Distributed Consensus Strategy for Platooning of Vehicles in the Presence of Time-Varying Heterogeneous Communication Delays

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***Abstract*—We analyze and solve the platooning problem by treating it as the problem of achieving consensus in a network of dynamical systems affected by time-varying heterogeneous delays due to wireless communication among vehicles. Specifically, a platoon is modeled as a dynamical network where: 1) each vehicle, with its own dynamics, is a node; 2) the presence of communication links between neighboring vehicles is represented by edges; and 3) the structure of the intervehicle communication is encoded in the network topology. A distributed control protocol, which acts on every vehicle in the platoon, is derived. It is composed of two terms: a local action depending on the state variables of the vehicle itself (measured onboard) and an action depending on the information received from neighboring vehicles through the communication network. The stability of the platoon is proven by using Lyapunov–Razumikhin theorem. Numerical results are included to confirm and illustrate the theoretical derivation.**

***Index Terms*—Agent-based control, consensus, control of complex network systems, intelligent transportation systems, platooning.**

I. Introduction

PLATOONING is the coordinated motion of groups of vehicles cooperating with each other to reach the same destination with a common velocity. It has been shown that platooning can effectively improve safety, efficiency, and travel duration while decreasing traffic congestion, pollution, and stress for passengers [1].

From a control viewpoint, the main goal for vehicles is to form a platoon and then maintain an optimal spacing policy (e.g., in terms of the relative distance and velocity between vehicles). Currently, the stability and robustness of the platoon are guaranteed by implementing a local cooperative adaptive control action [cooperative adaptive cruise control (CACC)]. Common CACC strategies rely on pairwise interactions based on local measurements by onboard sensors. That is, each vehicle only uses proximity information transmitted from its preceding agent in the platoon [2], [3]. The typical aim of the theoretical analysis is to prove robustness, i.e., that perturbations on the leader vehicle are not amplified when propagating downstream through the follower vehicles (e.g., see [4] and the references therein). This property is also known in the technical literature as string stability [5], [6]. It has been noticed that pairwise interactions between vehicles can make long strings sensitive to perturbations [7], [8]. Hence, there is a need to explore the use of more generic communication structures among vehicles.

These can be achieved through wireless vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communication. The reference behavior is dispatched to all vehicles in the network by *a leader vehicle* belonging to the platoon (typically the first vehicle in the group) [1]. New challenges arise due to uncertainties and time-varying communication delays. Recent work addresses this problem within the CACC framework, investigating the effect of network-induced constant delays or sample-and-hold devices on string stability within a networked control system perspective [9], [10].

The design of coupling protocols for the coordination of a group of agents exchanging information in the presence of limited and uncertain communication is a well-known problem known as *consensus*, [11], [12]. Usually, consensus is solved under the assumption of constant homogeneous or heterogeneous delays affecting communication among agents [13]. Fewer approaches have been proposed to cope with time-varying heterogeneous communication delays for first- and second-order linear systems [14], [15].

The aim of this paper is to solve platooning by treating it as the problem of achieving second-order consensus in a network of dynamical systems with a more generic structure and in the presence of heterogeneous delays. Following the paradigm of dynamical networks, the platoon is represented as a network where: 1) nodes are vehicles characterized by their own dynamics; 2) edges model the presence of communication links between neighboring vehicles; and 3) the structure of intervehicle communication is encoded in the network topology. Note that the use of the network paradigm allows for the exploration of communication strategies alternative to pairwise interactions. For example, the possibility can be investigated of vehicles communicating in clusters because of conditions due to their proximity and practical communication constraints. Furthermore, switching topologies in network control design can be used to model and compensate for the effect of packet

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losses, communications failures, or automated vehicle maneuvers, such as leaving the platoon.

In our approach, we assume that the control input is acting nb on every vehicle in the platoon. The control is designed as a coupling protocol composed of two terms: a local action depending on the state variables of the vehicle itself (measured onboard) and an action depending on information received from neighboring vehicles through the communication network. The resulting overall control architecture is decentralized and distributed. The platoon formation and its stability in the presence of time-varying delays is guaranteed by using Lyapunov–Razumikhin theorem (see [16] for further information). Numerical results confirm the achievement of consensus for a string of vehicles forming a platoon in the extraurban technological scenario described in [1] despite rapid variations of the delays along the string. The robustness of the approach with respect to communication losses considering both a switching topology and time-varying delays is investigated numerically.

Note that other attempts at solving platooning as that of achieving consensus in a dynamic network have been independently presented in literature. For example, in [17], a leaderless strategy is proposed for three autonomous vehicles that are ideally moving in a circle and sharing information across an all-to-all configuration via V2V communication affected by a constant and common delay. Platooning as a weighted and constrained consensus control problem is also discussed in [18] and [19], where the aim is to understand the influence of time-varying network structures on the platooning dynamics. More specifically, switchings in the communication network are taken into account by using a discrete-time Markov chain. The presence of noise due to the communication channel is also explicitly taken into account while other communication effects, such as the presence of time-varying delays, are not considered. Finally, in [20] platooning has been treated as a synchronization problem when the communication structure among vehicles can be freely chosen, and vehicles only share partial (output) information but, again, in the case of ideal communication [20].

II. Mathematical Preliminaries

If every vehicle/agent is regarded as a node of a network, then the intervehicle communication structure can be described by a graph. (Some basic notions on graph theory can be found in [21] and references therein.) Specifically, a platoon of N vehicles can be described by a weighted directed graph (digraph) $\mathcal{G}_N = (\mathcal{V}_N, \mathcal{E}_N, \mathcal{A}_N)$ of order N characterized by a set of nodes $\mathcal{V}_N = \{1, \ldots, N\}$ and a set of edges $\mathcal{E}_N \subseteq \mathcal{V}_N \times \mathcal{V}_N$. The topology of the graph is associated to a weighted adjacency matrix with nonnegative elements $\mathcal{A}_N = [a_{N,ij}]_{N\times N}$. In general, we assume that $a_{N,ii} = 0$ (i.e., self-edges (i, i) are not allowed unless indicated otherwise). The edge (i, j) in $\mathcal{E}_N$ denotes that vehicle i can obtain information from vehicle j but not necessarily *vice versa*, with j being the *parent node* and i the *child node*.

The set of neighbors of node i is denoted as $\mathcal{N}_i = \{i \in \mathcal{V}_N : e_{ij} = (i, j) \in \mathcal{E}_{N,j} \neq i\}$, $\mathcal{E}_N \subset \mathcal{V}_N \times \mathcal{V}_N$. A sequence $1, 2, \ldots, l$ of distinct nodes is a *directed path* if $(i-1, i) \in \mathcal{E}_N$, $i = 2, \ldots, l$.

A digraph is *strongly connected* (SC) if there is a path from every node to every other node. A *strong component* of a digraph is an induced subgraph that is maximal, which is subject to being SC. A *directed tree* is a digraph in which every node has exactly one parent node with the exception of one node, which is called the *root*, which has no parent and has a directed path to every other node. A subgraph $(\mathcal{V}_N^s, \mathcal{E}_N^s)$ of $(\mathcal{V}_N, \mathcal{E}_N)$ is a graph such that $\mathcal{V}_N^s \subseteq \mathcal{V}_N$ and $\mathcal{E}_N^s \subseteq \mathcal{E}_N \cap (\mathcal{V}_N^s \times \mathcal{V}_N^s)$. A *rooted directed spanning tree* $(\mathcal{V}_N^s, \mathcal{E}_N^s)$ of digraph $(\mathcal{V}_N, \mathcal{E}_N)$ is a subgraph of $(\mathcal{V}_N, \mathcal{E}_N)$ such that $(\mathcal{V}_N^s, \mathcal{E}_N^s)$ is a directed tree and $\mathcal{V}_N^s = \mathcal{V}_N$.

