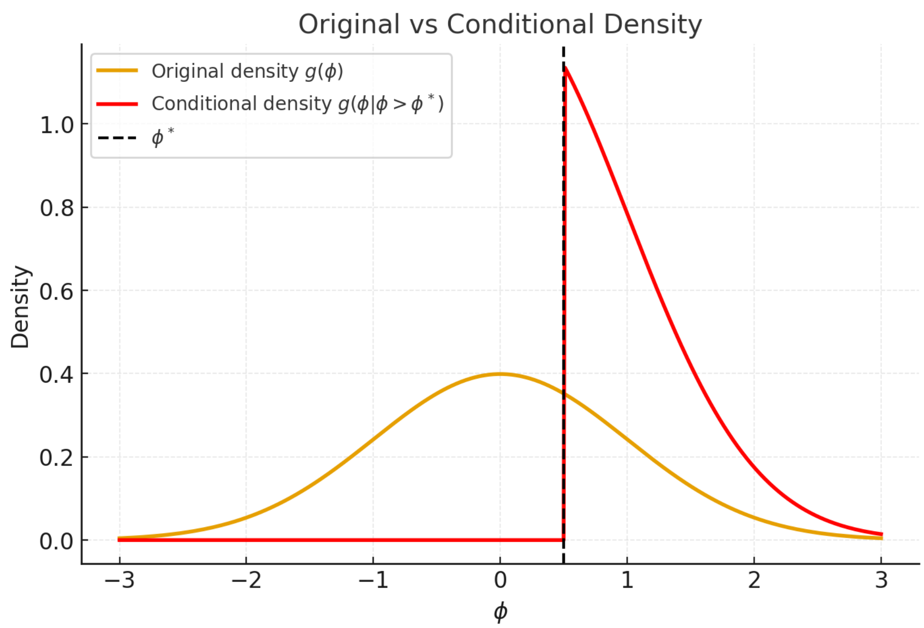

Figure A1 — PDF and Conditional PDF of Pareto Distribution

Notes. This figure shows PDF and Conditional PDF of Pareto Distribution using code in Code 1 which assumes $\phi^{\star od,s} = 0.5$.

A.2.5. Simplifying Free-entry Condition

The free-entry condition can be further simplified after deriving $P_{d,s}$ and bilateral trade flow $X_{od,s}$.

Deriving $P_{d,s}$:

Rewrite Equation (A.3) to $P_{d,s}^{1-\sigma_s}$

$$
\begin{aligned}
P_{d,s}^{1-\sigma_s} &= \sum_o \int_0^{M_{od,s}} \left(p_{od,s}\right)^{1-\sigma_s} dv \\
&= \sum_o M_{od,s} \mathbf{E}\left[p_{od,s}^{1-\sigma_s} | \phi > \phi_{od,s}^{\star}\right] \\
&= \sum_o \mathrm{Pr}(\phi > \phi_{od,s}^{\star}) M_{o,s}^{e} \int_{\phi_{od,s}^{\star}}^{\infty} p_{od,s}^{1-\sigma_s} g(\phi|\phi > \phi_{od,s}^{\star}) d\phi \\
&= \sum_o M^e \left[1 - G(\phi^{\star})\right] \int_{\phi^{\star}}^{\infty} p^{1-\sigma} \theta \left(\phi^{\star}\right)^{\theta} \phi^{-\theta-1} d\phi \\
&= \sum_o M^e b^{\theta} (\phi^{\star})^{-\theta} \int_{\phi^{\star}}^{\infty} \left(\frac{\sigma}{\sigma-1} \frac{c\tau}{\phi^{1-\alpha}}\right)^{1-\sigma} \theta \left(\phi^{\star}\right)^{\theta} \phi^{-\theta-1} d\phi \\
&= \sum_o M^e \left(\frac{\sigma}{\sigma-1} c\tau\right)^{1-\sigma} \theta b^{\theta} \frac{1}{\theta - (1-\alpha)(\sigma-1)} \left(\phi^{\star}\right)^{(1-\alpha)(\sigma-1)-\theta} \\
&= \sum_o M^e \left[\frac{\sigma}{\sigma-1} \frac{t^{\alpha} w_o^{1-\alpha} \tau}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\right]^{1-\sigma} \theta \beta^{\theta} \frac{1}{\theta - (1-\alpha)(\sigma-1)} \\
&\cdot \left[\left(\frac{\sigma}{\sigma-1} \cdot \frac{t^{\alpha} w_o^{1-\alpha} \tau}{\alpha^{\alpha}(1-\alpha)^{1-\alpha} P_{d,s}}\right) \left(\frac{\sigma w_d f_{od,s}}{E_{d,s}}\right)^{\frac{1}{\sigma-1}}\right]^{\frac{(1-\alpha)(\sigma-1)-\theta}{1-\alpha}} \\
&= \sigma^{1-\frac{\sigma\theta}{(1-\alpha)(\sigma-1)}} \left(\sigma-1\right)^{\frac{\theta}{1-\alpha}} \alpha^{\frac{\alpha\theta}{1-\alpha}} (1-\alpha)^{\theta} \\
&\cdot \sum_o M_{o,s}^{e} (w_o)^{-\theta_s} (b_{o,s})^{\theta} (\tau_{od,s})^{-\frac{\theta}{1-\alpha}} \left(f_{od,s}\right)^{1-\frac{\theta}{(1-\alpha)(\sigma-1)}} \left(E_{d,s}\right)^{\frac{\theta}{(1-\alpha)(\sigma-1)}-1} (w_d)^{1-\frac{\theta}{(1-\alpha)(\sigma-1)}} \left(t_{o,s}\right)^{\frac{\alpha\theta}{1-\alpha}} \\
&\cdot P_{d,s}^{-\frac{(1-\alpha)(\sigma-1)-\theta}{1-\alpha}}
\end{aligned}
$$

The second equality follows from firms' sourcing $p_{od} \leq min\{p_{o1}, p_{o2}, ..., p_{od}\}$ (See international trade literature such as Eaton and Kortum (2002)). The fifth equality follows by substituting in Equation (A.21) of $c_{o,s}$ and Equation (A.31) of cutoff productivity $\phi_{o,s}^{\star}$.

Define constant χ_s:

$$
\chi_s \equiv \sigma_s^{1-\frac{\sigma_s\theta_s}{(1-\alpha_s)(\sigma_s-1)}} \left(\sigma_s-1\right)^{\frac{\theta_s}{1-\alpha_s}} \alpha_s^{\frac{\alpha_s\theta_s}{1-\alpha_s}} \left(1-\alpha_s\right)^{\theta_s} \tag{A.53}
$$

Solving for the last equality derives $P_{d,s}$:

$$P_{d,s}^{-\frac{\theta_s}{1-\alpha_s}} = \chi_s \sum_o M_{o,s}^e \left(\frac{w_o}{b_{o,s}}\right)^{-\theta_s} (\tau_{od,s})^{-\frac{\theta_s}{1-\alpha_s}} (f_{od,s})^{1-\frac{\theta_s}{(1-\alpha_s)(\sigma_s-1)}} (t_{o,s})^{\frac{\alpha_s\theta_s}{1-\alpha_s}} \left(\frac{E_{d,s}}{w_d}\right)^{\frac{\theta_s}{(1-\alpha_s)(\sigma_s-1)}-1} \tag{A.54}$$

