2024 年 CFA 一级——Derivatives 学习笔记

【本科目的知识，重新做了梳理，以具体衍生产品为维度而非原版书的 module。大家在使用这个科目的原版书课后题时需注意。】

目录

1. 基本概念

- derivative 衍生品：一个金融工具（合同），依赖于标的资产（underlying asset）价格变动的合约。
- 交易双方：
  - ✓ The long: buyer, holder
  - ✓ The short: seller, writer
- Underlying asset 不同
  - ✓ Financial asset 金融资产: equity, fixed-income security, currency;
  - ✓ Physical asset 实务资产: commodity;
  - ✓ Other: interest rate, credit, weather, other derivatives

- 衍生品分类
  - ✓ Forward commitment 远期承诺：交易双方权利义务对等，必须履行
    - ◆ Forward/futures/swap
  - ✓ Contingent claim 或有索取权：双方权利义务不对等，long 方可以行使权利也可以不行使
    - ◆ Option
  - ✓ ETD&OTC
    - ◆ Exchange-traded derivative (ETD) markets：交易所交易市场
      - Futures/ Some options
    - ◆ Over-the-counter (OTC) markets OTC 市场/场外市场
      - Forward/Swap/Some options
  - ✓ OTC 和交易所对比

OTC (over-the-counter)	Exchange Traded
Customized	Standardized
Trade with counterparty (default risk)	Backed by a clearing house
Not trade in a central location	Trade in a physical exchange
Unregulated	Regulated

Large trading volume	Small trading volume

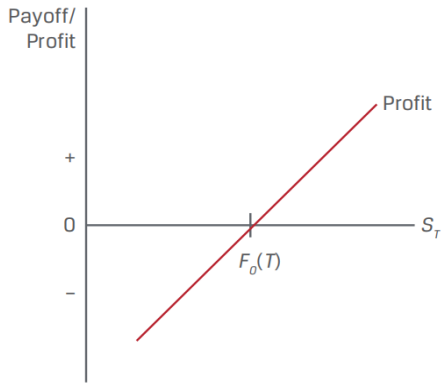
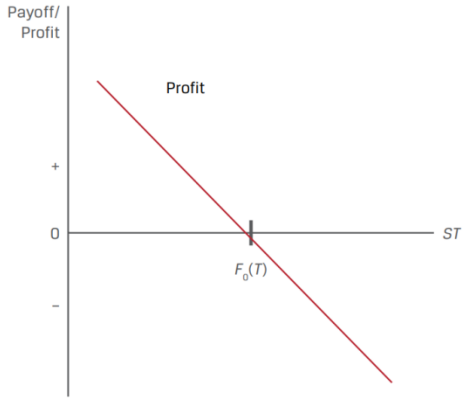
- 衍生品的 Benefits
  - ✓ Risk Allocation, Transfer, and Management 风险分配、转移和管理
  - ✓ Information Discovery 信息发现
  - ✓ Operational Advantages 运营优势
    - ■ 减少现金支出，降低交易成本，提高流动性和“做空”能力
  - ✓ Market Efficiency 市场效率
    - ■ 利用套利机会或错误定价的成本更低
- 衍生品的 risk
  - ✓ Greater Potential for Speculative Use 导致比较多的投机
    - ■ 某些衍生工具策略的高度隐性杠杆可能会增加财务困境的可能性。
  - ✓ Lack of Transparency 缺乏透明度
    - ■ 衍生工具增加了投资组合的复杂性，可能会造成利益相关者不太了解的风险状况。
  - ✓ Basis Risk 基差风险
    - ■ 衍生工具的预期价值与标的资产价格变化不一致的风险
  - ✓ Liquidity Risk 流动性风险
  - ✓ Counterparty Credit Risk 交易对手信用风险
  - ✓ Destabilization and Systemic Risk
    - ■ 过度冒险和在衍生品市场使用杠杆可能会加剧市场压力

2. Forward 远期

1) 基本概念

- 远期合约指双方约定未来特定时间以约定价格买卖一定数量的标的资产的合约
  - ✓ 未来特定时间：远期到期时间 T
  - ✓ 约定价格：forward price 远期价格 $F_0(T)$
  - ✓ 买资产：long forward
  - ✓ 卖资产：short forward