We say that j is *reachable* from i if there exists a path from node i to node j. A node is said to be globally reachable if it is reachable from any other node in the graph. A *cluster* is any subset $\mathcal{V}_N^s \subset \mathcal{V}_N$ of the nodes of the digraph. The set of neighbors of a cluster $\mathcal{V}_N^s$ is defined as $\mathcal{N}_{\mathcal{V}_N^s} = \bigcup_{i\in\mathcal{V}_N^s} \mathcal{N}_i = \{j \in \mathcal{V}_N : i \in \mathcal{V}_N^s, (i, j) \in \mathcal{E}_N\}$. Defining the degree matrix as $D = \mathrm{diag}\{\bar{d}_1, \bar{d}_2, \ldots, \bar{d}_N\}$, with $\bar{d}_i = \sum_{j\in\mathcal{N}_i} a_{N,ij}$, the Laplacian of weighted digraph $\mathcal{G}_N$ can be defined as $L = D - \mathcal{A}_N$. N+1

In what follows, we consider N vehicles (agents) together with a leader vehicle, which is taken as an additional agent labeled with the index zero, i.e., node 0. We use an augmented weighted digraph $\mathcal{G}_{N+1}$ to model the resulting leader–follower network topology. We assume that node 0 is *globally reachable* in $\mathcal{G}_{N+1}$ if there is a path in $\mathcal{G}_{N+1}$ from every node i in $\mathcal{G}_N$ to node 0 [22].

Definition 1: A complex square matrix is said to be negative stable (positive stable) if its spectrum lies in the open left (right) half-plane [23].

Definition 2: The square matrix $\tilde{A} = [\tilde{a}_{ij}]$ of order $N \times N$ is said to be SC if for every pair of distinct integers p, q, with $1 \le p$ and $q \le N$, there is a sequence of distinct integers $p = o_1, o_2, o_3, \ldots, o_{m-1}, o_m = q, 1 \le m \le N$ such that all of the matrix entries $\tilde{a}_{o_1,o_2}, \tilde{a}_{o_2,o_3}, \ldots, \tilde{a}_{o_{m-1},o_m}$ are nonzero [22].

In addition, the following Lemmas hold (see [24] for the proofs).

Lemma 1: A digraph $\mathcal{G}_N = (\mathcal{V}_N, \mathcal{E}_N, \mathcal{A}_N)$ has a globally reachable node if and only if every pair of nonempty and disjoint subset $\mathcal{V}_N^1, \mathcal{V}_N^2 \subset \mathcal{V}_N$ satisfies $\mathcal{N}_{\mathcal{S}_i} \cup \mathcal{N}_{\mathcal{S}_j} \neq \emptyset$.

Let $S_1, S_2, \ldots, S_p$ be the strong components of $\mathcal{G}_N = (\mathcal{V}_N, \mathcal{E}_N, \mathcal{A}_N)$ and $\mathcal{N}_{S_i}$ be the neighbor sets for $S_i, i = 1, \ldots, p, p > 1$.

Lemma 2: Digraph $\mathcal{G}_N$ has a globally reachable node if and only if the Laplacian of $\mathcal{G}_N$ has a simple zero eigenvalue (with eigenvector $\mathbf{1} = (1, \ldots, 1) \in \mathbb{R}^N$).

Furthermore, let $C([-r, 0], \mathbb{R}^n)$ be a Banach space of continuous functions defined on interval $[-r, 0]$, taking values in $\mathbb{R}^n$ with a norm $\|\varphi\|_c = \max_{\theta\in[-r,0]} \|\varphi(\theta)\|$, with $\|\cdot\|$ being the Euclidean norm. Given a system of the following form:

$$\begin{aligned} \dot{x} &= f(x_t), \quad t > 0 \\ x(\theta) &= \varphi(\theta), \quad \theta \in [-r, 0] \end{aligned} \tag{1}$$

where $x_t(\theta) = x(t + \theta)$, $\forall\theta \in [-r, 0]$, and $f(0) = 0$, the following result holds.

Theorem 1 (Lyapunov–Razumikhin) [16]: Given system (1), suppose that function $f : C([-r, 0], \mathbb{R}^n) \to \mathbb{R}^n$ maps the bounded sets of $C([-r, 0], \mathbb{R}^n)$ into the bounded sets of $\mathbb{R}^n$. Let

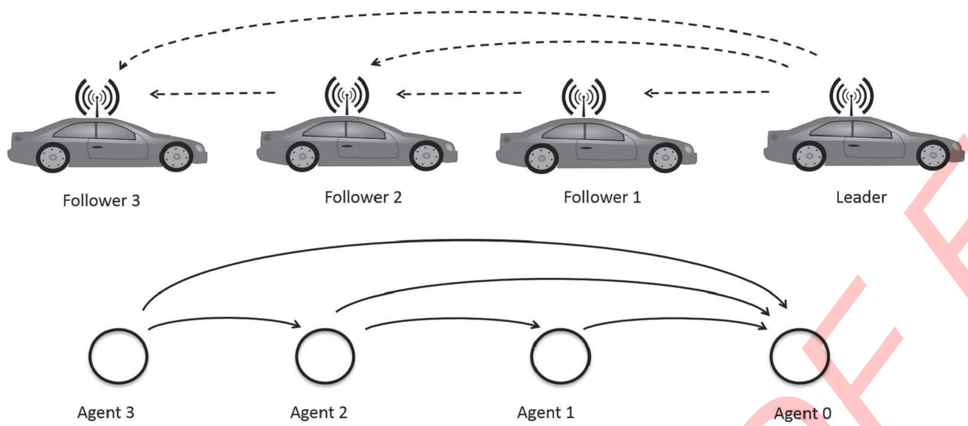

Fig. 1. Representative platoon configurations consisting of a leader and three followers $F1-F2-F3$. The arrows in the top panel denote the information flow among vehicles and with the leader, whereas those in the associated network graph indicate the edges directed according to the definition given in Section II and used in the literature (e.g., see [15] and [27]).

ψ_1, ψ_2, and ψ_3 be continuous, nonnegative, and nondecreasing functions, with $\psi_1(s) > 0$, $\psi_2(s) > 0$, $\psi_3(s) > 0$ for $s > 0$ and with $\psi_1(0) = \psi_2(0) = 0$. If there is a continuous function $V(t,x)$ (the Lyapunov–Razumikhin function) such that

$$\psi_1(\|x\|) \le V(t,x) \le \psi_2(\|x\|), \qquad t \in \mathbb{R}; x \in \mathbb{R}^n \tag{2}$$

and there exists a continuous nondecreasing function $\psi_4(s)$ with $\psi_4(s) > s$, $s > 0$ such that

$$\dot{V}(t,x) \le -\psi_3(\|x\|) \tag{3}$$

when $V(t+\theta, x(t+\theta)) < \psi_4(V(t,x(t)))$, $\theta \in [-r, 0]$, then the origin $x = 0$ is uniformly and asymptotically stable.

