ZAD. 1 | ZESTAW B | ŁUKASZ TWORZYDŁO - gd29623
BADANIA OPERACYJNE (WYKŁADY) - INIS3

$f(x_1, x_2) = 2(1-x_1)^2 + (3-x_2)^2$

x_1 - ZUŻYCIE PALIWA W ELEKTROWNI I

x_2 - ZUŻYCIE PALIWA W ELEKTROWNI II

I 5 MWh
II 3 MWh

DZIENNA PRODUKCJA ENERGII TO 100 MWh

1 t paliwa w I 5 MWh
1 t paliwa w II 3 MWh

1-7

$5x_1 + 3x_2 = 100$
$5x_1 + 3x_2 - 100 = 0$
$g(x_1, x_2) = 0$

$L(x_1, x_2, \lambda) = 2(1-x_1)^2 + (3-x_2)^2 + \lambda(5x_1 + 3x_2 - 100)$

$$\begin{cases} L_{x_1} = 4(1-x_1) + 5\lambda \\ L_{x_2} = 2(3-x_2) + 3\lambda \\ L_\lambda = 5x_1 + 3x_2 - 100 \end{cases}$$

$$\begin{cases} 4(1-x_1) + 5\lambda = 0 \,/\cdot 3 \\ 2(3-x_2) + 3\lambda = 0 \,/\cdot(-5) \\ 5x_1 + 3x_2 - 100 = 0 \end{cases}$$

$12(1-x_1) + 15\lambda = 0$
$-10(3-x_2) - 15\lambda = 0$

$12(1-x_1) - 10(3-x_2) = 0 \,/:2$
$6(1-x_1) - 5(3-x_2) = 0$
$6 - 6x_1 - 15 + 5x_2 = 0$
$-6x_1 + 5x_2 - 9 = 0$
$-6x_1 + 5x_2 = 9 \,/\cdot(-1)$

$$\begin{cases} 6x_1 - 5x_2 = -9 \\ 5x_1 + 3x_2 = 100 \end{cases}$$

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$$\begin{cases} 6x_1 - 5x_2 = -9 \\ 5x_1 + 3x_2 = 100 \end{cases}$$

$$W = \begin{vmatrix} 6 & -5 \\ 5 & 3 \end{vmatrix} = 6 \cdot 3 - 5 \cdot (-5) = 18 + 25 = 43$$

$$W_{x_1} = \begin{vmatrix} -9 & -5 \\ 100 & 3 \end{vmatrix} = (-9) \cdot 3 - 100 \cdot (-5) = -27 + 500 = 473$$

$$W_{x_2} = \begin{vmatrix} 6 & -9 \\ 5 & 100 \end{vmatrix} = 6 \cdot 100 - 5 \cdot (-9) = 600 + 45 = 645$$

2 - 7

$$x_1 = \frac{W_{x_1}}{W} = \frac{473}{43} = 11 \qquad \underline{x_1 = 11}$$

$$x_2 = \frac{W_{x_2}}{W} = \frac{645}{43} = 15 \qquad \underline{x_2 = 15}$$

$P_E(x_1, x_2)$ - ROZDZIELONA PRODUKCJA ENERGII MIĘDZY DWOMA ELEKTROWNIAMI

$\underline{P_E(11, 15)}$

$$\begin{cases} L_{x_1} = 4(1 - x_1) + 5\lambda \\ L_{x_2} = 2(3 - x_2) + 3\lambda \\ L_\lambda = 5x_1 + 3x_2 - 100 \end{cases}$$

$$L_{x_1 x_1} = 4 \qquad L_{x_1 x_2} = 0$$

$$L_{x_2 x_1} = 0 \qquad L_{x_2 x_2} = 2$$

$$\Delta = \begin{vmatrix} 0 & g_{x_1} & g_{x_2} \\ g_{x_1} & L_{x_1 x_1} & L_{x_1 x_2} \\ g_{x_2} & L_{x_2 x_1} & L_{x_2 x_2} \end{vmatrix}$$

$$\Delta(11, 15) = \begin{vmatrix} 0 & 5 & 3 \\ 5 & 4 & 0 \\ 3 & 0 & 2 \end{vmatrix} = (0 \cdot 4 \cdot 2) + (5 \cdot 0 \cdot 3) + (3 \cdot 5 \cdot 0) - (3 \cdot 4 \cdot 3) - (0 \cdot 0 \cdot 0) - (2 \cdot 5 \cdot 5) = 0 + 0 + 0 - 36 - 0 - 50 =$$