Deriving $X_{od,s}$:

$$\begin{aligned} X_{od,s} &= \Pr\left(\phi > \phi^\star_{od,s}\right) M_{o,s}^e \mathbf{E}\left[r_{od,s} | \phi > \phi^\star_{od,s}\right] \\ &= \left[1 - G(\phi^\star)\right] M^e \int_{\phi^\star}^\infty pq g(\phi|\phi > \phi^\star) d\phi \\ &= \left(\frac{\phi}{b_{o,s}}\right)^{-\theta} M^e \int_{\phi^\star}^\infty \left[\frac{\sigma}{\sigma-1}\frac{c\tau}{\phi^{1-\alpha}}\right]^{1-\sigma} P^{\sigma-1} E_{d,s} \theta \left(\phi^\star\right)^\theta \phi^{-\theta-1} d\phi \\ &= \frac{\theta b_{o,s}^\theta M^e}{\theta - (1-\alpha)(\sigma-1)} \left(\frac{\sigma}{\sigma-1} c\tau\right)^{1=\sigma} P^{\sigma-1} E_{d,s} \left(\phi^\star\right)^{(1-\alpha)(\sigma-1)-\theta} \\ &= b_{o,s}^\theta M_{o,s}^e w_o^{-\theta_s} (\tau_{od,s})^{-\frac{\theta_s}{1-\alpha_s}} (f_{od,s})^{1-\frac{\theta_s}{(1-\alpha_s)(\sigma_s-1)}} (t_{o,s})^{\frac{\alpha_s\theta_s}{1-\alpha_s}} \left(\frac{E_{d,s}}{w_d}\right)^{\frac{\theta_s}{(1-\alpha_s)(\sigma_s-1)}} w_d \chi_s P_{d,s}^{\frac{\theta_s}{1-\alpha_s}} \end{aligned} \tag{A.55}$$

The fourth equality follows by substituting in Equation (A.21) of $c_{o,s}$ and Equation (A.31) of cutoff productivity $\phi^\star_{o,s}$.

Comment: $X_{od,s}$ is a sector-level aggregates of firms' revenue.

Simplifying Free-entry Condition

Rearrange free-entry condition Equation A.52 by multiplying w_0, then substitute in Equation

(A.31) of cutoff productivity $\phi^{\star}_{o,s}$, finally simply using $P_{d,s}$ and $X_{od,s}$:

$$
\begin{aligned}
& w_o f^e_{o,s} \frac{\theta-(\sigma-1)(\alpha-1)}{(\sigma-1)(1-\alpha)} \\
&= \sum_d \left(\frac{b_{o,s}}{\phi^{\star}_{od,s}}\right)^{\theta_s} w_d f_{od,s} \\
&= \sum_d b^{\theta}_{o,s} \left[\frac{\sigma}{\sigma-1}\frac{c\tau}{P}\left(\frac{\sigma w_d f_{od}}{E_{d,s}}\right)^{\frac{1}{\sigma-1}}\right]^{-\frac{\theta}{1-\alpha}} w_d f_{od} \\
&= \sum_d b^{\theta}_{o,s} \left(\frac{\sigma}{\sigma-1}\right)^{-\frac{\theta}{1-\alpha}} (c\tau)^{-\frac{\theta}{1-\alpha}} (P_{d,s})^{\frac{\theta}{1-\alpha}} \sigma^{-\frac{\theta}{(1-\alpha)(\sigma-1)}} (w_d f_{od,s})^{1-\frac{\theta}{(1-\alpha)(\sigma-1)}} (E_{d,s})^{\frac{\theta}{(1-\alpha)(\sigma-1)}} \\
&= \sum_d b^{\theta}_{o,s} \chi_s w_o^{-\theta} \tau_{od,s}^{-\frac{\theta}{1-\alpha}} (w_d f_{od,s})^{1-\frac{\theta}{(1-\alpha)(\sigma-1)}} (E_{d,s})^{\frac{\theta}{(1-\alpha)(\sigma-1)}} \\
&\cdot \left[\sum_o M^e_{o,s}\left(\frac{w_o}{b_{o,s}}\right)^{-\theta} (\tau_{od,s})^{-\frac{\theta}{1-\alpha}} (w_d f_{od,s})^{1-\frac{\theta}{(1-\alpha)(\sigma-1)}} \chi_s (E_{d,s})^{\frac{\theta}{(1-\alpha)(\sigma-1)}-1}\right]^{-1} \\
&= \sum_d \frac{X_{od,s}\left(M^e_{o,s}\right)^{-1} P_{d,s}^{-\frac{\theta}{1-\alpha}} w_d^{-1} \left(\frac{E_{d,s}}{w_d}\right)^{-\frac{\theta}{(1-\alpha)(\sigma-1)}} \chi_s^{-1} \cdot \frac{\theta-(\sigma-1)(\alpha-1)}{\theta\sigma} E_d}{P_{d,s}^{-\frac{\theta}{1-\alpha}} \chi_s^{-1} \left(\frac{E_{d,s}}{w_d}\right)^{1-\frac{\theta}{(1-\alpha)(\sigma-1)}}} \\
&= \sum_d \frac{X_{od,s}}{M^e_{o,s}} \cdot \frac{\theta-(\sigma-1)(\alpha-1)}{\theta\sigma} \\
&= \frac{R_{o,s}}{M^e_{o,s}} \cdot \frac{\theta-(\sigma-1)(\alpha-1)}{\theta\sigma}
\end{aligned}
$$

The second equality follows by substituting in Equation (A.31) of cutoff productivity $\phi^{\star}_{o,s}$; The fourth equality follows by substituting in Equation (A.21) of $c_{o,s}$ and Equation (A.54) of $P_{d,s}$, then rearrange; The fifth equality follows by substituting $X_{od,s}$ and $P_{d,s}$; The last equality follows by the definition of $R_{o,s}$:

$$R_{o,s} = \sum_d X_{od,s} \tag{A.56}$$

Rearrangement derives Equation (11) in SW(2018):

$$\frac{1-\alpha_s}{\theta_s}\frac{\sigma_s-1}{\sigma_s} R_{o,s} = w_o f_{o,s^e} M^e_{o,s} \tag{A.57}$$

A.3. Derivation of Labor Market Clearing

Labor supply, $L_d = \frac{R_d}{w_d}$, is used for five purposes. We first derive sector-specific labor demand in origin/destination country, then sum them together across sectors.

A.3.1. Labor for fixed costs of drawing productivity

By definition, labor for fixed costs of drawing productivity equals the mass of firms attempting to entry times the sunk fixed costs of entry:

$$L^e_{o,s} = M^e_{o,s} f^e_{o,s} \tag{A.58}$$

A.3.2. Labor for production and abatement

Labor for production and abatement equals the mass of successful entrants times the expected labor demand, and sum it across total markets:

$$
\begin{aligned}
L^p_{o,s} &= \sum_d \Pr(\phi > \phi^\star_{od,s}) M^e_{o,s} \mathbf{E}\left[l_{od,s}\tau_{od,s} | \phi > \phi^\star_{od,s}\right] \\
&= \sum_d \left[1 - G(\phi^\star)\right] M^e_{o,s} \int_{\phi^\star}^{\infty} l\tau g(\phi|\phi > \phi^\star) d\phi \\
&= \sum_d M^e_{o,s} b^\theta \left(\phi^\star\right)^{-\theta} \int_{\phi^\star}^{\infty} q\phi^{-1}(1-a)^{-1}\tau\theta \left(\phi^\star\right)^\theta \phi^{-\theta-1} d\phi \\
&= \sum_d M^e_{o,s} b^\theta \int_{\phi^\star}^{\infty} p^{-\sigma} P^{\sigma-1} E_{d,s} \phi^{-1} \left(\frac{w_o}{t_{o,s}} \frac{\alpha}{1-\alpha}\right)^{-\alpha} \phi^\alpha \tau_{od,s} \theta \phi^{-\theta-1} d\phi \\
&= \sum_d M^e_{o,s} \theta b^\theta \int_{\phi^\star}^{\infty} \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} (c\tau)^{-\sigma} \phi^{1-\alpha} P^{\sigma-1} E_{d,s} \left(\frac{w_o}{t_{o,s}} \frac{\alpha}{1-\alpha}\right)^{-\alpha} \phi^\alpha \tau_{od,s} \theta \phi^{-\theta-1} \\
&= \sum_d M^e_{o,s} \theta b^\theta \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} c^{-\sigma} \tau^{1-\sigma} \left(\frac{w_o}{t_{o,s}} \frac{\alpha}{1-\alpha}\right)^{-\alpha} P^{\sigma-1} E_{d,s} \cdot \frac{\left(\phi^\star\right)^{(1-\alpha)(\sigma-1)-\theta}}{\theta - (1-\alpha)(\sigma-1)} \\
&= \sum_d \frac{M^e_{o,s} \theta b^\theta}{\left[\theta - (1-\alpha)(\sigma-1)\right] \left(\phi^\star\right)^\theta} \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} c^{-\sigma} \tau^{1-\sigma} \left(\frac{w_o}{t_{o,s}} \frac{\alpha}{1-\alpha}\right)^{-\alpha} P^{\sigma-1} E_{d,s} \\
&\cdot \left[\frac{\sigma}{\sigma-1} \frac{c\tau}{P}\right]^{\sigma-1} \frac{\sigma w_d f_{od,s}}{E} \\
&= M^e_{o,s} \theta \sum_d \frac{b^\theta (\sigma-1)(1-\alpha)}{\left(\phi^\star\right)^\theta \left[\theta - (1-\alpha)(\sigma-1)\right]} \frac{w_d}{w_o} f_{od,s} \\
&= M^e_{o,s} \theta_s f^e_{o,s}
\end{aligned}
$$

