Lemma 1. *For a task set Γ with a total utilization of $m * ut$ $(0 < ut \leq 1)$, if execution assignment is rational in each zone Z_i $(i \in [0,k), k \leq znum)$, and overload effect occurs in zone Z_k, then $\exists Z_d$ is a destination zone, $d \neq k$, $Z_k \Rightarrow Z_d$.*

Proof. We consider interval $[ZEnd_k, hp)$ as a special big zone Z_B, $ZStart_B = ZEnd_k$; $ZEnd_B = hp$; $ZWidth_B = hp - ZEnd_k$; $Cap_B = ZWidth_B \cdot m$.
$Zones = \{Z_i : i = 0,1,2,...,k,B\}$; $S = \{Z : Z \in Zones, Z_k \Rightarrow Z\}$; $\bar{S} = \{Z : Z \in Zones, Z \notin S, Z \neq Z_k\}$.
$overflow = \sum_{i=0}^{n-1} alloc_{i,k} - m \cdot ZWidth_k$; $tr = \sum_{i=0}^{k-1} Rem_i$.
For a job $J_{i,j}$, $S_1 = J_{i,j} \cap S$, $S_2 = J_{i,j} \cap \bar{S} \neq \emptyset$.

$$\sum_{Z_l \in S_2} alloc_{i,l} = MIN(C_i, \sum_{Z_l \in S_2} ZWidth_l) \geq \frac{C_i}{T_i} \sum_{Z_l \in S_2} ZWidth_l$$

The equality holds only if $S_1 = \emptyset$. So for all jobs having an intersection with $\bar{S}$, the total executing time in $\bar{S}$:

$$E(\bar{S}) = \sum_{J_{i,j} \cap \bar{S} \neq \emptyset} \sum_{Z_l \in J_{i,j} \cap \bar{S}} alloc_{i,l} \geq \sum_{Z_l \in \bar{S}} ZWidth_l \sum_{i=0}^{n-1} \frac{C_i}{T_i} = m \cdot ut \sum_{Z_l \in \bar{S}} ZWidth_l.$$

The equality holds only if for each task $\tau_i \in \Gamma$, any job of τ_i locates complete in either S or $\bar{S}$. It is known that only time 0 and hp can be such boundaries, so we can get that

$$E(\bar{S}) > m \cdot ut \sum_{Z_l \in \bar{S}} ZWidth_l. \tag{1}$$

On the other hand, the assigned time units of Z_l is

$$A(Z_l) = \begin{cases} m \cdot ZWidth_l - Rem_l & l = 0,1,2,...,k-1 \\ m \cdot ut \cdot hp - m \sum_{i=0}^{k} ZWidth_i + tr - overflow & l = B \end{cases}$$

case1: $Z_B \in S$
In this case, $Z_k \Rightarrow Z_B$. Assuming that $Z_k \rightarrow Z_B$, then $\exists \tau_i \in \Gamma$, the current job $J_{i,j}$ satisfys the conditions that $JStart_{i,j} \leq ZStart_k$, $JEnd_{i,j} \geq ZEnd_l$, $alloc_{i,k} > 0$, and $alloc_{i,l} < ZWidth_l$. This contradict to the definition of overload effect. So $\exists Z_{o1}, Z_{o2}, ..., Z_{oz} \in Zones : Z_k \rightarrow Z_{o1}, Z_{o1} \rightarrow Z_{o2}, ..., Z_{oz} \rightarrow Z_B$, $Z_{oz} \neq Z_k, Z_{oz} \neq Z_B$.
$\exists J_{i,j}$ is a current job, $alloc_{oz} > 0$ and $L_{i,k} > ZWidth_k$, which means Z_{oz} is a destination zone.
case2: $Z_B \in \bar{S}$, $\bar{S}' = \bar{S} - Z_B$

$$\begin{aligned} A(\bar{S}) &= m \cdot ut \cdot hp - m \sum_{Z_l \in \bar{S}' \cup Z_k} ZWidth_l + tr - \sum_{Z_l \in \bar{S}'} Rem_l - overflow \\ &\leq m \cdot ut \sum_{Z_l \in \bar{S}} ZWidth_l + tr - \sum_{Z_l \in \bar{S}'} Rem_l - overflow \end{aligned}$$

If $\exists Z_d \in S : Rem_d > 0$, Z_d is the destination zone. Assume that $\forall Z_l \in S, Rem_l = 0$, we can get that $tr = \sum\limits_{Z_l \in \bar{S}'} Rem_l$, $A(\bar{S}) \leq m \cdot ut \sum\limits_{Z_l \in \bar{S}} ZWidth_l$. This is contradict to inequation 1.

In conclusion, $\exists Z_d$ is a destination zone, $d \neq k$, $Z_k \Rightarrow Z_d$. $\square$