

Definition 1. The minimum load of τ_i in Z_k is the minimum units of execution time that must be assigned to τ_i in Z_k to meet the deadline.

$$Lt_{i,k} = \text{MAX}(0, Z\text{Width}_k - L_{i,k})$$

Definition 2. The maximum load of τ_i in Z_k is the maximum units of execution time that can be assigned to τ_i in Z_k considering the WCET of a job and the width of a Zone (to ensure a task can only be executed in a processor at a time).

$$Mt_{i,k} = \text{MIN}(R_{i,k}, Z\text{Width}_k)$$

Definition 3. The execution assignment is **rational** in Z_k if execution time assigned to each task is between minimum load and maximum load, and the sum of execution time of tasks in Γ doesn't exceed the capacity of Z_k .

$$\forall i \in [0, n) : Lt_{i,k} \leq alloc_{i,k} \leq Mt_{i,k}$$

$$\sum_{i=0}^{n-1} alloc_{i,k} \leq Cap_k$$

Definition 4. **Overload effect** occurs if the total minimum load of tasks in Γ is more than the capacity of Z_k .

$$\sum_{i=0}^{n-1} Lt_{i,k} > Cap_k$$

Definition 5. Zone Z_p **zone-connects** to zone Z_q in $J_{i,k}$, denoted as $Z_p \rightarrow Z_q$, if

$$Z_p \in J_{i,k}, Z_q \in J_{i,k}$$

and

$$alloc_{i,p} > 0, alloc_{i,q} < Z\text{Width}_q.$$

The connection between Z_p and Z_q is called a **zone-connection**.

Definition 6. Zone Z_p **connects** to zone Z_q denoted as $Z_p \Rightarrow Z_q$ if

$$Z_p \rightarrow Z_q$$

or

$$\exists k : Z_p \Rightarrow Z_k, Z_k \Rightarrow Z_q.$$

Definition 7. During scheduling, the current zone Z_s is called a **source zone** if overload effect occurred in Z_s .

Definition 8. In a source zone Z_s , a **source point** $alloc_{i,s}$ is an element in scheduling matrix such that: $alloc_{i,s} > 0$ and $L_{i,s} < T_i - C_i$.

Definition 9. During scheduling, the overload effect occurs in the current zone Z_s . A zone Z_d is called a **destination zone** if

- (1) $\exists J_{i,j}$ is a current job ($JStart_{i,j} \leq ZStart_s, JEnd_{i,j} \geq ZEnd_s$), $Z_d \in J_{i,j}$, $alloc_{i,d} > 0$, $L_{i,s} > 0$. or
(2) $Rem_d > 0$.

The element $alloc_{i,d}$ in scheduling matrix is called a **destination point**.

Lemma 1. When overload effect occurs in Z_k , there exist a source point $alloc_{i,k}$.

Proof. Γ_l is the task set that $\forall \tau_i \in \Gamma_l : alloc_{i,k} > 0$. It is obvious that $\Gamma_l \neq \emptyset$. □

Lemma 2. When overload effect occurs in Z_k , there exist a destination zone Z_d , if $Rem_d = 0$, there exist a destination point $alloc_{i,d}$.

Proof. ... □

Lemma 3. For a taskset Γ with a total utilization of $m * ut$ ($0 < ut \leq 1$), if execution assignment is rational in each zone Z_i ($i \in [0, k), k \leq znum$), and overload effect occurs in zone Z_k , then $\exists Z_d$ is a destination zone, $d \neq k$, $Z_k \Rightarrow Z_d$.

Proof. ... □