The seventh equality follows by substituting in cutoff productivity $\phi^\star$ (Equation A.31); The eighth equality follows by substituting in Equation (A.21), and the last equality follows by the free-entry condition (Equation A.52).

Rewrite the last equality here:

$$L^p_{o,s} = \theta_s M^e_{o,s} f^e_{o,s} \tag{A.59}$$

A.3.3. Labor for paying pollution tax

Labor for paying pollution tax equals the sum of pollution tax normalized by wage:

Comment: Below $Z_{o,s}$ is the sum of $z_{od,s}$ across d

$$
\begin{aligned}
L_{o,s}^{t} &= \frac{t_{o,s} Z_{o,s}}{w_o} && \text{(A.60)}\\
&= \frac{t_{o,s}}{w_o} \sum_d \Pr\left(\phi > \phi_{od,s}^{\star}\right) M_{o,s}^{e} \mathbf{E}\left[z_{od,s}\tau_{od,s} | \phi > \phi_{od,s}^{\star}\right] && \text{(A.61)}\\
&= \frac{t_{o,s}}{w_o} \sum_d \left[1 - G\left(\phi_{od,s}^{\star}\right)\right] M_{o,s}^{e} \int_{\phi_{od,s}^{\star}}^{\infty} (1-a)^{\frac{1}{\alpha_s}} \phi_{od,s} l_{od,s} \tau_{od,s} g\left(\phi | \phi > \phi_{od,s}^{\star}\right) d\phi && \text{(A.62)}\\
&= \frac{t_{o,s}}{w_o} \sum_d \left[1 - G\left(\phi_{od,s}^{\star}\right)\right] M_{o,s}^{e} \int_{\phi_{od,s}^{\star}}^{\infty} \frac{w_o}{t_{o,s}} \frac{\alpha_s}{1-\alpha_s} l_{od,s} \tau_{od,s} g\left(\phi | \phi > \phi_{od,s}^{\star}\right) d\phi && \text{(A.63)}\\
&= \frac{t_{o,s}}{w_o} \cdot \frac{w_o}{t_{o,s}} \frac{\alpha_s}{1-\alpha_s} \cdot L_{o,s}^{p} && \text{(A.64)}\\
&= \frac{\alpha_s}{1-\alpha_s} \theta_s M_{o,s}^{e} f_{o,s}^{e} && \text{(A.65)}
\end{aligned}
$$

The fifth equality follows by the second equality in deriving $L_{o,s}^{p}$.

A.3.4. Labor for fixed costs of entering domestic/foreign markets

Labor for entering a domestic/foreign country market equals the mass of successful entrants times the fixed costs of entry:

$$
L_{d,s}^{m} = \sum_o M_{od,s} f_{od,s}
$$

We find $M_{od,s} f_{od,s}$ from the definition of $X_{od,s}$:

$$
\begin{aligned}
X_{od,s} &= M_{od,s} \cdot \mathbf{E}\left[r_{od,s} | \phi > \phi_{od,s}^{\star}\right] && \text{(A.66)}\\
&= M_{od,s} \cdot w_d f_{od,s} \cdot \frac{\theta_s \sigma_s}{\theta_s - (1-\alpha_s)(\sigma_s - 1)} && \text{(A.67)}\\
\Rightarrow M_{od,s} \cdot w_d f_{od,s} &= \frac{\theta_s - (1-\alpha_s)(\sigma_s - 1)}{\theta_s \sigma_s} \cdot \frac{X_{od,s}}{w_d} && \text{(A.68)}
\end{aligned}
$$

The second equality follows by the former part of Equation (A.49) calculating $\mathbf{E}\left[\frac{r_{od,s}}{\sigma_s} | \phi > \phi_{od,s}^{\star}\right]$

Therefore,

$$L_{d,s}^m = \sum_o M_{od,s} f_{od,s} \tag{A.69}$$

$$= \sum_o \frac{X_{od,s}}{w_d} \frac{\theta_s - (1-\alpha_s)(\sigma_s - 1)}{\theta_s \sigma_s} \tag{A.70}$$

$$= \frac{E_{d,s}}{w_d} \frac{\theta_s - (1-\alpha_s)(\sigma_s - 1)}{\theta_s \sigma_s} \tag{A.71}$$

A.3.5. Labor for paying trade deficits

Labor for paying trade deficits is defined by assuming there exits trade imbalance $R_{d,s} \neq E_{d,s}$ and $NX_{d,s} = R_{d,s} - E_{d,s}$, the difference between total labor supply $\frac{R_d}{w_d}$ and the above four labor demand $\sum_s L_{d,s}^e + L_{d,s}^p + L_{d,s}^t + L_{d,s}^m$.

We first calculate the sum of the first four labor demand. Substitute in Equation (A.57)

$$\sum_s L_{d,s}^e + L_{d,s}^p + L_{d,s}^t + L_{d,s}^m = \sum_s \frac{R_{d,s}}{w_d} \left[\frac{(1-\alpha)(\sigma-1)}{\sigma\theta} + \frac{(1-\alpha)(\sigma-1)}{\sigma} + \frac{(\sigma-1)\alpha}{\sigma} \right] \tag{A.72}$$

$$+ \sum_s \frac{E_{d,s}}{w_d} \left[\frac{\theta - (1-\alpha)(\sigma-1)}{\sigma\theta} \right] \tag{A.73}$$

Note that if we assume no trade deficits, i.e., $E_{d,s} = Rd, s$, the above equation becomes $\frac{R_d}{w_d} = L_d$.

We now introduce trade deficits. Define NX as net exports:

$$E_{d,s} = R_{d,s} - NX_{d,s} \tag{A.74}$$

$$E_d = R_d - NX_d \tag{A.75}$$

recall that the Cobb-Douglas expenditure share $\beta_{d,s}$ implies:

$$E_{d,s} = \beta_{d,s} E_d \tag{A.76}$$

so that

$$\sum_s L^e_{d,s} + L^p_{d,s} + L^t_{d,s} + L^m_{d,s} = \sum_s \frac{R_{d,s}}{w_d} - \frac{NX_{d,s}}{w_d}\frac{\theta - (1-\alpha)(\sigma-1)}{\sigma\theta} \tag{A.77}$$

$$= \sum_s \frac{E_{d,s}}{w_d} - \frac{NX_{d,s}}{w_d}\frac{(\sigma-1)(\theta-\alpha+1)}{\sigma\theta} \tag{A.78}$$

$$= \sum_s \beta_{d,s}\frac{E_d}{w_d} + \sum_s \frac{NX_{d,s}}{w_d}\frac{(\sigma-1)(\theta-\alpha+1)}{\sigma\theta} \tag{A.79}$$

$$= \sum_s \beta_{d,s}\frac{R_d}{w_d} - \sum_s \beta_{d,s}\frac{NX_d}{w_d} + \sum_s \frac{NX_{d,s}}{w_d}\frac{(\sigma-1)(\theta-\alpha+1)}{\sigma\theta} \tag{A.80}$$

$$= L_d - \sum_s \beta_{d,s}\frac{NX_d}{w_d} + \sum_s \frac{NX_{d,s}}{w_d}\frac{(\sigma-1)(\theta-\alpha+1)}{\sigma\theta} \tag{A.81}$$