$$= -36 - 50 = \underline{-86}$$

ZAD. 2

ZESTAW B

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BADANIA OPERACYJNE (WYKŁADY) - INIS3

i	x_i	y_i	x_i^2	$(x_i)\cdot(y_i)$	$\hat{y}(x)$	$\hat{y}_i - y_i$	$(\hat{y}_i - y_i)^2$	$(y_i - \bar{y})^2$
1	1	8	1	8	9,5714	1,57	2,4694	64
2	2	13	4	26	11,7143	-1,29	1,6531	9
3	3	14	9	42	13,8571	-0,14	0,0204	4
4	4	17	16	68	16,0000	-1,00	1,0000	1
5	5	18	25	90	18,1429	0,14	0,0204	4
6	6	20	36	120	20,2857	0,29	0,0816	16
7	7	22	49	154	22,4286	0,43	0,1837	36
SUMA	28	112	140	508	112	0	5,4286	134

3-7

$$\begin{cases} a\sum_{i=1}^{n} x_i^2 + b\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i \cdot y_i \\ a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} 1 = \sum_{i=1}^{n} y_i \end{cases}$$

$$\begin{cases} 140a + 28b = 508 \\ 28a + 7b = 112 \end{cases}$$

$$W = \begin{vmatrix} 140 & 28 \\ 28 & 7 \end{vmatrix} = 140 \cdot 7 - 28 \cdot 28 = 980 - 784 = 196$$

$$W_a = \begin{vmatrix} 508 & 28 \\ 112 & 7 \end{vmatrix} = 508 \cdot 7 - 112 \cdot 28 = 3556 - 3136 = 420$$

$$W_b = \begin{vmatrix} 140 & 508 \\ 28 & 112 \end{vmatrix} = 140 \cdot 112 - 28 \cdot 508 = 15680 - 14224 = 1456$$

$$a = \frac{W_a}{W} = \frac{420}{196} = \frac{210}{98} = \frac{105}{49} = \frac{15}{7} \approx 2,1429$$

$$b = \frac{W_b}{W} = \frac{1456}{196} = \frac{728}{98} = \frac{364}{49} = \frac{52}{7} \approx 7,4286$$

$$\hat{y} = ax + b$$

$$\hat{y} = \frac{15}{7}x + \frac{52}{7}$$

$$\hat{y} \approx 2,1429x + 7,4286$$

ZAD. 2 /col/ | ZESTAW B | ŁUKASZ TWORZYDŁO-gd29623 BADANIA OPERACYJNE (WYKŁADY)-INIS3

$\hat{y} = \frac{15}{7}x + \frac{52}{7}$

$\hat{y} \approx 2{,}1429x + 7{,}4286$

$\hat{y}(1) = \frac{15}{7} \cdot 1 + \frac{52}{7} = \frac{67}{7} \approx 9{,}5714$

$\hat{y}(2) = \frac{15}{7} \cdot 2 + \frac{52}{7} = \frac{30}{7} + \frac{52}{7} = \frac{82}{7} \approx 11{,}7143$

$\hat{y}(3) = \frac{15}{7} \cdot 3 + \frac{52}{7} = \frac{45}{7} + \frac{52}{7} = \frac{97}{7} \approx 13{,}8571$

$\hat{y}(4) = \frac{15}{7} \cdot 4 + \frac{52}{7} = \frac{60}{7} + \frac{52}{7} = \frac{112}{7} = 16 = 16{,}0000$

$\hat{y}(5) = \frac{15}{7} \cdot 5 + \frac{52}{7} = \frac{75}{7} + \frac{52}{7} = \frac{127}{7} \approx 18{,}1429$

$\hat{y}(6) = \frac{15}{7} \cdot 6 + \frac{52}{7} = \frac{90}{7} + \frac{52}{7} = \frac{142}{7} \approx 20{,}2857$

$\hat{y}(7) = \frac{15}{7} \cdot 7 + \frac{52}{7} = \frac{105}{7} + \frac{52}{7} = \frac{157}{7} \approx 22{,}4286$