2) 远期 Payoff：S_T：T 时标的资产价格

Long Forward	Short Forward
	
Payoff from Buying = $S_T - F_0(T)$ 标的资产价格上升，赚钱	Payoff from Selling = $-[S_T - F_0(T)]$ 标的资产价格下降，赚钱

3) 到期交割方式

- 实物交割 Physical settlement：一手交钱一手交货
- 现金交割 Cash settlement：补差价

4) 定价

【远期和期货的定价相同。考虑不同的远期和期货类型，其定价公式有很多种，不需要一一记住，需记住基本公式再加上“+成本，-收益”口诀即可。】

- 基本公式：
  - ✓ 离散复利: $\mathbf{F_0(T) = S_0\,(1 + R)^T}$
  - ✓ 连续复利：$F_0(T) = S_0e^{rt}$

    无风险利率：Opportunity cost (risk-free interest rate, r)
- **口诀：+成本，-收益**
  - ✓ **成本类：Storage cost、Financing costs 等**
  - ✓ **收益类：dividend、便利性收益（convenience yield）等**
    - ■ 便利性收益：is a non-cash benefit associated with owning an underlying physical commodity that arises under certain economic conditions.持有现货资产带来的好处。
- 小结：

	定价公式

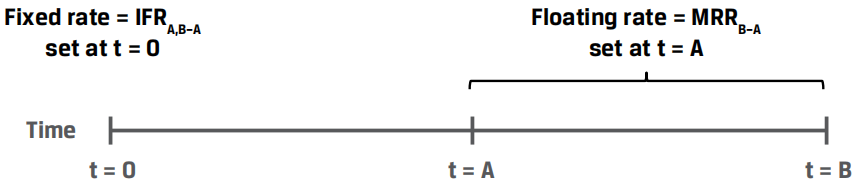
成本（C）收益（I）现金形式	$F_0(T) = (S_0 + C_0 - I_0)(1+R)^T$ —C_0、I_0指的是其现值
成本（c）收益(q)“%”形式	$F0(T) = S_0 e^{(r+c-q)t}$

5) 远期估值

At initiation	$\mathbf{V_{Forward} = 0}$
During its life ($t < T$)	$V_{Forward} = S_t - \frac{F_0(T)}{(1+R)^{T-t}}$
At expiration ($t = T$)	$V_{Forward} = S_T - F_0(T)$

6) FRA 远期利率协议

- Forward rate agreement(FRA)远期利率协议:A forward rate agreement (FRA) is a contract in which counterparties agree to apply a specific interest rate to a future period.
  - ✓ 是一个远期合约，标的资产是利率。
- FRA 的报价形式：“3×9FRA”表示：3 个月以后开始借钱，9 个月以后还钱，借款期限为 6 个月；**0-3 个月这段时间为 FRA 的合约期间**
- 交易双方
  - ✓ **Long FRA(收浮动利率 MRR，支固定利率)→long market reference rate(MRR)→borrower:利率上升，赚钱**
  - ✓ **Short FRA(收固定利率，支浮动利率 MRR)→short market reference rate(MRR)→lender:利率下降，赚钱**
- 交易机制



t=0	签订 FRA，确定 fixed rate	
t=A	settlement 结算时间	**Long FRA 角度：** **实际上的结算金额：PV(Net Payment)**
t=B	B-A，MRR 的开始时间及	Long FRA 角度：

	期限	Net Payment = $(MRR_{B\text{-}A} - IFR_{A,B\text{-}A}) \times$ Notional Principal × Period

3. Future 期货

- Future 期货：和远期非常类似，只不过是在交易所交易。

1) 与 forward 的区别

Forward（OTC）	**Futures (Exchange Traded)**
Private contracts	Exchange-traded
Unique customized contracts	Standardized contracts
Default risk is present	Guaranteed by clearinghouse
Little or no regulation	Regulated
No margin deposit required	Margin required and adjusted
Settlement at maturity	Daily settlement (mark to market)
Delivery usually happens	Closed out before maturity
Large trading volume	Small trading volume