The first equality follows by substituting in $E_{d,s} = R_{d,s} - NX_{d,s}$; The second equality follows by substituting in $R_{d,s} = E_{d,s} + NX_{d,s}$; The third equality follows by substituting in $E_{d,s} = \beta_{d,s}E_d$; The fourth equality follows by substituting in $E_d = R_d - NX_d$. The last equality follows by utilizing $\sum_d \beta_{d,s} = 1$ and $L_d = \frac{R_d}{w_d}$. The latter two terms of the last equality are the definition of $-L^{NX}_d$, so that

$$L^{NX}_{d,s} \equiv \beta_{d,s}\frac{NX_d}{w_d} - \frac{NX_{d,s}}{w_d}\frac{(\sigma_s-1)(\theta_s-\alpha_s+1)}{\sigma_s\theta_s} \tag{A.82}$$

Finally, summing across the five sector-specific labor demand and simplifying:

$$
\begin{aligned}
L_d &= \sum_s L^e_{d,s} + L^p_{d,s} + L^t_{d,s} + L^m_{d,s} + L^{NX}_{d,s} \\
&= \sum_s \left[M^e_{d,s} f^e_{od,s} \left(\theta_s + 1 + \frac{\alpha_s \theta_s}{1-\alpha_s} \right) \right] \\
&\quad + \sum_s \left[\frac{\theta_s - (1-\alpha_s)(\sigma_s - 1)}{\sigma_s \theta_s} \cdot \beta_{d,s} \frac{R_d - NX_d}{w_d} \right] \\
&\quad + \sum_s \left[\beta_{d,s} \frac{NX_d}{w_d} - \frac{NX_{d,s}}{w_d} \frac{(\sigma_s - 1)(\theta_s - \alpha_s + 1)}{\sigma_s \theta_s} \right] \\
&= \sum_s \left[M^e_{d,s} f^e_{od,s} \left(\theta_s + 1 + \frac{\alpha_s \theta_s}{1-\alpha_s} \right) \right] \\
&\quad + \sum_s \left[\frac{\theta_s - (1-\alpha_s)(\sigma_s - 1)}{\sigma_s \theta_s} \cdot \beta_{d,s} L_d \right] \\
&\quad + \sum_s \left[\left(\frac{\theta_s - (1-\alpha_s)(\sigma_s - 1)}{\sigma_s \theta_s} - 1 \right) \beta_{d,s} \frac{NX_d}{w_d} - \frac{NX_{d,s}}{w_d} \frac{(\sigma_s - 1)(\theta_s - \alpha_s + 1)}{\sigma_s \theta_s} \right]
\end{aligned}
$$

The first equality follows by substituting in Equation (A.57) and $E_{d,s} = \beta_{d,s} E_d = \beta_{d,s}(R_d - NX_d)$; The second equality follows by substituting $\frac{R_d}{w_d}$ by L_d.

Then, the last equality solves L_d:

$$
\begin{aligned}
L_d \cdot \left(1 - \sum_s \frac{\theta_s - (1-\alpha_s)(\sigma_s - 1)}{\sigma_s \theta_s} \beta_{d,s} \right) &= \sum_s M^e_{d,s} f^e_{od,s} \left(\theta_s + 1 + \frac{\alpha_s \theta_s}{1-\alpha_s} \right) \\
&\quad - \left(\frac{\theta_s - (1-\alpha_s)(\sigma_s - 1)}{\sigma_s \theta_s} - 1 \right) \beta_{d,s} \frac{NX_d}{w_d} \\
&\quad - \frac{(\sigma_s - 1)(\theta_s - \alpha_s + 1)}{\sigma_s \theta_s} \frac{NX_{d,s}}{w_d}
\end{aligned}
\tag{A.83}
$$

Comment: One appealing thing of the expression for L_d is that it is a function of w_d, $M^e_{d,s}$ and $\beta_{d,s}$, NX, where the former two are endogenous parameters in the model, and the latter two can be observed in the data.

A.4. Derivation of Hat Algebra

Using *hat algebra* to denote changes,

$$
\hat{x} \equiv \frac{x'}{x}
$$

where x' is the counterfactual value.

For the ease of exposition and calculation, sunk fixed costs of drawing a productivity $f^e od, s$ are assumed to be unchanged.

A.4.1. Hat Algebra of Free-entry Condition (Equation A.57)

We first write Equation (A.57) in the hat form:

$$\frac{w_o'}{w_o} = \frac{R_{o,s}'}{R_{o,s}} \frac{M_{o,s}^e}{M_{o,s}^{e'}} \Rightarrow \hat{w}_o \cdot \hat{M}_{o,s}^e = \hat{R}_{o,s} = \sum_d \zeta_{od,s} \hat{X}_{od,s} \tag{A.84}$$

where $\zeta_{od,s}$ measures the bilateral trade flow share from o to d (i.e., export share over revenue):

$$\zeta_{od,s} = \frac{X_{od,s}}{\sum_d X_{od,s}} = \frac{X_{od,s}}{R_{o,s}} \tag{A.85}$$

Comment: In practice, $\hat{R}_{o,s}$ is hard to measure, so that we need to convert it to some function of "shocks".

We then calculate $\hat{X}_{od,s}$:

Define $\Upsilon_{od,s}$

$$\Upsilon_{od,s} \equiv \left(\frac{w_o}{b_{o,s}}\right)^{-\theta_s} (\tau_{od,s})^{-\frac{\theta_s}{1-\alpha_s}} (f_{od,s})^{1-\frac{\theta_s}{(1-\alpha_s)(\sigma_s-1)}} (t_{o,s})^{\frac{\alpha_s\theta_s}{1-\alpha_s}} \tag{A.86}$$

such that

$$X_{od,s} = M_{o,s}^e \Upsilon_{od,s} P_d^{-\frac{\theta}{1-\alpha}} \left(\frac{E_d}{w_d}\right)^{\frac{\theta}{(1-\alpha)(\sigma-1)}} w_d \chi_s \tag{A.87}$$

$$= \frac{M_{o,s}^e \Upsilon_{od,s}}{\sum_o M_{o,s}^e \Upsilon_{od,s}} \cdot E_{d,s} \tag{A.88}$$

To simplify the fraction, define $\lambda_{od,s}$ measuring the bilateral trade flow share from d to o (i.e., import share over expenditure):

$$\lambda_{od,s} = \frac{X_{od,s}}{\sum_o X_{od,s}} = \frac{X_{od,s}}{E_{d,s}} \tag{A.89}$$

By definition,

$$\lambda_{od,s} = \frac{M^e_{o,s}\Upsilon_{od,s}}{\sum_o M^e_{o,s}\Upsilon_{od,s}} \equiv \frac{M^e_{o,s}\Upsilon_{od,s}}{\Omega_{d,s}} \tag{A.90}$$

where

$$\Omega'_{d,s} = \sum_o M^{e'}_{o,s}\Upsilon'_{od,s} \tag{A.91}$$
$$= \sum_o \hat{M}^e_{o,s}M^e_{o,s} \cdot \hat{\Upsilon}_{od,s}\Upsilon_{od,s} \tag{A.92}$$
$$= \sum_o \hat{M}^e_{o,s}\hat{\Upsilon}_{od,s} \cdot M^e_{o,s}\Upsilon_{od,s} \tag{A.93}$$
$$= \sum_o \hat{M}^e_{o,s}\hat{\Upsilon}_{od,s} \cdot \Omega_{d,s} \cdot \lambda_{od,s} \tag{A.94}$$
$$\Rightarrow \hat{\Omega}_{d,s} = \frac{\Omega'_{d,s}}{\Omega_{d,s}} = \frac{1}{\sum_o \lambda_{od,s}\hat{M}^e_{o,s}\hat{\Upsilon}_{od,s}} \tag{A.95}$$