III. Problem Statement

Consider a group of N vehicles moving along a single lane. In the scenario under investigation, vehicles are organized as a *string*, with vehicles following one another along a straight line and sharing their state information (e.g., the absolute position, the velocity, and the acceleration) with all other agents communicating through a V2V communication paradigm, as described in [1]. The onboard integration of inertial sensors with a GPS receiver allows each vehicle to measure its absolute position and velocity [25]. The vehicles are also equipped with onboard units that were configured as receiving and transmitting hosts. The reference trajectory according to the required spacing policy is provided by the first vehicle in the platoon, as shown in Fig. 1. Note that, although the string of vehicles is always in a leader–follower configuration, different topologies may arise depending on the communication technology and its specific transmission ranges [26].

A. Platooning as a Consensus Problem

Within our framework, the behavior of the generic ith (vehicle) inertial agent ($i = 1, \ldots, N$) in the platoon is mathematically described by a simple linear model that was possibly obtained by applying input–output feedback linearization to simplify the complexity of the model describing the longitudinal vehicle dynamics, and without considering parasitic time delays and lags [28], [29], as follows:

basic model

$$\boxed{\dot{r}_i(t) = v_i(t), \quad \dot{v}_i(t) = \frac{1}{M_i} u_i(t)} \tag{4}$$

where r_i (in meters) and v_i (in meters per second) are the ith vehicle position and velocity, respectively, measured with respect to a given reference framework, M_i (in kilograms) is the mass of the ith vehicle assumed to be constant, and u_i denotes the control input. We denote the state vector of the ith vehicle as $\eta_i = [\eta_i^{(1)}, \eta_i^{(2)}]^\top = [r_i, v_i]^\top \in \mathbb{R}^2$. In the leader–follower topology [30], the spacing policy is determined according to the reference dynamics provided by the leader. Usually, when the platoon has to move with a reference constant velocity, e.g., v_0, such dynamics can be described as

$$\dot{r}_0(t) = v_0, \quad \dot{v}_0 = 0. \tag{5}$$

We label the leader state vector as $\eta_0 = [r_0, v_0]^\top \in \mathbb{R}^2$.

Due to the presence of limited communication, the control input to the ith vehicle (4) has to be determined by choosing an appropriate *decentralized coupling protocol* of the following form:

you cannot just send u_i

$$u_i = u_i\left(\eta_i(t), \eta_j\left(t, \tau_{ij}(t)\right), \eta_0\left(t, \tau_{i0}(t)\right)\right) \tag{6}$$

3 arguments

where $\tau_{ij}(t)$ and $\tau_{i0}(t)$ are the unavoidable time-varying communication delays affecting the ith agent when information is transmitted from its neighbor j (within the transmission range) and from the leader, respectively. Note that communication is assumed to be such that, in general, delay $\tau_{ij}(t) \neq \tau_{ji}(t)$.

Given (4) and (5), the problem of maintaining a desired intervehicle spacing policy and a common velocity of the platoon of vehicles in the presence of delays can be formulated as a second-order consensus problem with the aim of driving the positions and velocities of all vehicles toward the following desired steady-state values:

$$\boxed{r_i(t) \to \frac{1}{d_i}\left\{\sum_{j=0}^{N} a_{ij} \cdot (r_j(t) + d_{ij})\right\}}, \quad \boxed{v_i(t) \to v_0} \tag{7}$$

where v_0 is the desired constant velocity [the solution of the reference leader dynamics (5)], $d_i = \sum_{j=0}^{N} a_{ij}$ is the degree of agent i, d_{ij} are the desired spacing errors between agents i and j (for $i = 1, \dots, N$ and $j = 0, \dots, N$), and a_{ij} are the elements of the adjacency matrix of graph $\mathcal{G}_{N+1}$ assumed to be time invariant. In platooning, we assume that $a_{i0} = 1$, $\forall i = 1, \dots, N$ as all vehicles have a direct communication with the leader and that $a_{0j} = 0$, $\forall j = 0, \dots, N$, given that the leader does not receive data from any other vehicle. We assume that $d_{ij} = h_{ij} v_0$, with h_{ij} being the *constant time headway* for a given spacing policy, e.g., the time that the ith vehicle takes to arrive at the position of its predecessor. Note that the reference velocity $v_0 > 0$ is selected so that vehicles cannot crash into each other. Setting $h_{ij} = -h_{ji}$ (in order to increase safety with respect to the vehicles following the ith agent, as done in [31]), the consensus target (7) can be rewritten as

this is the goal for each vehicle

$$r_i(t) \to r_0(t) + d_{i0}, \quad v_i(t) \to v_0 \tag{8}$$

where $r_0(t)$ is the desired position trajectory [the solution of the reference leader dynamics (5)].

The consensus goal (8) can be achieved using an appropriate distributed strategy [13]. Here, we propose to make use of a coupling protocol inspired from the work in [15] but with the notable difference of embedding the spacing policy information into the protocol to solve the platooning scenario. Specifically, we assume that all neighboring vehicles transmit their state variables (the position and the velocity) to the ith vehicle, together with their ID and time stamp $\bar{t}$ (representing the time instant at which information was sent [32]). (Clock synchronization is guaranteed across all the strings via GPS [33].) Then, the ith vehicle adjusts its dynamics through the following decentralized coupling protocol computed onboard as

$$u_i = -b\left[v_i(t) - v_0\right] + \frac{1}{d_i}\sum_{j=0}^{N} k_{ij}a_{ij} \cdot (\tau_i(t)v_0) - \frac{1}{d_i}\sum_{j=0}^{N} k_{ij}a_{ij} \cdot \left(r_i(t) - r_j\left(t - \tau_i(t)\right) - h_{ij}v_0\right) \tag{9}$$

internal adjusting

where $k_{ij} > 0$ and $b > 0$ are the stiffness and damping coefficients to be appropriately tuned to regulate the mutual behavior among neighboring inertial agents (i.e., the decentralized controller parameters), and $\tau_i(t)$ is the *aggregate delay* computed by node i using the time stamps sent by its neighboring vehicles. That is, the aggregate delay is obtained by an onboard strategy (e.g., see [34]) that uses the delays τ_{ij} that were estimated by vehicle i from the time stamps received from all neighboring nodes. Note that, typically, τ_i is updated over finite time intervals and is therefore a piecewise constant function. In the rest of this paper, we will assume that $\tau_i(t)$ is a bounded stochastic piecewise constant function. Indeed, focusing on the road segment close to the receiver vehicle, delay time $\tau_i(t)$ can be assumed bounded between a maximum constant value and a minimal constant value, e.g., $\tau_{\min} \leq \tau_i(t) \leq \tau_{\max}$ [35].

IV. Closed-Loop Vehicular Network

To prove consensus of systems (4) and (5) under the action of coupling protocol (9), we define the error state with respect to reference signals $r_0(t)$, v_0 as

this is the error vector components

components

$$\bar{r} = [\bar{r}_1, \dots, \bar{r}_i \dots, \bar{r}_N]^\top$$
$$\bar{v} = [\bar{v}_1, \dots, \bar{v}_i \dots, \bar{v}_N]^\top \tag{10}$$

with $\bar{r}_i = (r_i(t) - r_0(t) - h_{i0}v_0)$, and $\bar{v}_i = (v_i(t) - v_0)$.