2) Futures 的 Key Features

- 合约规模 contract size:例如 S&P 500 futures 有两种。$250*点位；$50*点位（mini S&P index futures，市场上交易最活跃。）
- Price limit 价格限制：限制价格变化的规定，基于前一天的结算价格设定一个价格区间
- Mark to market (MTM)逐日盯市：根据结算价格确定每日保证金账户的损益。
  - ✓ Settlement price 结算价格：当天最终期货交易的平均值。
- Margin 保证金
  - ✓ Initial margin（初始保证金）
    - ■ The amount of money which must be deposited when contract is initiated.
  - ✓ Maintenance margin（维持保证金）
    - ■ 维持保证金，是一种临界值。
    - ■ 初始保证金大于维持保证金

- 保证金账户余额达不到维持保证金水平，就需要补充保证金。
- The balance account falls below the maintenance margin, the investor receives a margin call and is expected to top up the margin account to the initial margin level.

✓ Variation margin （变动保证金）

- The extra funds deposited by the investor after receiving a **margin call**（保证金催缴电话）.
- 收到保证金催缴电话，如果不能补充到初始保证金，头寸会被强制平仓。
- Variation margin = initial margin - margin account balance

3) 期货定价

- 满足下面两个条件，期货和远期的定价公式是一样的。但是由于期货每日结算（货币时间价值），使得期货价格和远期价格略有差异。
  - if interest rates are constant, or
  - if futures prices and interest rates are uncorrelated.
- 期货价格和远期价格不同(具体体现在利率和期货价格相关性上)

	结论	解释
正相关	Futures price will be higher	Futures 价格上升，期货赚钱。同时，利率上升。期货赚来的钱可以以一个更高的利率投资，整体收益更高。市场更偏好期货。
负相关	Forward price will be higher	Futures 价格下降，期货亏钱，需要补充保证金。同时，利率上升。保证金融资成本更高。期货整体收益下降。市场更偏好远期。

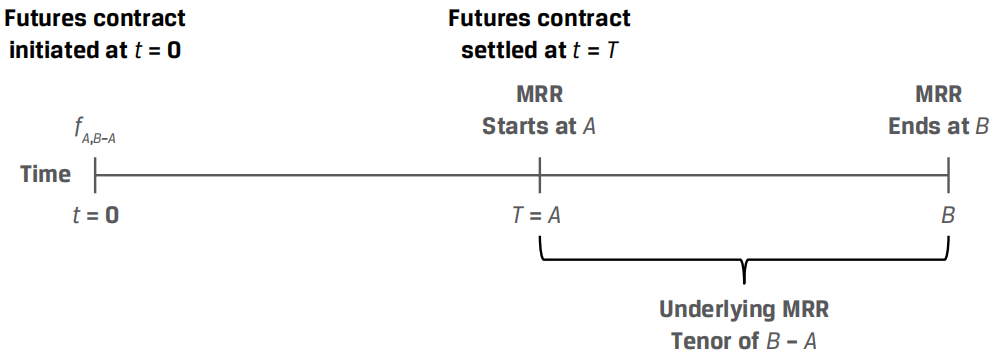
4) 期货估值

- Futures 每日结算，无需对其估值。

5) Interest rate futures 利率期货

【不同国家、不同交易所的利率期货种类有很多，比如 CME group 的长期国债期货、欧洲美元期货等，但 CFA 一级中没有涉及具体的利率期货种类。这里主要掌握期货价格和利率之间的关系】

- 期货报价$= 100 - (100 \times MRR_{A,B-A})$
  - ✓ 根据期货报价，可以得出对应期限的市场利率。
  - ✓ 期货价格和利率正向关系。



- 例如，we may solve for the implied three-month MRR rate in three months' time (where A = 3m, B = 6m, B − A = 3m) if an interest rate futures contract is trading at a price of 98.25:

  $98.25 = 100 - (100 \times MRR_{A,B-A}) \rightarrow MRR_{3m,3m} = 1.75\%$

- 交易双方：
  - ✓ Long futures contract (lender): MRR 下降，利率期货价格上升，赚钱
  - ✓ Short futures contract (borrower): MRR 上升，利率期货价格下降，赚钱
- Futures contract basis point value (BPV)
  - ✓ 利率变化一个 BP，利率期货合约价值变动
  - ✓ Futures Contract BPV = Notional Principal × 0.01% × Period
- FRA&利率期货