Therefore,

$$\hat{X}_{od,s} = \frac{X'_{od,s}}{X_{od,s}} \tag{A.96}$$
$$= \hat{M}^e_{o,s}\hat{\Upsilon}_{od,s}\hat{E}_{d,s} \cdot \frac{\Omega_{d,s}}{\Omega'_{d,s}} \tag{A.97}$$
$$= \frac{\hat{M}^e_{o,s}\hat{\Upsilon}_{od,s}}{\sum_o \lambda_{od,s}\hat{M}^e_{o,s}\hat{\Upsilon}_{od,s}} \cdot \hat{E}_{d,s} \tag{A.98}$$
$$= \frac{\hat{M}^e_{o,s}\hat{\Upsilon}_{od,s}}{\sum_o \lambda_{od,s}\hat{M}^e_{o,s}\hat{\Upsilon}_{od,s}} \cdot \hat{\beta}_{d,s} \cdot \frac{\hat{R}_d R_d - \hat{NX}_d NX_d}{R_d - NX_d} \tag{A.99}$$

Substitute Equation (A.96) back to Equation (A.84) derives Equation (3):

$$\hat{w}_o - \sum_d \frac{\zeta_{od,s}\hat{w}_o^{-\theta_s}\hat{\Gamma}_{od,s}}{\sum_o \lambda_{od,s}\hat{w}_o^{-\theta_s}\hat{\Gamma}_{od,s}} \cdot \hat{\beta}_{d,s} \cdot \frac{\hat{w}_d R_d - \hat{NX}_d NX_d}{R_d - NX_d} = 0 \tag{A.100}$$

where $\hat{R}_d R_d = \hat{w}_d R_d$ utilizes $\hat{L}_d = \frac{R'_d}{w'_d} \cdot \frac{w_d}{R_d} = \frac{\hat{R}_d}{\hat{w}_d} = 1$.

A.4.2. Recover Historical Value of $\hat{\Gamma}_{od,s}$

To derive foreign/domestic competition $\hat{\Gamma}_{od,s}$, rewrite Equation (18) in SW(2018) by definition,

$$\hat{\Gamma}_{od,s} \equiv \left(\frac{1}{\hat{b}_{o,s}}\right)^{-\theta_s} (\hat{\tau}_{od,s})^{-\frac{\theta_s}{1-\alpha_s}} (\hat{f}_{od,s})^{1-\frac{\theta_s}{(1-\alpha_s)(\sigma_s-1)}} (\hat{t}_{o,s})^{\frac{\alpha_s\theta_s}{1-\alpha_s}} \tag{A.101}$$

Rewrite Equation (A.55) of $X_{od,s}$ in the hat form and contrast it with $\hat{\Gamma}_{od,s}$:

$$\hat{X}_{od,s} = \hat{\Gamma}_{od,s} \cdot \hat{M}^e_{o,s} \hat{w}_o^{-\theta_s} \hat{P}_{d,s}^{\frac{\theta_s}{1-\alpha_s}} \hat{E}_{d,s}^{\frac{\theta_s}{(1-\alpha_s)(\sigma_s-1)}} \hat{w}_d^{\frac{\theta_s}{(1-\alpha_s)(\sigma_s-1)}-1} \tag{A.102}$$

so that we can derive $\hat{\Gamma}_{od,s}$, then simplify by substituting Equation (A.89) of $\lambda_{od,s}$ in the hat form:

$$\begin{aligned}
\hat{\Gamma}_{od,s} &= \hat{X}_{od,s} \left(\hat{M}^e_{o,s}\right)^{-1} \hat{w}_o^{\theta_s} \hat{P}_{d,s}^{-\frac{\theta_s}{1-\alpha_s}} \hat{E}_{d,s}^{-\frac{\theta_s}{(1-\alpha_s)(\sigma_s-1)}} \hat{w}_d^{1-\frac{\theta_s}{(1-\alpha_s)(\sigma_s-1)}} && \text{(A.103)}\\
&= \frac{\hat{\lambda}_{od,s}}{\hat{M}^e_{o,s}\hat{w}_o^{-\theta_s}} \hat{P}_{d,s}^{-\frac{\theta_s}{1-\alpha_s}} \hat{E}_{d,s}^{-\frac{\theta_s}{(1-\alpha_s)(\sigma_s-1)}} \hat{w}_d^{1-\frac{\theta_s}{(1-\alpha_s)(\sigma_s-1)}} && \text{(A.104)}\\
&= \frac{\hat{\lambda}_{od,s}}{\hat{M}^e_{o,s}\hat{w}_o^{-\theta_s}} \hat{P}_{d,s}^{-\frac{\theta_s}{1-\alpha_s}} \left(\frac{\hat{\beta}_{d,s}}{\hat{w}_d} \frac{\hat{R}_d R_d - \hat{NX}_d NX_d}{R_d - NX_d}\right)^{1-\frac{\theta_s}{(1-\alpha_s)(\sigma_s-1)}} && \text{(A.105)}
\end{aligned}$$

The second equality follows by substituting in $\hat{\lambda}_{od,s} = \frac{\hat{X}_{od,s}}{\hat{E}_{d,s}}$; The third equality follows by substituting in $\hat{E}_{d,s} = \hat{\beta}_{d,s}\hat{E}_d = \hat{\beta}_{d,s}\frac{R'_d - NX'_d}{R_d - NX_d}$.

A.4.3. Hat Algebra of Labor Market Clearing (Equation A.83)

Define two terms as in SW(2018):

$$\eta_{o,s} \equiv \left(\frac{\theta_s - (1-\alpha_s)(\sigma_s-1)}{\sigma_s\theta_s} - 1\right)\beta_{d,s} NX_d - \frac{(\sigma_s-1)(\theta_s-\alpha_s+1)}{\sigma_s\theta_s} NX_{d,s} \tag{A.106}$$

$$\Psi_o \equiv \left[1 - \sum_s \frac{\theta_s - (1-\alpha_s)(\sigma_s-1)}{\sigma_s\theta_s}\beta_{d,s}\right] \cdot \left[1 - \sum_s \frac{\theta_s - (1-\alpha_s)(\sigma_s-1)}{\sigma_s\theta_s}\beta'_{d,s}\right]^{-1} \tag{A.107}$$

Write Equation (A.83) into the hat form:

$$\frac{L_o'}{L_o} = \frac{R_o'}{R_o} \cdot \frac{w_o}{w_o'} = \Psi_o \left[\frac{\sum_s \frac{R_{o,s}'}{w_o'} \cdot \frac{(1-\alpha)(\sigma-1)}{\sigma\theta} \cdot \left(\theta + 1 + \frac{\alpha\theta}{1-\alpha}\right) + \frac{\eta_o'}{w_o'}}{\sum_s \frac{R_{o,s}}{w_o} \cdot \frac{(1-\alpha)(\sigma-1)}{\sigma\theta} \cdot \left(\theta + 1 + \frac{\alpha\theta}{1-\alpha}\right) + \frac{\eta_o}{w_o}} \right] \tag{A.108}$$

$$\Rightarrow \hat{R}_o = \Psi_o \left[\frac{\sum_s R_{o,s}' \cdot \frac{(\sigma-1)(\theta-\alpha+1)}{\sigma\theta} + \eta_o'}{\sum_s R_{o,s} \cdot \frac{(\sigma-1)(\theta-\alpha+1)}{\sigma\theta} + \eta_o} \right] \tag{A.109}$$

$$\Rightarrow \hat{R}_o = \Psi_o \left[\frac{\sum_s \hat{R}_{o,s} R_{o,s} \cdot \frac{(\sigma-1)(\theta-\alpha+1)}{\sigma\theta} + \eta_o'}{\sum_s R_{o,s} \cdot \frac{(\sigma-1)(\theta-\alpha+1)}{\sigma\theta} + \eta_o} \right] \tag{A.110}$$