4-7

$\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$

$\bar{y} = \frac{1}{7} \cdot 112 = \frac{112}{7} = 16$

$\bar{y} = 16$

$\hat{y}(x) = \hat{y}_i$

$\hat{y}_i - y_i = ?$

$\hat{y}_1 - y_1 = \frac{67}{7} - 8 = \frac{67}{7} - \frac{56}{7} = \frac{11}{7} \approx 1{,}57$

$\hat{y}_2 - y_2 = \frac{82}{7} - 13 = \frac{82}{7} - \frac{91}{7} = -\frac{9}{7} \approx -1{,}29$

$\hat{y}_3 - y_3 = \frac{97}{7} - 14 = \frac{97}{7} - \frac{98}{7} = -\frac{1}{7} \approx -0{,}14$

$\hat{y}_4 - y_4 = \frac{112}{7} - 17 = \frac{112}{7} - \frac{119}{7} = -\frac{7}{7} = -1{,}00$

$\hat{y}_5 - y_5 = \frac{127}{7} - 18 = \frac{127}{7} - \frac{126}{7} = \frac{1}{7} \approx 0{,}14$

$\hat{y}_6 - y_6 = \frac{142}{7} - 20 = \frac{142}{7} - \frac{140}{7} = \frac{2}{7} \approx 0{,}29$

$\hat{y}_7 - y_7 = \frac{157}{7} - 22 = \frac{157}{7} - \frac{154}{7} = \frac{3}{7} \approx 0{,}43$

$\hat{y}_i - y_i = 0$

$(\hat{y}_1 - y_1)^2 \approx 2{,}4694$

$(\hat{y}_2 - y_2)^2 \approx 1{,}6531$

$(\hat{y}_3 - y_3)^2 \approx 0{,}0204$

$(\hat{y}_4 - y_4)^2 = 1{,}000$

$(\hat{y}_5 - y_5)^2 \approx 0{,}0204$

$(\hat{y}_6 - y_6)^2 \approx 0{,}0816$

$(\hat{y}_7 - y_7)^2 \approx 0{,}1837$

$(\hat{y}_i - y_i)^2 \approx 5{,}4286$

ZAD. 2 /cd/ | ZESTAW B | ŁUKASZ TWORZYDŁO - gd29623
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$(y_i - \bar{y})^2 = ?$

$\bar{y} = 16$

$(y_1 - \bar{y})^2 = (8-16)^2 = (-8)^2 = 64$

$(y_2 - \bar{y})^2 = (13-16)^2 = (-3)^2 = 9$

$(y_3 - \bar{y})^2 = (14-16)^2 = (-2)^2 = 4$

$(y_4 - \bar{y})^2 = (17-16)^2 = (1)^2 = 1$

$(y_5 - \bar{y})^2 = (18-16)^2 = (2)^2 = 4$

$(y_6 - \bar{y})^2 = (20-16)^2 = (4)^2 = 16$

$(y_7 - \bar{y})^2 = (22-16)^2 = (6)^2 = 36$

$\sum_{i=1}^{n} (y_i - \bar{y})^2 = 134$

$R^2 = 1 - \frac{\sum_{i=1}^{n} (\hat{y}_i - y_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2}, \quad 0 \le R^2 \le 1$