After some algebraic manipulations, the coupling protocol (9) can be expressed in terms of the state error (10) as

$$\boxed{u_i = -b\bar{v}_i(t) - \frac{1}{d_i}\sum_{j=0}^{N} k_{ij}a_{ij} \cdot \left(\bar{r}_i(t) - \bar{r}_j\left(t - \tau_i(t)\right)\right).} \tag{11}$$

Then, the dynamics of the error variables for the generic ith vehicle in the platoon can be written as

$$\begin{cases} \dot{\bar{r}}_i = \bar{v}_i \\ M_i\dot{\bar{v}}_i = -b\bar{v}_i(t) - \frac{1}{d_i}\sum_{j=0}^{N} k_{ij}a_{ij} \cdot \left(\bar{r}_i(t) - \bar{r}_j\left(t - \tau_i(t)\right)\right). \end{cases} \tag{12}$$

From (12), the defining the error vector $\bar{x}(t) = [\bar{r}^\top(t)\ \bar{v}^\top(t)]^\top$, the dynamics of the closed-loop vehicular network can be recast in a compact form as:

compact form

$$\boxed{\dot{\bar{x}}(t) = A_0\bar{x}(t) + \sum_{p=1}^{N} A_p\bar{x}\left(t - \tau_p(t)\right)} \tag{13}$$

where for $p = 1, 2, \dots N$,

$$A_0 = \begin{bmatrix} 0_{N\times N} & I_{N\times N} \\ -M\widetilde{K} & -M\widetilde{B} \end{bmatrix} \qquad A_p = \begin{bmatrix} 0_{N\times N} & 0_{N\times N} \\ M\widetilde{K}_p & 0_{N\times N} \end{bmatrix} \tag{14}$$

with

calcoli

$$M = \operatorname{diag}\left\{\frac{1}{M_1}, \dots, \frac{1}{M_N}\right\} \in \mathbb{R}^{N\times N} \tag{15}$$

$$\widetilde{K} = \operatorname{diag}\{\tilde{k}_{11}, \dots, \tilde{k}_{NN}\} \in \mathbb{R}^{N\times N} \tag{16}$$

and $\tilde{k}_{ii} = (1/d_i)\sum_{j=0}^{N} k_{ij}a_{ij}$, $i = 1, \dots, N$; moreover, we have

$$\widetilde{B} = \operatorname{diag}\{b, \dots, b\} \in \mathbb{R}^{N\times N} \tag{17}$$

$$\widetilde{K}_p = \begin{bmatrix} 0 & \tilde{k}_{12,p} & \cdots & \cdots & \tilde{k}_{1N,p} \\ \tilde{k}_{21,p} & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \tilde{k}_{(N-1)N,p} \\ \tilde{k}_{N1,p} & \cdots & \cdots & \tilde{k}_{N(N-1),p} & 0 \end{bmatrix} \tag{18}$$

with $\tilde{k}_{ij,p} = \begin{cases} a_{ij}k_{ij}/d_i, & p = i \\ 0, & p \neq i \end{cases}$, $i, j = 1, \dots, N$, and $\widetilde{K}_p \in \mathbb{R}^{N\times N}$, $p = 1, \dots, N$.

Note that matrix $\widetilde{K}$ in (16) can be recast as follows:

$$\widetilde{K} = K + \bar{K} \tag{19}$$

where

$$K = \operatorname{diag}\{k_1, \dots, k_N\}, \quad k_i = \frac{k_{i0}a_{i0}}{d_i} \quad (i = 1, \dots, N) \tag{20}$$

$$\bar{K} = \operatorname{diag}\{\bar{l}_{11}, \dots, \bar{l}_{NN}\} \tag{21}$$

with $\bar{l}_{ii}$ being the diagonal elements of the *normalized* weighted Laplacian matrix $\bar{L}$ associated with graph $\mathcal{G}_N$ [27] defined as

$$\bar{l}_{ii} = \frac{1}{d_i} l_{ii} = \frac{1}{d_i} \sum_{p=1}^{N} k_{ip} a_{ip} \quad (i = 1, \ldots, N) \tag{22}$$

where l_{ii} are the diagonal elements of the weighted Laplacian matrix L of $\mathcal{G}_N$.

From the definition of $\widetilde{K}_p$ and $\bar{K}$ given in (18) and (19), respectively, the normalized weighted Laplacian $\bar{L}$ can be also recast as

$$\bar{L} = [\bar{l}_{ij}]_{N\times N} = \bar{K} - \sum_{p=1}^{N} \widetilde{K}_p \quad (i, j = 1, \ldots, N; j \neq 0). \tag{23}$$

V. Convergence Analysis

Next, in order to solve the consensus problem in the presence of time-varying communication delays, we introduce a model transformation. From the Leibniz–Newton formula [36], it is known that

$$\bar{x}\left(t - \tau_p(t)\right) = \bar{x}(t) - \int_{-\tau_p(t)}^{0} \dot{\bar{x}}(t+s) ds. \tag{24}$$

Hence, substituting expression (13) in (24), we have $(p = 1, \ldots, N)$

$$\bar{x}\left(t - \tau_p(t)\right) = \bar{x}(t) - \sum_{j=0}^{N} A_j \int_{-\tau_p(t)}^{0} \bar{x}\left(t + s - \tau_j(t+s)\right) ds \tag{25}$$

where matrices $A_0, A_1, \ldots, A_N$ are defined in (14), and $\tau_0(t+s) \equiv 0$.

Expressing the delayed state, as in (25), the time-delayed model (13) can be transformed into

$$\dot{\bar{x}}(t) = A_0 \bar{x} + \sum_{p=1}^{N} A_p \bar{x}(t) - \sum_{p=1}^{N} \sum_{j=0}^{N} A_p A_j \int_{-\tau_p(t)}^{0} \bar{x}\left(t + s - \tau_j(t+s)\right) ds. \tag{26}$$

From definition (14), it follows that $A_p A_j = 0$ when $p = 1, \ldots N$ and $j \neq 0$. Hence, system (13) can be rewritten as

this should be the final model form

$$\boxed{\dot{\bar{x}}(t) = F\bar{x}(t) - \sum_{p=1}^{N} C_p \int_{-\tau_p(t)}^{0} \bar{x}(t+s) ds} \tag{27}$$

where

$$C_p = A_p A_0 = \begin{bmatrix} 0_{N\times N} & 0_{N\times N} \\ 0_{N\times N} & M\widetilde{K}_p \end{bmatrix} \tag{28}$$

and

$$F = A_0 + \sum_{p=1}^{N} A_p = \begin{bmatrix} 0_{N\times N} & I_{N\times N} \\ -M\widehat{K} & -M\widetilde{B} \end{bmatrix} \tag{29}$$

with

$$\widehat{K} = -\sum_{p=1}^{N} \widetilde{K}_p + \widetilde{K}. \tag{30}$$

~~We now introduce some lemmas that will be instrumental for the proof of convergence presented later in this section.~~

~~*Lemma 3:* Supposing that $k_i \geq 0$ in (20) $(i = 1, \ldots, N)$, matrix $\widehat{K}$ in (30) is positive stable [23] if and only if node 0 is globally reachable in $\mathcal{G}_{N+1}$.~~

~~*Proof (Sufficiency):* First, substituting expression (19) into (30), from (23), we have $\widehat{K} = \bar{L} + K$. According to the Geršgorin disk theorem [22], all the eigenvalues of $\widehat{K}$ are located in the union of N disks, which is the so called Geršgorin region given by~~

$$\cancel{G(\widehat{K}) = \bigcup_{i=1}^{N} \left\{ z \in \mathbb{C} : |z - \bar{l}_{ii} - k_i| \leq \sum_{\substack{j \neq i \\ j \neq 0}}^{N} |\bar{l}_{ij}| \right\}.} \tag{31}$$