$$\Rightarrow \frac{\hat{R}_o}{\hat{w}_o} = \Psi_o \left[\frac{\sum_s \frac{\hat{R}_{o,s}}{\hat{w}_o} R_{o,s} \cdot \frac{(\sigma-1)(\theta-\alpha+1)}{\sigma\theta} + \eta_o'}{\sum_s R_{o,s} \cdot \frac{(\sigma-1)(\theta-\alpha+1)}{\sigma\theta} + \eta_o} \right] \tag{A.111}$$

$$\Rightarrow \hat{L}_o = \Psi_o \left[\frac{\sum_s \frac{\hat{R}_{o,s}}{\hat{w}_o} R_{o,s} \cdot \frac{(\sigma-1)(\theta-\alpha+1)}{\sigma\theta} + \frac{\eta_o'}{\hat{w}_o}}{\sum_s R_{o,s} \cdot \frac{(\sigma-1)(\theta-\alpha+1)}{\sigma\theta} + \eta_o} \right] \tag{A.112}$$

$$\Rightarrow 1 = \Psi_o \left[\frac{\sum_s \hat{M}_{o,s}^e R_{o,s} \cdot \frac{(\sigma-1)(\theta-\alpha+1)}{\sigma\theta} + \frac{\eta_o'}{\hat{w}_o}}{\sum_s R_{o,s} \cdot \frac{(\sigma-1)(\theta-\alpha+1)}{\sigma\theta} + \eta_o} \right] \tag{A.113}$$

The first equality follows by substituting in Equation (A.57); The fourth equality follows by multiplying LHS and RHS with $\frac{1}{w_o}$; The fifth equality follows by substituting in the hat algebra form of Equation (A.57) and utilizes the normalized labor supply/demand $\frac{L_d'}{L_d} = 1$.

A.4.4. Hat Algebra of Pollution Emissions (Equation 1)

$Z_{o,s}$ appears in Equation (A.65) where we derive labor demand for paying pollution tax

$$Z_{o,s} = \frac{w_o}{t_{o,s}} \cdot \frac{\alpha_s}{1-\alpha_s} \theta_s M_{o,s}^e f_{o,s}^e \tag{A.114}$$

$$\Rightarrow \hat{Z}_{o,s} = \frac{\hat{w}_o \hat{M}_{o,s}^e}{\hat{t}_{o,s}} \tag{A.115}$$

B — Code Appendix

B.1. Code for conditional CDF

```python
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm

# Define the original distribution: standard normal
phi = np.linspace(-3, 3, 500)
g_phi = norm.pdf(phi)
G_phi = norm.cdf(phi)

# Threshold
phi_star = 0.5
mask = phi > phi_star

# Conditional density
g_cond = np.zeros_like(phi)
g_cond[mask] = g_phi[mask] / (1 - norm.cdf(phi_star))

# Plot
plt.figure(figsize=(8,5))
plt.plot(phi, g_phi, label="Original density $g(\\phi)$", lw=2)
plt.plot(phi, g_cond, label="Conditional density $g(\\phi | \\phi > \\phi^*)$", lw
    =2, color="red")
plt.axvline(phi_star, color="black", ls="--", label="$\\phi^*$")

plt.title("Original vs Conditional Density", fontsize=14)
plt.xlabel("$\\phi$", fontsize=12)
plt.ylabel("Density", fontsize=12)
plt.legend()
plt.grid(alpha=0.3)
plt.show()
```

Code 1: Code for PDF and Conditional PDF of Pareto Distribution

B.2. Code for Algorithm to solve $\hat{w}_o$ and $\hat{M}^e_{o,s}$

```
function diff = solveWhatMhat(guess, baseline, shocks, loop_shock, n, N, J,
    parameter)

%============================================================
% Purpose: Solve the nonlinear system of Eq.(12) + Eq.(13)
% Inputs:
%   guess      = Initial guesses (hats of wages and firm numbers)
%   baseline   = Baseline data (year 1990)
%   shocks     = Various shocks (Gamma_hat_foreign, Gamma_hat_US, etc.)
%   loop_shock = Type of shock (1=Foreign, 2=US, 3=Regulation, 4=Expenditure)
%   n          = Year index
%   N          = Number of countries
%   J          = Number of industries
%   parameter  = Model parameters (alpha, sigma, theta)
% Output:
%   diff = Residuals of Eq.(12) + Eq.(13) (passed to fsolve to approach zero)
%============================================================


%% -------- 1. Initialize shock variables (Appendix Step 1) --------
Gamma_hat = ones(N,N,J); % bilateral productivity shifter
beta_hat  = ones(N,J);   % expenditure share shock
NX_hat    = ones(N,1);   % net exports shock (agg.)
NXs_hat   = ones(N,J);   % net exports shock (sectoral)

if loop_shock == 1
Gamma_hat = shocks.Gamma_hat_foreign(:,:,:,n);   % Foreign competitiveness shock
elseif loop_shock == 2
Gamma_hat = shocks.Gamma_hat_US(:,:,:,n);        % U.S. competitiveness shock
elseif loop_shock == 3
Gamma_hat = shocks.t_hat(:,:,:,n);               % Environmental regulation shock
elseif loop_shock == 4
beta_hat  = squeeze(shocks.beta_hat(:,:,:,n));   % Expenditure share shock
end
```

```
%% -------- 2. Recover wages & firm numbers from guess --------
% guess = [ w_US ; M_hat(N*J) ]
w_US = guess(1);

% Foreign wage determined by numeraire constraint:
% w_US * (R_US / total R) + w_F * (R_F / total R) = 1
w_foreign = (1 - w_US .* baseline.R(2, :)) ./ baseline.R(1, :);

w_hat = [w_foreign; w_US];   % N×1

% Firm number change (N×J)
M_hat = reshape(guess(2:end), [N J]);

%% -------- 3. Eq.(13): Wage condition --------

w_hat_nnj   = reshape(w_hat, [N 1 1]);
w_d_hat_nnj = reshape(w_hat, [1 N 1]);
M_hat_nnj   = reshape(M_hat, [N 1 J]);
beta_hat_nnj= reshape(beta_hat,[1 N J]);
NX_d_hat_nnj = reshape(NX_hat, [N 1 1]);

baseline.R_d_nnj = reshape(baseline.R, [1 N 1]);
baseline.NX_nj = reshape(baseline.NX, [N 1]);
baseline.NX_d_nnj = reshape(baseline.NX, [1 N 1]);

parameter.theta_nnj = reshape(parameter.theta, [1 1 J]);

diff1 = w_hat_nnj - sum(     (baseline.zeta .* w_hat_nnj.^(-parameter.theta_nnj) .*
    Gamma_hat) ...
./ (sum(baseline.lambda .* M_hat_nnj .* w_hat_nnj.^(-parameter.theta_nnj) .*
    Gamma_hat, 1)) ...
.* beta_hat_nnj ...
.* (    (w_d_hat_nnj .* baseline.R_d_nnj - NX_d_hat_nnj .* baseline.NX_d_nnj) ...
./ (baseline.R_d_nnj - baseline.NX_d_nnj) ...
) ...
```

```
,2);

diff1 = squeeze(diff1);
diff1 = reshape(diff1, [N*J 1]);

%% -------- 4. Eq.(12): Market equilibrium condition (resource condition) --------
% Parameter constants
cons1 = (parameter.theta - (parameter.sigma-1).*(1-parameter.alpha)) ...
./ (parameter.sigma .* parameter.theta);
cons2 = ((parameter.sigma-1).*(parameter.theta-parameter.alpha+1)) ...
./ (parameter.sigma .* parameter.theta);

cons1 = reshape(cons1, [1 J]);
cons2 = reshape(cons2, [1 J]);

% □:  (Footnote 16)
psi =  (1 - sum(cons1 .* baseline.beta, 2)) ...
./ (1 - sum(cons1 .* beta_hat .* baseline.beta, 2));
psi = reshape(psi, [N 1]);

% □:  (Footnote 16)
eta_0 = sum( (-cons1+1) .* baseline.beta .* baseline.NX_nj ...
- baseline.NXs .* cons2 , 2);

% □_post:
eta_0_post = sum( (-cons1+1).*beta_hat.*baseline.beta.*NX_hat.*baseline.NX_nj ...
- NXs_hat.*baseline.NXs.*cons2, 2);
eta_0_post = eta_0_post ./ w_hat;



eta_0      = reshape(eta_0, [N 1]);
eta_0_post = reshape(eta_0_post, [N 1]);

% Eq.(12): resource condition
diff2 = 1 - psi .* ( sum(M_hat .* baseline.Rs .* cons2, 2) + eta_0_post ) ...
./ ( sum(baseline.Rs .* cons2, 2) + eta_0 );
```

```
diff2 = diff2(2:end);   % drop one equation (numeraire country)

%% -------- 5. Combine residuals of Eq.(12)+(13) --------
diff = [diff2; diff1];

end
```