Contract Type	Gains from Rising MRR	Gains from Falling MRR
Interest rate futures	Short futures contract	Long futures contract
Forward rate agreement	Long FRA: FRA fixed-rate payer (FRA floating-rate receiver)	Short FRA: FRA floating-rate payer (FRA fixed-rate receiver)

4. Swap 互换

1) 基本概念

- 互换约定的是未来一定时间内一系列现金流的交换，可以看成是一系列的远期合约。

- 互换在 OTC 市场交易，利率互换（interest rate swap）是最流行的场外衍生品合约。

2) 利率互换（interest rate swap）

- Fixed interest rate payments are exchanged for floating-rate payments.
- **固定利率(Fixed interest rate)和浮动利率(floating-rate)交换**
  - ✓ **Fixed interest rate:**互换合约中事先约定好的利率，也叫做 **swap rate**
  - ✓ **Floating-rate：市场利率 MRR**
- 交易双方
  - ✓ **Floating-rate payer (or fixed-rate receiver)：支浮动利率，收固定利率；市场利率下降，赚钱→看空利率**
    - ■ short a floating-rate note (FRN) priced at the MRR and long a fixed-rate bond with a coupon equal to the fixed swap rate
    - ■ **即 short 浮动利率债券，long 固定利率债券**
  - ✓ **Fixed-rate payer (floating-rate receiver)：支固定利率，收浮动利率；市场利率上升，赚钱→看多利率**
    - ■ long a floating-rate note (FRN) priced at the MRR and short a fixed-rate bond with a coupon equal to the fixed swap rate
    - ■ **即 long 浮动利率债券，short 固定利率债券**
- 利率互换机制
  - ✓ 起初和期末不互换本金，期间交换利息。
  - ✓ 互换期间，每一期互换的金额，以净值结算。
  - ✓ 以 fixed-rate payer 为例，

    第 t 期 net payment/Periodic settlement value

    $$=(\text{Swap fixed rate} - \text{MRR}) \times \text{Principal} \times \frac{\#\text{ days per period}}{360}$$
- 利率互换定价
  - ✓ 确定的是固定利率，即 **swap rate**。
- 利率互换估值

T=0 时	互换合约价值$V_{Swap} = 0$

T=t 时 【注意区分收的是固定利率还是浮动利率】	1.固定端价值: 看做是固定利率债券，把未来现金流折现加总。 2.浮动端价值: 看做是浮动利率债券，基于回归面值的特点，PV=面值。 3.$V_{Swap} = V_{收} - V_{支}$

3) 其他互换

- Equity swap/ index swap
  - ✓ The return on a stock index for fixed【fixed return 换浮动收益】
  - ✓ The return on a stock index for the return on another index【浮动收益换浮动收益】
- Currency swap 货币互换
  - ✓ 不同货币之间的互换，一般是固定换固定

5. Option 期权

1) 基本概念

- 期权可以在 OTC 市场，也可以在交易所交易
- Short 期权的一方需要交保证金，long 方不需要交保证金
- 期权的价格 option price=期权的价值 option value=期权的期权费 premium

2) Option 分类（1）

- Long call: 有权买
- Short call: 有义务卖
- Long put: 有权卖
- Short put: 有义务买
- 【long 方是有权利，short 方是有配合的义务】
- writer of the option：是指期权的 short 方

3) Option 分类（2）

- European options 欧式期权：到期日才能选择行权或不行权【下面没有特殊说明的话，主要指的是欧式期权】
- American options 美式期权：到期日之前任何时候都可以选择行权或不

行权

4) Moneyness 价值状态

- In the money: immediate exercise would generate a positive payoff.
- At the money: immediate exercise would generate no payoff.
- Out of the money: immediate exercise would result in a loss.