~~From the assumptions that $k_i \geq 0$ and the protocol gains $k_{ij} > 0$, $(i = 1, \ldots, N$ and $j = 0, \ldots, N)$, each Geršgorin disk is therefore located in the right hand side of the complex plane, and matrix $\widehat{K}$ has either a zero eigenvalue or an eigenvalue with a positive real part. Furthermore, from the assumption that node 0 is globally reachable in $\mathcal{G}_{N+1}$, there exist at least one $k_i > 0$ and, therefore, at least one Geršgorin circle bounding a corresponding Geršgorin disk, which does not pass through the origin of the complex plane.~~

~~Now, to prove the sufficient condition, we consider the following two cases.~~ ealcoli

~~*Case 1: $\mathcal{G}_N$ has a globally reachable node.* Let $S_1, \ldots, S_p$ $(p \in \mathbb{Z}^+)$ be the strong components of $\mathcal{G}_N$. If $p = 1$, $\mathcal{G}_N$ is SC, and its weighted adjacency matrix, i.e., $(\sum_{p=1}^{N} \widetilde{K}_p)$, is also SC. Since $(K + \bar{K})$ is a diagonal matrix with nonnegative diagonal entries, matrix $\widehat{K}$ is SC. According to [22, pp. 365], if zero is a simple eigenvalue of $\widehat{K}$, then every Geršgorin circle of $\widehat{K}$ passes through 0, which leads to a contradiction. It follows that zero cannot be an eigenvalue of $\widehat{K}$.~~

~~If $p > 1$, there is one strong component, e.g., S_1, having no neighbor set (see [37, Lemma 1]). We can then rewrite the normalized weighted Laplacian matrix $\bar{L}$ in the following block form:~~

$$\cancel{\bar{L} = \begin{pmatrix} \bar{L}^{(11)} & 0 \\ \bar{L}^{(21)} & \bar{L}^{(22)} \end{pmatrix}} \tag{32}$$

~~where $\bar{L}^{(11)} \in \mathbb{R}^{\iota\times\iota}$ $(\iota < N)$ is the normalized weighted Laplacian matrix of S_1.~~

~~From Lemma 2, it follows that zero is a simple eigenvalue of $\bar{L}^{(11)}$ (and therefore $\bar{L}$), whereas $\bar{L}^{(22)}$ is nonsingular. Node 0 is globally reachable; hence, the block corresponding to $\bar{L}^{(11)}$ in matrix K, e.g., K_1, is not null. As done for the case when $p = 1$, it follows that zero is not an eigenvalue of $\bar{L}^{(11)} + K_1$; therefore, it is not an eigenvalue of $\widehat{K}_M$.~~

~~*Case 2: $\mathcal{G}_N$ has no globally reachable node.* Let $S_1, \ldots, S_p$ $(p \in \mathbb{Z}^+)$ be the strong components, with $\mathcal{N}_{S_i} = \emptyset$ for $i = 1$;~~

$\ldots, p$ and $p > 1$. As $\bigcup_{i=1}^{p} \mathcal{V}(S_i) \subset \mathcal{V}(\mathcal{G}_N)$, the matrix $\bar{L}$ that is associated to $\mathcal{G}_N$ can be recast in the following form:

$$\bar{L} = \begin{pmatrix} \bar{L}^{(11)} & 0 & \cdots & 0 & 0 \\ \vdots & \bar{L}^{(22)} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \bar{L}^{(pp)} & \vdots \\ \bar{L}^{(p+1,1)} & \cdots & \cdots & \bar{L}^{(p+1,p)} & \bar{L}^{(p+1,p+1)} \end{pmatrix} \tag{33}$$

where $\bar{L}^{(ii)}$ is the normalized weighted Laplacian matrix of S_i $(i = 1, \ldots, p)$, and $\bar{L}^{(p+1,p+1)}$ is nonsingular. Node 0 is globally reachable in $\mathcal{G}_{N+1}$; then, blocks K_i in K correspond ing to blocks $\bar{L}^{(ii)}$ in $\bar{L}$ are nonnull. Following the same steps taken to prove *Case 1*, we can conclude that zero is not an eigenvalue of $\bar{L}^{(ii)} + K_i$ or, equivalently, $\hat{K}$.

(Necessity). If node 0 is not globally reachable in $\mathcal{G}_{N+1}$, then $\mathcal{G}_N$ might have or not have a globally reachable node.

Consider first the case when $\mathcal{G}_N$ has a globally reachable node. Following the approach used to prove *Case 1* and assuming that $\mathcal{V}_1$ has no neighbor set, we can again derive (32), with $\bar{L}^{(11)} \in \mathbb{R}^{t \times t}$ $(t < N)$ being the weighted Laplacian matrix corresponding to $\mathcal{V}_1$. Again, from Lemma 2, it follows that zero is an eigenvalue of $\bar{L}^{(11)}$ and $\bar{L}$, whereas $\bar{L}^{(22)}$ is nonsingular. Now, as node 0 is not globally reachable in $\mathcal{G}_{N+1}$, we have $K_1 = 0$; hence, zero is a simple eigenvalue of $\bar{L}^{(11)} + K_1$ and a simple eigenvalue of $\hat{K}$, leading to a contradiction.

Instead, when $\mathcal{G}_N$ has no globally reachable node, we consider (33), as in *Case 2*. Node 0 is not globally reachable in $\mathcal{G}_{N+1}$; hence, there exist at least one $K_i = 0$ (corresponding to $\bar{L}^{(ii)}$) for $i = 1, \ldots, p$. Thus, $\bar{L}^{(ii)} + K_i$ and, correspondingly, $\hat{K}$ have more than one zero eigenvalue, and this implies a contradiction. ∎

Remark 1: Notice that according to Lemma 3, the following matrix is also positive stable:

$$\hat{K}_M = M\hat{K} \tag{34}$$

since $M > 0$ [see (15)].

Lemma 4: Let F be the matrix defined in (29). F is Hurwitz stable if and only if $\hat{K}_M$ [see (34)] in Lemma 3 is positive stable and if the control gain in (11) is such that

$$b > \max_i \left\{ \frac{|\mathrm{Im}(\mu_i)|}{\sqrt{\mathrm{Re}(\mu_i)}} M_i \right\} \tag{35}$$

with μ_i being the ith eigenvalue of $\hat{K}_M$ $(i = 1, \ldots, N)$.