Code 2: Matlab code for Algorithm to solve $\hat{w}_o$ and $\hat{M}^e_{o,s}$

B.3. Code for Data Processing $X_{od,s}$

```
%======================================================================
% Date:         2025-10-10
% Author:       Wenjie Luo
%
% Purpose:
%   This script solves the nonlinear system consisting of Eq. (12) and
%   Eq. (13) in the model. The system determines the equilibrium wage
%   and the number of firms (in hat values) under various counterfactual
%   scenarios. Numerical methods (fsolve) are used to find the solution
%   based on the baseline calibration and specified shocks.
%
% Reference:
%   - Shapiro, J.S. and Walker, R. (2018). “Why Is Pollution from U.S.
%     Manufacturing Declining? The Roles of Environmental Regulation,
%     Productivity, and Trade”. *American Economic Review*, 108(12),
% –    38143854.
%
% Notes:
%   - Input data "rawFile.mat" comes from replication file in SW2018
%   - CHANGE "pollution_index" \in [1, 7] in Line 41 for different pollutants
%======================================================================

clear all;
clc;

```

```
% Try to set the working directory automatically
if isempty(mfilename('fullpath'))
% Fall back to current folder if script is run interactively
currentPath = pwd;
else
currentPath = fileparts(mfilename('fullpath'));
cd(currentPath);
end



%% ==================== Basic Settings ====================
load rawFile.mat

% Select the index of the pollutant (1=CO, 2=NOx, 3=PM10, …)
pollution_index = 1;

% Data structure: xbilat(o,d,j,t)
% o: origin countries (2: foreign, US)
% d: destination countries (2: foreign, US)
% j: industries (17 industries)
% t: years (19 years, −19902008)
%
% Original data:
% co2_poll(j,t) - CO2 emissions (6 observed years)
% poll(j,t,p) - emissions of 7 pollutants (6 observed years)
% vship(o,j,t) - production by country and industry (equivalent to Rs)
% xbilat(o,d,j,t) - bilateral trade flows

%% ==================== Parameter Settings ====================
N  = 2;       % Number of countries (1=foreign, 2=US)
us = 2;       % US index
J  = 17;      % Number of industries
Y  = 19;      % Number of years
yvec = 1990:2008;
pollutants = {'co','nox','pm10','pm25','so2','voc','co2'};
```

```
% run plot_pollution_trend.m

% parameter: model parameters
% alpha: pollution elasticity: Table 2, Column (2)
parameter.alpha = [.0040; .0022; .0103; .0223; .0212; .0205; .0048; .0303; .0557;
    .0019; .0015; .0023; .0005; .0014; .0016; .0019; .0047];

% inputshare: wL/R: Table 2, Column (3)
inputshare = [.74;.79;.83;.79;.88;.70;.78;.73;.85;.79;.76;.81;.79;.65; .82;.74;.73];

% sigma: elasticity of substitution <- Footnote 8, App p19
parameter.sigma = (1-parameter.alpha)./((1-parameter.alpha)-inputshare);

% theta: Pareto shape parameter: Table 2, Column (5)
parameter.theta = [4.81; 5.38; 8.30; 4.29; 17.52; 4.13; 5.02; 3.39; 9.72; 5.60;
    4.30; 5.07; 4.13; 2.09; 5.29; 3.27; 4.77];

% pm.dirty: dirty industries; pm.dirty: clean industries
% parameter.dirty = [4 5 6 8 9];
% parameter.clean = [1 2 3 7 10 11 12 13 14 15 16 17];

%% ==================== Data Processing ====================
% Total import expenditure and total revenue
Es = squeeze(sum(xbilat,1));     % destination-sector-year ∑(origin)
Rs = squeeze(sum(xbilat,2));     % origin-sector-year ∑(destination)
NXs = Rs - Es;                   % Net exports by industry

E = squeeze(sum(Es,2));          % destination-year
R = squeeze(sum(Rs,2));          % origin-year
NX = R - E;                      % Net exports by country

% □: CES expenditure share
beta   = Es ./ reshape(E, [N 1 Y]);            % d×j×t
lambda = xbilat ./ reshape(Es, [1 N J Y]);     % o×d×j×t, expenditure share
zeta   = xbilat ./ reshape(Rs, [N 1 J Y]);     % o×d×j×t, revenue share
```

```

% extrapolate CO2 emissions
%   [1991,1994,1998,2002,2006,2010] -> (1990:2008)
for j = 1:J
Z_co2(j,:) = interp1([1991,1994,1998,2002,2006,2010]',co2_poll(j,:)',(1990:2008)','linear','extrap')';
for n = 1:length(pollutants)
Z(j,:,n) = interp1q([1990,1996,1999,2002,2005,2008]',poll(j,:,n)',(1990:2008)')';
end
end
Z(:,:,7) = Z_co2;

%% ==================== Baseline Data (t=1, 1990) ====================
baseline.xbilat = xbilat(:,:,:,1);
baseline.lambda = lambda(:,:,:,1);
baseline.zeta   = zeta(:,:,:,1);
baseline.Rs     = Rs(:,:,1);        % origin×sector
baseline.R      = R(:,1);           % origin
baseline.NXs    = NXs(:,:,1);       % origin×sector
baseline.NX     = NX(:,1);          % origin
baseline.Z      = Z(:,1,:);         % sector×pollutant
baseline.wage   = baseline.R;       % Baseline wage □ income
baseline.beta   = beta(:,:,1);      % destination×sector

%% ==================== Hat Variables ====================
lambda_hat = lambda ./ baseline.lambda;
Z_hat      = Z ./ baseline.Z;
Rs_hat     = Rs ./ baseline.Rs;
R_hat      = R ./ baseline.R;
NXs_hat    = NXs ./ baseline.NXs;
NX_hat     = NX ./ baseline.NX;
w_hat      = R ./ baseline.wage;                   % Equation (13), wage hats
M_hat      = Rs_hat ./ reshape(w_hat, [N 1 Y]);    % Equation (11), mass of firms hats

%% ==================== Shocks ====================
```

```
beta_hat = beta ./ baseline.beta;

% Pollution tax (Eq. 15)
t_hat = reshape(M_hat, [N 1 J Y]) ...
.* reshape(w_hat, [N 1 1 Y]) ...
./ reshape(Z_hat(:,:,pollution_index), [1 1 J Y]);
t_hat = squeeze(t_hat);

% Construct 4D arrays (o×d×j×t) to be substituted into Eq.19 / Eq.21
M_hat_4D     = reshape(M_hat, [N 1 J Y]);
w_hat_4D     = reshape(w_hat, [N 1 1 Y]);
w_d_hat_4D   = reshape(w_hat, [1 N 1 Y]);
beta_hat_4D  = reshape(beta_hat, [1 N J Y]);
R_hat_4D     = reshape(R_hat, [N 1 1 Y]);
R_d_hat_4D = reshape(R_hat, [1 N 1 Y]);
NX_hat_4D    = reshape(NX_hat,[N 1 1 Y]);
NX_d_hat_4D = reshape(NX_hat, [1 N 1 Y]);
t_hat_4D = reshape(t_hat, [N 1 J Y]);

baseline.R_d_4D = reshape(baseline.R, [1 N 1 1]);
baseline.NX_d_4D = reshape(baseline.NX,[1 N 1 1]);

parameter.theta_4D = reshape(parameter.theta,[1 1 J 1]);
parameter.sigma_4D = reshape(parameter.sigma,[1 1 J 1]);