$R^2 = 1 - \frac{5,4286}{134}$

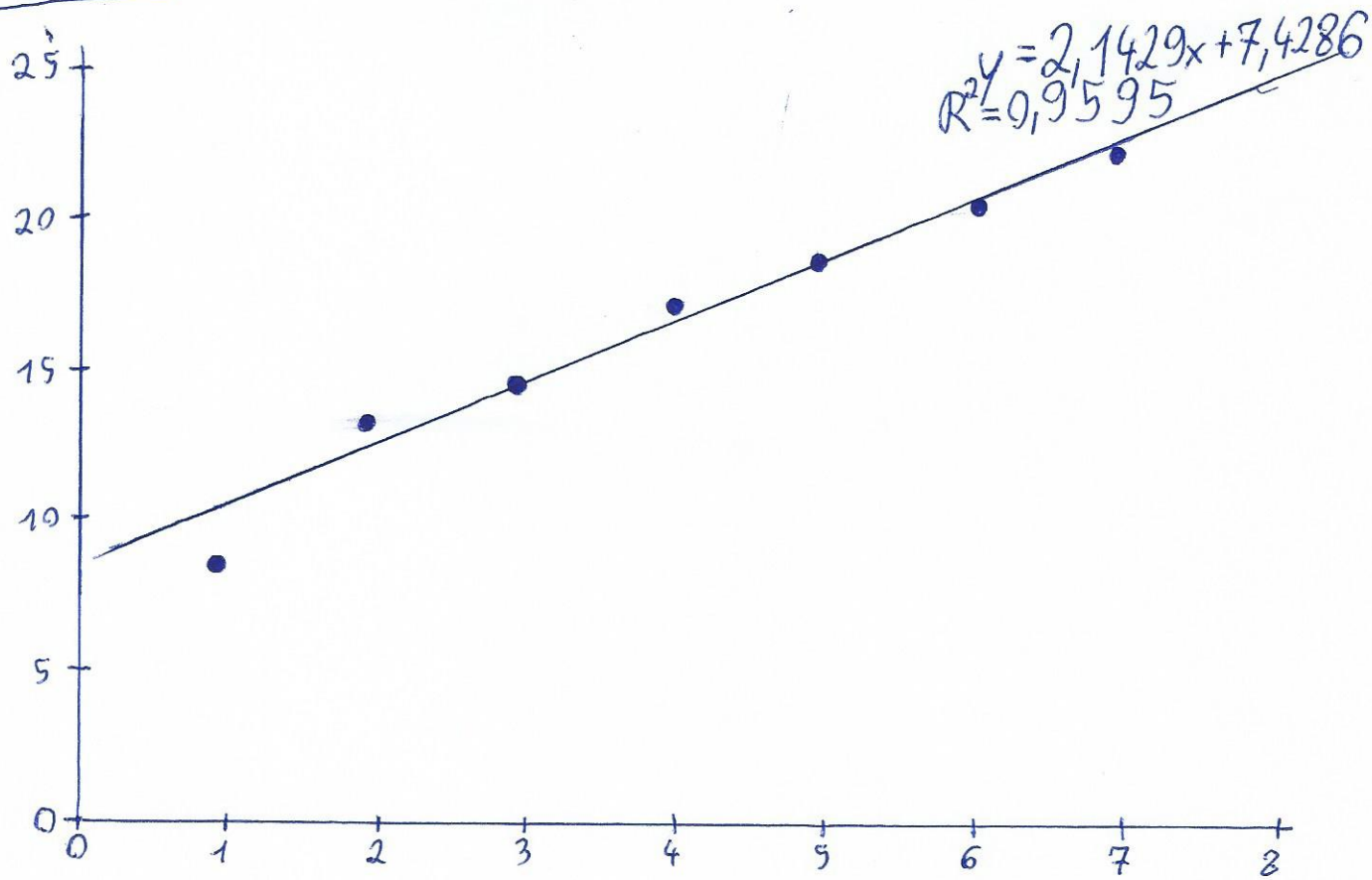
[illegible]

[illegible]

$R^2 \approx 1 - 0,0405$

$R^2 \approx 0,9595$

5 - 7



$a = \frac{15}{7} \approx 2,1429$

$b = \frac{52}{7} \approx 7,4286$

$\hat{y} = \frac{15}{7}x + \frac{52}{7}$

$y \approx 2,1429x + 7,4286$

$\bar{y} = 16$

$\sum_{i=1}^{n} (\hat{y}_i - y_i)^2 = 5,4286$

$\sum_{i=1}^{n} (y_i - \bar{y}) = 134$

$R^2 \approx 0,9595$

ZAD. 3 | ZESTAW B | ŁUKASZ TWORZYDŁO-gd29623 BADANIA OPERACYJNE (WYKŁADY)-INIS3

SUROWCE PRZEDSIĘBIORSTWA	WYROBY A	WYROBY B	WYROBY C	LIMITY DZIENNEGO ZUŻYCIA
I	1,5	3	4	1500 kg
II	3	2	1	1200 kg
ZYSK OSIĄGANY NA JEDNOSTCE WYROBU	12 zł	18 zł	12 zł	

$f(x_1, x_2, x_3) = 12x_1 + 18x_2 + 12x_3 \rightarrow \max$ (dążenie do maksymalizacji zysków)

$$\begin{cases} 1{,}5x_1 + 3x_2 + 4x_3 \leq 1500 \\ 3x_1 + 2x_2 + 1x_3 \leq 1200 \end{cases}$$