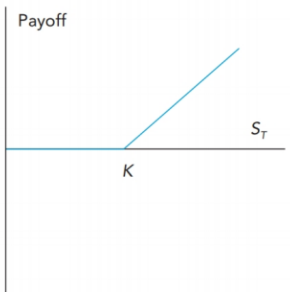
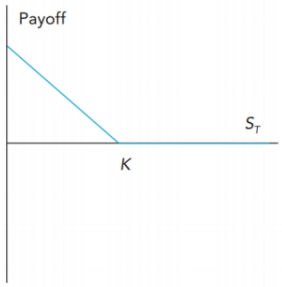
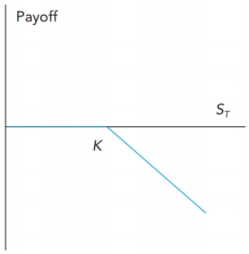
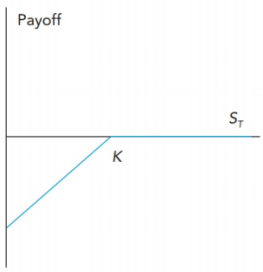
Moneyness	Call option	Put Option
In-the-money(ITM)赚钱	S > K	S < K
At-the-money(ATM) 不赚不亏	S = K	S = K
Out-the-money(OTM)亏钱	S < K	S > K

5) Intrinsic Value and Time Value 内在价值和时间价值

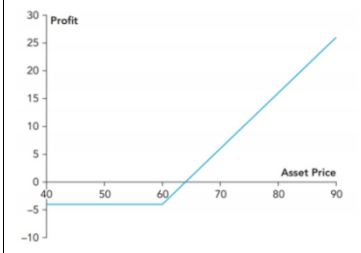
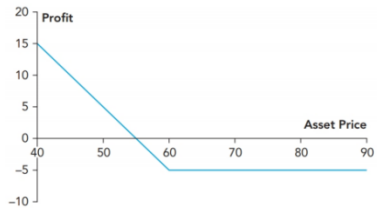
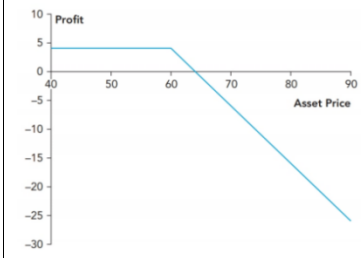
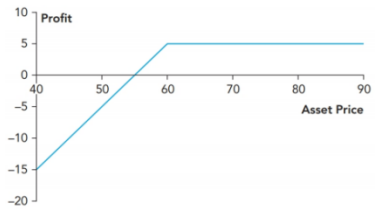
- Intrinsic Value： the amount that it is in the money, and zero otherwise.
  - ✓ Intrinsic value of call option: C= Max(S- X,0)
  - ✓ Intrinsic value of put option: P= Max(X - S ,0)
- Time Value 无法直接计算
- Option value=intrinsic value + time value

6) Payoff and Profit of Option

- Payoffs of Options

Long call Payoff=Max (S_T-X, 0)		Long put Payoff=Max(X-S_T, 0)	
Short call Payoff=-Max(S_T-X, 0)		Short put Payoff=-Max(X-S_T, 0)	

- Profits of Options

Long call Profit = Max(0, S_T - X) - c_0		Long put Profit = Max(X-S_T, 0) - p_0	
Short call Profit = -Max(S_T-X, 0) + c_0		Short put Profit = -Max(X-S_T, 0) + p_0	

7) 影响期权价格的因素

影响因素	Call Value	Put Value
Price of the underlying asset 标的资产价格 S	+	-
exercise price/strike price K/X 执行价格	-	+
Time to expiration (T-t)到期时间	+	+，有特例（deep ITM put）
risk-free rate of interest (r)无风险利率	+	-
Volatility of the underlying price (σ)标的资产价格波动率	+	+
标的资产的 income	-	+
标的资产的 cost	+	-

“+”:正向影响； “-”：反向影响

8) Put-Call Parity 买卖权平价定理

【注意：call 和 put 指的是欧式期权；相同的到期时间 T，相同的执行价格 X，相同的标的资产 stock。】

- **Fiduciary call**

- ✓ Combination of a European call option with exercise price of X and a pure-discount, riskless bond with face value of X.
- ✓ 即 long call，同时 long 面值为 X 的零息债券。