Proof: According to Lemma 3, one can select gains $k_i \geq 0$ $(i = 1, \ldots, N)$ so that matrix $\hat{K}_M$ defined as in (34) is positive stable, i.e., $\mathrm{Re}(\mu_i) > 0$ for any $\mu_i \in \Lambda(\hat{K}_M)$ [23]. Exploiting Schur's formula [22], the characteristic polynomial of F can be computed as

$$\begin{aligned} \det(sI_{2N\times 2N} - F) &= \det\left(\begin{bmatrix} sI_{N\times N} & -I_{N\times N} \\ \hat{K}_M & (M\hat{B} + sI_{N\times N}) \end{bmatrix}\right) \\ &= \det\left(\begin{bmatrix} sI_{N\times N} & -I_{N\times N} \\ \hat{K}_M & (bMI_{N\times N} + sI_{N\times N}) \end{bmatrix}\right) \\ &= \det(s^2 I_{N\times N} + bMI_{N\times N}s + \hat{K}_M). \end{aligned} \tag{36}$$

From (36), we have

$$\det(sI_{2N\times 2N} - F) = \prod_{i=1}^{N} \left(s^2 + \frac{b}{M_i} s + \mu_i \right) \tag{37}$$

where μ_i is the ith eigenvalue of $\hat{K}_M$. Since the polynomial $\pi(s, \mu_i) = s^2 + (b/M_i)s + \mu_i$ is Hurwitz stable if and only if $\mathrm{Re}(\mu_i) > 0$ and $b > (|\mathrm{Im}(\mu_i)|/\sqrt{\mathrm{Re}(\mu_i)})M_i$ [22], it follows that all the eigenvalues of F have negative real parts if and only if inequality (35) is fulfilled. ∎

The consensus of the vehicular network in the presence of heterogeneous time-varying delays can be guaranteed under the hypothesis of the following theorem.

Theorem 2: Consider system (13) and take the control parameters in (9) as $k_{ij} > 0$ and b such that

$$b > b^* = \max_i \left\{ \frac{|\mathrm{Im}(\mu_i)|}{\sqrt{\mathrm{Re}(\mu_i)}} M_i \right\} \tag{38}$$

where $\hat{K}_M$ is defined in (34). Then, there exists a constant $\tau^* > 0$ such that, for $\tau_p(t) < \tau^*$, $\forall p$, $\forall t$, we have

$$\lim_{t\to\infty} \bar{x}(t) = 0 \tag{39}$$

if and only if node 0 is globally reachable in $\mathcal{G}_{N+1}$.

Proof (Sufficiency): Since node 0 is globally reachable in $\mathcal{G}_{N+1}$, from Lemma 3, it follows that matrix $\hat{K}_M$ is positive stable. Setting b as in (38), the hypothesis of Lemma 4 is satisfied; hence, matrix F defined in (29) is Hurwitz stable, and from the Lyapunov theorem, there exists a positive definite matrix $P \in \mathbb{R}^{2N\times 2N}$ such that

$$PF + F^\top P = -I_{2N\times 2N}. \tag{40}$$

Consider the following Lyapunov–Razumikhin candidate function (satisfying condition (2) in Theorem 1):

$$V(\bar{x}) = \bar{x}^\top P \bar{x}. \tag{41}$$

From (27), taking the derivative of V along (13) gives

$$\dot{V}(\bar{x}) = \bar{x}^\top (PF + F^\top P)\bar{x} - \sum_{p=1}^{N} 2\bar{x}^\top PC_p \int_{-\tau_p(t)}^{0} \dot{\bar{x}}(t+s)ds. \tag{42}$$

Now, for any positive definite matrix Ξ, it is possible to show that $2a^\top c \leq a^\top \Xi a + c^\top \Xi^{-1} c$ according to [22]. Therefore, setting $a^\top = -\bar{x}^\top PC_p$, $c = \dot{\bar{x}}(t+s)$, and $\Xi = P^{-1}$, and integrating both sides of the inequality, we can write

$$\begin{aligned} \dot{V}(\bar{x}) \leq\ & \bar{x}^\top (PF + F^\top P)\bar{x} \\ & + \sum_{p=1}^{N} \left[\tau_p(t) \bar{x}^\top PC_p P^{-1} C_p^\top P\bar{x} + \int_{-\tau_p(t)}^{0} \dot{\bar{x}}^\top(t+s) P \dot{\bar{x}}(t+s) ds \right]. \end{aligned} \tag{43}$$

Choose $\psi_4(s) = qs$ for some constant $q > 1$. According to condition (3) in Theorem 1, when

$$V(\bar{x}(t+\theta)) < \psi_4(V(\bar{x})) = qV(\bar{x}(t)); \quad -\tau \leq \theta \leq 0 \tag{44}$$

~~inequality (43) becomes~~

$$\cancel{\dot{V}(\bar{x}) \leq -\psi_3\left(\|\bar{x}\|\right)} \tag{45}$$

~~with $\psi_3(\|\bar{x}\|) = \bar{x}^\top \bar{x} - \tau \sum_{p=1}^{N} \bar{x}^\top (PC_p P^{-1} C_p^\top P + qP)\bar{x}$ for some constant τ such that $\tau_p(t) \leq \tau$ $(p = 1, \ldots, N)$.~~

~~Hence, taking~~

$$\cancel{\tau < \tau^* = \frac{1}{\left\| \sum_{p=1}^{N} (PC_p P^{-1} C_p^\top P + qP) \right\|}} \tag{46}$$

~~yields~~

$$\cancel{\dot{V}(\bar{x}) \leq -\eta \bar{x}^\top \bar{x}} \tag{47}$$

~~for some constant $\eta > 0$. Therefore, the thesis follows from Theorem 1.~~

~~(Necessity). System (13) is asymptotically stable for any time delay $\tau_p(t) < \tau^*$, $p = 1, \ldots, N$. Letting $\tau_p(t) \equiv 0$ $(p = 1, \ldots, N)$ in (13), it follows from (27) that system $\dot{\bar{x}} = F\bar{x}$, with F defined in (29), is asymptotically stable. As all the eigenvalues of F have negative real parts, Lemma 4 implies that $\hat{K}_M$ is positive stable.~~ Now, applying Lemma 3, the theorem is proven. ■

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VI. Disturbance Propagation Through the String

As mentioned above, a common robustness issue in platooning is to ensure string stability. The key idea is to avoid that spacing errors are amplified downstream the traffic flow [5]. In what follows, we analyze the string stability of our V2V scenario following the approach in the Laplace domain presented in [38]. We focus on the case of homogenous traffic with a leader–predecessor configuration in the presence of a periodic perturbation acting on the leader motion. For the sake of simplicity, we assume that the time-varying delays are set to a unique constant value, which may also correspond to the maximum upper bound [39], i.e., $\tau_i(t) = \tau \leq \tau^*$.

The dynamics of the ith vehicle can be recast in the s domain as

Laplacian domain

$$X_i(s) = H_i(s)U_i(s) + \frac{x_i(0)}{s} \tag{48}$$

with $X_i = X_i(s) = \mathcal{L}(r_i)$, $U_i(s) = \mathcal{L}(u_i)$, $H_i(s) = 1/(Ms^2)$, and $x_i(0)$ being the initial condition.

The Laplace transform of the distributed coupling protocol (9) for the leader–predecessor topology is

$$U_1(s) = k_{10}E_1(s) + b(X_0 s - X_1 s) \tag{49}$$

and for $i = 2, \ldots, N$, we have

$$U_i(s) = \frac{k_{i0}}{d_i}\left(X_0 e^{-\tau s} - X_i + \tau X_0 s + h_{i0}X_0 s + \frac{d_{i0}}{s}\right) + \frac{k_{i,i-1}}{d_i}E_i(s) + b(X_0 s - X_i s) \tag{50}$$

where

$$E_i(s) = X_{i-1}e^{-\tau s} - X_i + \tau X_0 s + h_{i,i-1}X_0 s + \frac{d_{i,i-1}}{s} \tag{51}$$

is the spacing error dynamics with respect to the preceding vehicle $(i = 1, \ldots, N)$.

