Definition 1. *During scheduling, the overload effect occurs in the current zone Z_s. A zone Z_d is called a* ***destination zone*** *if*
(1) $\exists \tau_i \in \Gamma : R_{i,s} < C_i, L_{i,s} > ZWidth_s$, *$J_{i,j}$ is the current job of τ_i, $Z_d \in J_{i,j}$ and $alloc_{i,d} > 0$. or*
(2) $Rem_d > 0$.
The element $alloc_{i,d}$ in scheduling matrix is called a ***destination point****.*

Lemma 1. *When overload effect occurs in Z_k, there exist a destination zone Z_d, if $Rem_d = 0$, there exist a destination point $alloc_{i,d}$.*

Proof. (1)$\exists i \in [0, k), Rem_i > 0$, Z_i is a destination zone.
(2)$\forall i \in [0, k), Rem_i = 0$. We assume that $\forall \tau_i \in \Gamma : alloc_{i,k} > 0$, then:
for $\tau_i \in \Gamma$, $J_{i,j}$ is the current job. If $JEnd_{i,j} = ZEnd_k$, then

$$\frac{\sum\limits_{t=k+1}^{znum-1} alloc_{i,t}}{\sum\limits_{t=k+1}^{znum-1} ZWidth_t} = \frac{C_i}{T_i}$$

If $JEnd_{i,j} > ZEnd_k$, then

$$\frac{\sum\limits_{t=k+1}^{znum-1} alloc_{i,t}}{\sum\limits_{t=k+1}^{znum-1} ZWidth_t} = \frac{C_i \cdot rcj_i + JEnd_{i,j} - ZEnd_k}{T_i \cdot rcj_i + JEnd_{i,j} - ZEnd_k} > \frac{C_i}{T_i}$$

(rcj_i is the number of remaining complete job of τ_i).
So the total remaining execution units needed in the zones after Z_k is

$$\sum_{i=0}^{n-1} \sum_{t=k+1}^{znum-1} alloc_{i,t} \geq \sum_{i=0}^{n-1} \frac{C_i}{T_i} \cdot \sum_{t=k+1}^{znum-1} ZWidth_t = m \cdot \sum_{t=k+1}^{znum-1} ZWidth_t$$

This contratict to

$$\sum_{i=0}^{n-1} \sum_{t=0}^{znum-1} alloc_{i,t} = m * \sum_{t=0}^{znum-1} ZWidth_t.$$

So $\exists \tau_i \in \Gamma : alloc_{i,k} = 0$, which means $L_{i,s} > ZWidth_s$. $\Gamma_e = \{\tau_i : \tau_i \in \Gamma, alloc_{i,k} = 0\}$.

We assume that $\forall \tau_i \in \Gamma_e, R_{i,k} = C_i$. For $\tau_i \in \Gamma_e$,

$$\frac{\sum\limits_{t=k+1}^{znum-1} alloc_{i,t}}{\sum\limits_{t=k+1}^{znum-1} ZWidth_t} = \frac{C_i + C_i \cdot rcj}{T_i - (ZEnd_k - JStart_{i,j_i}) + T_i \cdot rcj} \leq \frac{C_i}{T_i}.$$

For $\tau_i \notin \Gamma_e$,

$$\frac{\sum_{t=k+1}^{znum-1} alloc_{i,t}}{\sum_{t=k+1}^{znum-1} ZWidth_t} = \frac{T_i - (ZEnd_k - JStart_{i,j_i} + C_i \cdot rcj}{T_i - (ZEnd_k - JStart_{i,j_i}) + T_i \cdot rcj} \leq \frac{C_i}{T_i}.$$

So the total remaining execution units needed in the zones after Z_k is

$$\sum_{i=0}^{n-1} \sum_{t=k+1}^{znum-1} alloc_{i,t} \geq \sum_{i=0}^{n-1} \frac{C_i}{T_i} \cdot \sum_{t=k+1}^{znum-1} ZWidth_t = m \cdot \sum_{t=k+1}^{znum-1} ZWidth_t$$

This contratict to

$$\sum_{i=0}^{n-1} \sum_{t=0}^{znum-1} alloc_{i,t} = m * \sum_{t=0}^{znum-1} ZWidth_t.$$

So $\exists \tau_i \in \Gamma_e : R_{i,k} < C_i$. In conclusion, $\exists \tau_i \in \Gamma : R_{i,k} < C_i, L_{i,s} > ZWidth_s$, there exist a destination zone. $\square$