parameter.alpha_4D = reshape(parameter.alpha,[1 1 J 1]);



%% ==================== Gamma (Eq.19, Eq.21) ====================
Gamma_hat_star = lambda_hat ...
./ (M_hat_4D .* w_hat_4D.^(-parameter.theta_4D)) ...
.* ( (beta_hat_4D ./ w_d_hat_4D) ...
.* ((R_d_hat_4D.*baseline.R_d_4D - NX_d_hat_4D.*baseline.NX_d_4D) ...
./ (baseline.R_d_4D - baseline.NX_d_4D)) ...
).^( 1 - parameter.theta_4D ./ ((parameter.sigma_4D - 1).*(1 - parameter.alpha_4D))
    );
% Foreign only
```

```
Gamma_hat_foreign = [Gamma_hat_star(1,:,:,:); ones(1,2,J,Y)];
% US only, with environmental tax shock (Eq.21)
Gamma_hat_US = Gamma_hat_star ./ t_hat_4D.^( -(parameter.alpha_4D.*parameter.theta_4D)./(1-parameter.alpha_4D) );
Gamma_hat_US = [ones(1,2,J,Y); Gamma_hat_US(2,:,:,:)];

% Save
shocks.Gamma_hat_foreign = Gamma_hat_foreign;
shocks.Gamma_hat_US = Gamma_hat_US;
shocks.beta_hat = reshape(beta_hat,[1 N J Y]);

t_hat_shcok = t_hat_4D.^( - (parameter.alpha_4D .* parameter.theta_4D) ...
./ (1 - parameter.alpha_4D) ...
);
t_hat_shcok = squeeze(t_hat_shcok);
shocks.t_hat = reshape([ones(1, 17, 19); t_hat_shcok(2, :, :)], [N 1 17 19]);

save('my_results.mat');

%% ==================== Step 1: Construct Initial Guess ====================
% Corresponds to Appendix p.22 Step 1
% Initial guess: current wage change (w_hat) and firm number change (M_hat)
% Dimension: [ (-N1 wages) + (N*J firms) ] × 1 vector
for n = 1:Y
guess(:, n) = [ w_hat(2, n); ...                          % US wage change (foreign wage as numeraire)
reshape(M_hat(:,:,n), [N*J, 1]) ];       % Firm number change for each country×industry
end

%% ==================== Step 2: Solve the Nonlinear System Using fsolve ====================
% Corresponds to Appendix p.22 Step 2
% Solve the system of equations (Eq.12, Eq.13), unknowns = w (-N1) + M (N*J)
% Number of equations = number of unknowns → system solvable
options = optimset('Display','off', ...
```

```
    'MaxFunEvals',60000, ...
    'MaxIter',4500, ...
    'TolFun',1e-14, ...
    'TolX',1e-14, ...
    'Algorithm','trust-region-dogleg');

    for n = 1:Y
    for loop_shock = 1:4   % Four types of shocks (Appendix p.22 Step 3)
    initial_guess = squeeze(guess(:, n));

    % Call fsolve to solve Eq.(12)+(13)
    [solver, fval, flag] = fsolve(@(g) solveWhatMhat(g, baseline, shocks, loop_shock, n,
         N, J, parameter), ...
    initial_guess, options);

    % ================== Step 3: Update Variables (Eq.12, Eq.13) ==================
    % Results from fsolve include:
    % solver(1) = w_US (wage change)
    % solver(2:end) = M_hat (firm number change, reshaped to N×J)

    % Recover wage changes (foreign wage determined by numeraire)
    w_US = solver(1);
    w_foreign = (1 - w_US .* (baseline.R(2) ./ sum(baseline.R))) ...
    ./ (baseline.R(1) ./ sum(baseline.R));
    % w_us * US's GDP(or wage) share + w_foreign * Foreign's GDP(or wage) share = 1
    w_hat = [w_foreign; w_US];  % N×1 vector

    % Firm number change (N×J matrix)
    M_hat = reshape(solver(N:end), [N J]);

    % Expand wages to N×J for emission calculation
    w_hat_nj = reshape(w_hat,[N 1]);

    % ================== Counterfactual Emissions (Eq.15, Eq.19, Eq.21)
        ==================
    if loop_shock == 3
```

```matlab
% Regulation shock
Z_hat_counterfactual(:,:,n,loop_shock) = (w_hat_nj .* M_hat) ./ squeeze(t_hat(:,:,n));
else
% Other shocks, pollution tax fixed at baseline
Z_hat_counterfactual(:,:,n,loop_shock) = (w_hat_nj .* M_hat) ./ squeeze(t_hat(:,:,1));
end
end
end

%% ==================== Calculate Aggregate Emissions (Weighted Average, Eq.22) ====================
% Weight = baseline pollution level (Z0)
Z0_sum = sum(baseline.Z(:,:,pollution_index),1);  % Sectoral baseline pollution ∑origin
Z0_sum = reshape(Z0_sum, [1 1 1]);

% Counterfactual US emission path (weighted sum, converted to %)
Z_hat_counterfactual_us = squeeze(Z_hat_counterfactual(us,:,:,:));
Z_hat_counterfactual_us = sum(Z_hat_counterfactual_us .* baseline.Z(:,:,pollution_index),1) ./ Z0_sum;
Z_hat_counterfactual_us = squeeze(Z_hat_counterfactual_us) .* 100;

% Actual data (observed emission trajectory)
Z_hat_real = squeeze(sum(Z(:,:,pollution_index),1))';
Z_hat_real = Z_hat_real ./ Z_hat_real(1) .* 100;

% Combine actual and counterfactual results
Z_hat_counterfactual_us = [Z_hat_real'; Z_hat_counterfactual_us']';

%% ==================== Plotting ====================
% Create a folder called "figure" (if it doesn't exist)
figDir = fullfile(currentPath, 'figure');
if ~exist(figDir, 'dir')
mkdir(figDir);
```

```
    end

    nei_yrs = [1990 1996 1999 2002 2005 2008];
    nei_yrsIndex = [1 7 10 13 16 19];

    f = figure('Visible','on');
    figHandles(pollution_index) = f; % store handle for later combination
    clf;

    plot(1990:2008,Z_hat_counterfactual_us(:,1),'-','LineWidth',2.5); hold on;

    plot(nei_yrs,Z_hat_counterfactual_us(nei_yrsIndex,2),'rp','LineWidth',1.5);
    plot(nei_yrs,Z_hat_counterfactual_us(nei_yrsIndex,3),'ro','LineWidth',1.5);
    plot(nei_yrs,Z_hat_counterfactual_us(nei_yrsIndex,4),'rs','LineWidth',1.5);
    plot(nei_yrs,Z_hat_counterfactual_us(nei_yrsIndex,5),'rv','LineWidth',1.5);

    plot(1990:2008,Z_hat_counterfactual_us(:,2),'--r','LineWidth',1.5);
    plot(1990:2008,Z_hat_counterfactual_us(:,3),'--r','LineWidth',1.5);
    plot(1990:2008,Z_hat_counterfactual_us(:,4),'--r','LineWidth',1.5);
    plot(1990:2008,Z_hat_counterfactual_us(:,5),'--r','LineWidth',1.5);

    set(gca,'FontSize',13,'YTick',[0 30 60 90 120 150],'XTick',[1990 1995 2000 2005
        2010]);
    axis([1990 2008 0 150]);
    xlabel('Year'); ylabel('1990=100'); box off;
    legend('Actual Data (All Shocks)', ...
    'Foreign Competitiveness Shocks Only', ...
    'U.S. Competitiveness Shocks Only', ...
    'U.S. Regulation Shocks Only', ...
    'U.S. Expenditure Share Shocks Only', ...
    'Location','Southwest');
    title(upper(pollutants{pollution_index}));
    saveas(f, fullfile(figDir, [pollutants{pollution_index}, '.png']));
```

Code 3: Matlab code for Data Processing $X_{od,s}$