Substituting (50) in (48), after some algebraic manipulation, we have

$$X_i(s) = \frac{k_{i0}H(s)}{d_i}\left[X_0 e^{-\tau s} - X_i + \tau X_0 s + h_{i0}X_0 s + \frac{d_{i0}}{s}\right] + \frac{k_{i,i-1}H(s)}{d_i}E_i(s) + bH(X_0 - X_i)s + \frac{x_i(0)}{s}. \tag{52}$$

Then, the spacing error can be computed in terms of sensitivity functions $T_i(s)$ and $S_i(s)$ $(i = 2, \ldots, N)$ as

$$E_i(s) = T_i(s)E_{i-1}(s) + S_i(s)\frac{d_{i,i-1}}{s} \tag{53}$$

where

$$T_i(s) = \frac{1}{(-1 - D_i)}\left[C_i + (C_i - e^{-\tau s})k_{i-1,0}\bar{H}\right] + \frac{1}{(-1 - D_i)}\left[W_1^{-1}F_i s + W_1^{-1}(C_i - e^{-\tau s})b\bar{H}s\right] \tag{54}$$

$$S_i(s) = \frac{1}{(-1 - D_i)}\left[\frac{(C_i - e^{-\tau s})}{1 + bHs} - 1 + C_i + \frac{2}{B_i}\right] + \frac{1}{(-1 - D_i)}\left[-W^{-1}\left(F_i s + (C_i - e^{-\tau s})b\bar{H}s\right)S_{i-1}\right] \tag{55}$$

with

$$\begin{aligned} C_i &= \frac{k_{i0}H}{d_i B_i} \\ D_i &= \frac{k_{i,i-1}H}{d_i B_i} \\ B_i &= 1 + \frac{k_{i0}H}{d_i} + bHs \\ F_i &= -\tau - h_{i,i-1} + C_i h_{i,i-1} + \frac{bH}{B_i} \\ \bar{H}(s) &= \frac{H(s)}{1 + bH(s)s}. \end{aligned} \tag{56}$$

For the details on the mathematical derivation, see [40].

The propagating errors along the string are then attenuated when $|T_i(j\omega)| < 1$ for all frequencies of interest [38]. Therefore, following a typical approach in the control design, the control parameters can be empirically selected among all the admissible values (belonging to the region in which consensus is analytically guaranteed) as those that also induce a string-stable behavior according to our theoretical investigation, e.g., $|T_i(j\omega)| < 1$.

Remark 2: Under the assumption of neglecting the communication delay (i.e., $\tau = 0$) and considering a well-known CACC strategy [38] that exploits a proportional controller on the position error with respect to both the leader and the predecessor (i.e., $b = 0$), our string stability analysis is the analysis described in [38] and [41]. Indeed, we have

$$T_i(s) = \frac{k}{2} \cdot \frac{H}{1 + Hk} \tag{57}$$

with the assumption that $k_{i0} = k$. Finally, (55) is

$$S_i(s) = 0. \tag{58}$$

Transfer functions $T_i(s)$ are the well-known complementary sensitivity functions, which were derived for a vehicle platoon in [38] and [41].

VII. Numerical Implementation

We refer to a platoon of four vehicles and a leader (as depicted in Fig. 1 with an additional follower) as a representative example. According to the intelligent-transportation-system extraurban scenario described in [1], the leader communicates with all the vehicles in the broadcast mode while every vehicle in the platoon shares information with its neighbors through wireless technology [1]. Simulations are carried out for a single-lane road. The leader vehicle imposes a common and constant fleet velocity equal to 20 m/s (i.e., 72 km/h). The spacing policy requires a constant time headway $h_{ij} = 0.8$ s for all the vehicles in the platoon $(i < j = i + 1)$, with $h_{ij} = -h_{ji}$. Note that the simulation scenario has been set according to the work in [42], where it was shown that, for this specific choice of parameters (the mean speed and the headway time), packet losses during communication are negligible. Furthermore, without loss of generality, we consider the case of homogeneous traffic, i.e., $M_i = M$ $(i = 0, \ldots, 4)$. Control parameters have been tuned inside the consensus region according to Theorem 2 and the conditions of Lemma 4. (Note that node 0, i.e., the leader, is globally reachable, as required in Theorem 2.)

In the simulation scenario, the heterogeneous time-varying delays are stochastic variables with a uniform discrete distribution, i.e., $\tau_i(t) \leq \tau^*; \tau_i(t) \in [\tau_{\min}, \tau_{\max}]$, with $\tau_{\min} = 0$ s, and $\tau_{\max} \leq \tau^* = 10.9 \cdot 10^{-2}$ s, where the theoretical upper bound τ^* is computed, as in Theorem 2 (choosing $q = 1.02 > 1$). Note that τ^* is within the average end-to-end communication delay that is typical of IEEE802.11p vehicular networks, which is of the order of hundredths of a second (i.e., 10^{-2} s) [43].

Furthermore, to test the ability of the platooning protocol to reject the variations of communication delays $\tau_i(t)$, we select $\tau_i(t)$ as a piecewise constant function whose value is selected with a uniform probability between 0 and τ^*, e.g., once every second.

As shown in Fig. 2, and as expected from the theoretical analysis, the strategy presented in this paper guarantees asymptotic stability of the closed-loop vehicular network and that the position and speed errors converge toward zero in the presence of network-induced time-varying delays. Note that the presence of noise in the position and velocity error signals (see the bottom panel in Fig. 2) depends on the rapid switching nature of the time-varying delay considered in the simulation run.

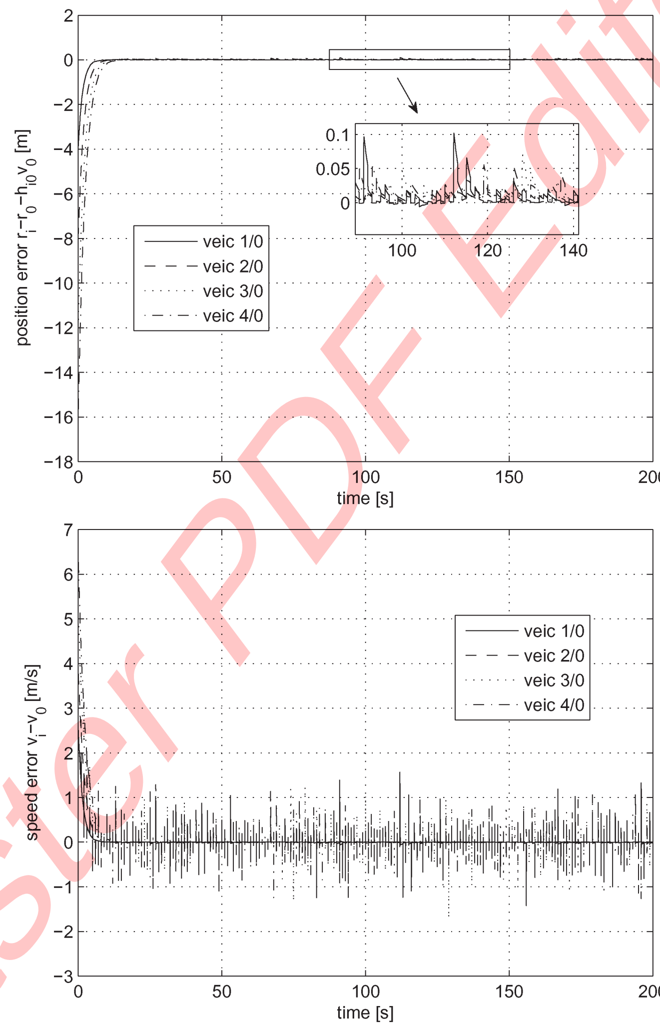

Fig. 2. Platooning in the presence of *time-varying* heterogeneous delays. (Top panel) Time history of the position error. (Bottom panel) Time history of the velocity error.