- **Protective put**
  - ✓ A share of stock together with a put option on the stock with exercise price of X.
  - ✓ 即 long stock，同时 long put。
- 公式：$p_0+S=c_0+X(1+r)^{-T}$

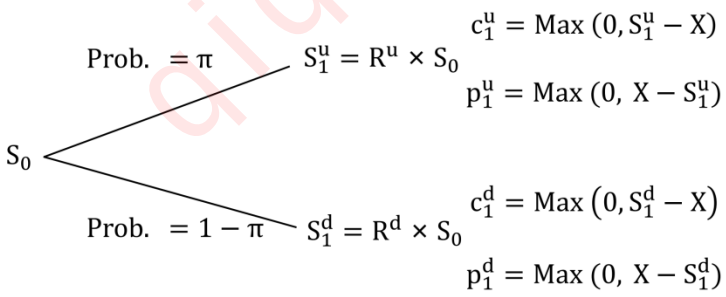
- **Put-call forward parity**
  - ✓ Put-call-forward parity is derived with a forward contract (forward price: F0(T)) rather than the underlying asset itself.

$$F_0(T)(1+r)^{-T}+p_0=c_0+X(1+r)^{-T}$$

9) 期权估值-Binomial Trees

One-period binomial model 一步二叉树

- 假设标的资产价格未来有两种可能，据此分析对应期权的价格。
- 【二叉树可以给欧式期权也可以个美式期权进行定价。我们这里主要针对的是欧式期权，且是一步二叉树。】



- ✓ R^U：标的资产价格上涨的幅度
- ✓ R^D：标的资产价格下降的幅度
- ✓ $R^U * R^D=1$

- No-arbitrage pricing 无套利定价
  - ✓ 通过构造无风险组合，求期权价格

$$\textbf{hedge ratio}=\frac{c_1^u-c_1^d}{s_1^u-s_1^d}=\frac{p_1^u-p_1^d}{s_1^u-s_1^d}$$

- Risk-neutral pricing 风险中性定价

【重点掌握此方法】

✓ 风险中性概率 risk-neutral probability：

$$\boldsymbol{\pi = \frac{1 + r - R^d}{R^u - R^d}}$$

✓ Call 的价格：

$$C_0 = \frac{\pi \times c_1^u + (1 - \pi) \times c_1^d}{(1 + r)^T}$$

✓ Put 的价格：

$$p_0 = \frac{\pi \times p_1^u + (1 - \pi) \times p_1^d}{(1 + r)^T}$$

- 例如：A stock's price is currently ¥8,000. At the end of one month when its options expire, the stock price is either up by 5% or down by 15%. If the risk-free rate is –0.20% for the period, what is the value of a put option with a strike price of ¥7,950?

A. ¥333.67
B. ¥299.60
C. ¥236. 93

解答: B

使用风险中性定价：

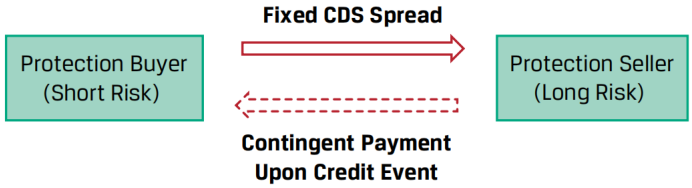
$$\pi = \frac{1 + r - R^d}{R^u - R^d} = \frac{1 - 0.002 - 0.85}{1.05 - 0.85} = 0.74$$

$$p_0 = \frac{0.74 \times \max(0, 7950 - 8400) + (1 - 0.74) \times \max(0, 7950 - 6800)}{(1 - 0.002)^1}$$

$$= 299.60$$

6. CDS 信用违约互换

- **Credit default swaps (CDS)** 信用违约互换 allow an investor to manage the risk of loss from borrower default separately from the bond market.
- In a CDS contract, the credit protection buyer pays the credit protection seller to assume the risk of loss from the default of an underlying third-party issuer.
- 一般来说是 CDS 的买方定期的支付一定的保费，当标的资产违约时，CDS 卖出方给予买方一定的赔付。



- ✓ Credit protection buyer:支付一定的保费 spread
- ✓ Credit protection seller:赔付（或有）

- CDS 类似于保险。