6-7

$$A = \begin{bmatrix} 1{,}5 & 3 & 4 \\ 3 & 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1500 \\ 1200 \end{bmatrix}$$

$$C = [12 \quad 18 \quad 12]$$

$W = [w_1, w_2]$

$g(w_1, w_2) = 1500w_1 + 1200w_2$

$$W^T = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

$$A^T W^T \geq C^T$$

$$A^T = \begin{bmatrix} 1{,}5 & 3 \\ 3 & 2 \\ 4 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1{,}5 & 3 \\ 3 & 2 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \geq \begin{bmatrix} 12 \\ 18 \\ 12 \end{bmatrix}$$

$$C^T = \begin{bmatrix} 12 \\ 18 \\ 12 \end{bmatrix}$$

$g(w_1, w_2) = 1500w_1 + 1200w_2$

$$\begin{cases} 1{,}5w_1 + 3w_2 \geq 12 \\ 3w_1 + 2w_2 \geq 18 \\ 4w_1 + 1w_2 \geq 12 \end{cases}$$

$$\begin{cases} 1{,}5w_1 + 3w_2 = 12 \\ 3w_1 + 2w_2 = 18 \end{cases}$$

$$\begin{cases} \frac{3}{2}w_1 + 3w_2 = 12 \,/\cdot 2 \\ 3w_1 + 2w_2 = 18 \end{cases}$$

$$-\begin{cases} 3w_1 + 6w_2 = 24 \\ 3w_1 + 2w_2 = 18 \end{cases}$$

$$4w_2 = 6$$

$$w_2 = \frac{6}{4}$$

$$\boxed{w_2 = \frac{3}{2}}$$

$$\frac{3}{2}w_1 + 3w_2 = 12$$

$$\frac{3}{2}w_1 + 3 \cdot \frac{3}{2} = 12$$

$$\frac{3}{2}w_1 + \frac{9}{2} = 12 \,/\cdot 2$$

$$3w_1 + 9 = 24$$

$$3w_1 = 24 - 9$$

$$3w_1 = 15$$

$$w_1 = \frac{15}{3}$$

$$\boxed{w_1 = 5}$$

ZAD. 3 /cd/ | ZESTAW B | ŁUKASZ TWORZYDŁO - gd 29623
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$g_{min}(5, \frac{3}{2}) = 1500 \cdot 5 + 1200 \cdot \frac{3}{2} = 7500 + \frac{3600}{2} = 7500 + 1800 = 9300$

$g_{min}(5, \frac{3}{2}) = 9300$

$L_1 = \frac{3}{2}w_1 + 3w_2 = \frac{3}{2} \cdot 5 + 3 \cdot \frac{3}{2} = \frac{15}{2} + \frac{9}{2} = \frac{24}{2} = 12$

$L_2 = 3w_1 + 2w_2 = 3 \cdot 5 + 2 \cdot \frac{3}{2} = 15 + \frac{6}{2} = 15 + 3 = 18$

$L_3 = 4w_1 + 1w_2 = 4 \cdot 5 + 1 \cdot \frac{3}{2} = 20 + 1\frac{1}{2} = 21{,}5 > P3$

7 - 7

$$\begin{cases} 1{,}5x_1 + 3x_2 + \cancel{x_3} = 1500 \\ 3x_1 + 2x_2 + \cancel{x_3} = 1200 \end{cases}$$

$$\begin{cases} \frac{3}{2}x_1 + 3x_2 = 1500 \,/\cdot 2 \\ 3x_1 + 2x_2 = 1200 \end{cases}$$

$$-\begin{cases} 3x_1 + 6x_2 = 3000 \\ 3x_1 + 2x_2 = 1200 \end{cases}$$

$4x_2 = 1800$

$2x_2 = 900$

$x_2 = 450$

$3x_1 + 2x_2 = 1200$

$3x_1 + 2 \cdot 450 = 1200$

$3x_1 + 900 = 1200$

$3x_1 = 1200 - 900$

$3x_1 = 300$

$x_1 = 100$

$x_1^\circ = 100$

$x_2^\circ = 450$

$x_3^\circ = 0$

Aby przedsiębiorstwo osiągnęło maksymalny zysk musi produkować wyrób A w ilości 100, oraz wyrób B w ilości 450. Poprzez taką produkcję przedsiębiorstwo będzie mogło uzyskać zysk w wysokości 9300.