Finally, we investigated the ability of the platooning strategy to guarantee convergence in the presence of time-varying delays whose upper bound exceeds the maximum theoretical bound, e.g., $\tau_{\max} > \tau^*$. We observed (simulations are omitted here for the sake of brevity) that the protocol still guarantees convergence but with a bounded tracking error.

A. Response to Perturbations and Communication Failures

Referring to the platoon topology depicted in Fig. 1, to assess its string stability, we analyze the propagation error back through the string (i.e., between following vehicles) due to the presence of a sinusoidal perturbation acting on the leader motion, i.e., $\delta(t) = A \sin \omega t$ being $A = 4$ and $\omega = \pi/5$, as in [4].

The results in Fig. 3 illustrate and numerically confirm the string stability of the platoon.

We now validate the robustness of our approach with respect to communication losses. Specifically, we investigate the effect of a sudden communication loss and its subsequent recovery among the leader and some of the followers, as well as the presence of losses during intervehicles communication.

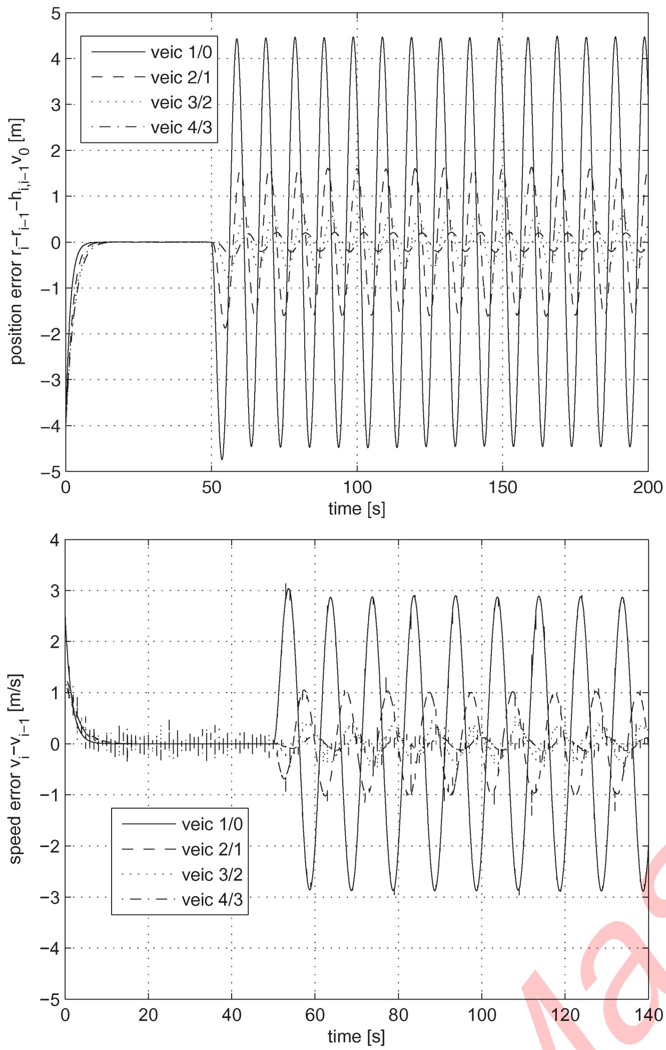

Fig. 3. Performance of the platooning algorithm in the presence of a sinusoidal disturbance acting on the leader. (Top panel) Spacing errors. (Bottom panel) Velocity errors.

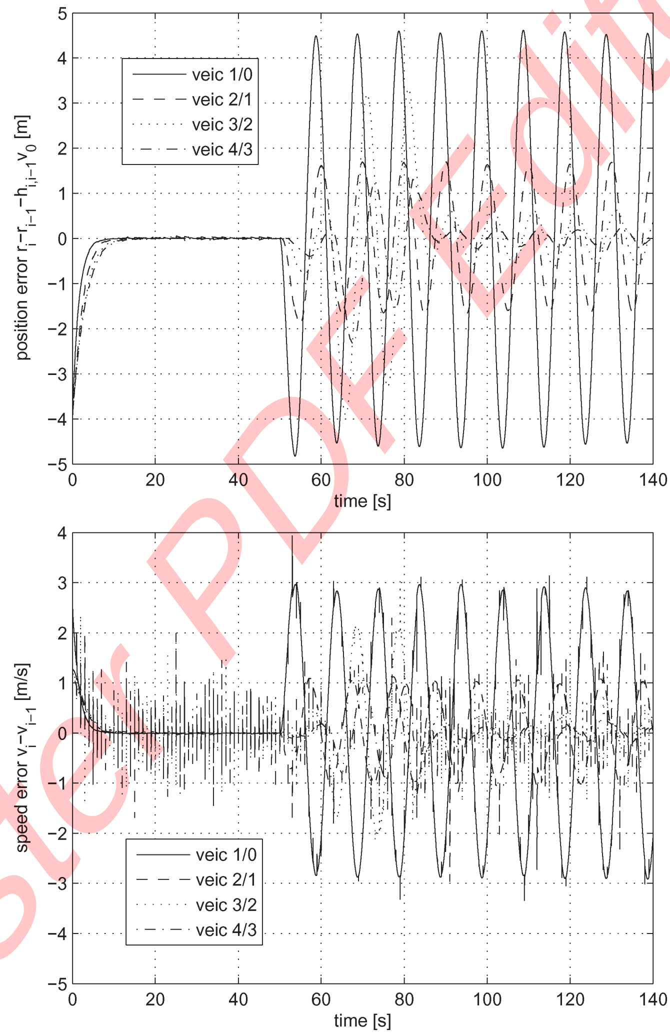

Fig. 4. Performance of the platooning algorithm in the presence of communication losses between the leader and some of the followers (vehicles 3 and 4 in Fig. 1). Time history of the (top) position and (bottom) velocity errors attenuating along the string.

To this aim, as a first representative case of study, we analyze the performance of the strategy when vehicles 3 and 4 in the platoon simultaneously loose connection with the leader at time instant $t = 62$ s, recovering it at $t = 82$ s. Moreover, to better test the effectiveness of the approach, the same periodic disturbance $\delta(t)$ is again added to the leader dynamics at $t = 50$ s. The results in Fig. 4 show the effectiveness of the approach in guaranteeing disturbance attenuation along the string. Note that the network switches among different string topologies that are still globally reachable. A similar behavior is achieved in the case of other kinds of communication losses, such as in the intervehicle communication among followers.

VIII. Conclusion

We presented a novel distributed control protocol to achieve platooning of vehicles in the presence of heterogeneous time-varying delays. By recasting the problem as that of achieving consensus in a closed-loop vehicular network, we were able to prove local convergence and stability despite the presence of delays. We found that the strategy is indeed effective in guaranteeing the stability of the platoon, even in the presence of disturbances. We wish to emphasize that the network paradigm proposed in this paper can be particularly suitable for exploring communication strategies alternative to pairwise interactions. The experimental implementation of the protocol is the subject of ongoing work.

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