(1) If κ, λ, and μ are cardinals, then $\kappa^{\lambda+\mu} = \kappa^\lambda \cdot \kappa^\mu$.

Proof. Given $\bar{\bar{K}} = \kappa$, $\bar{\bar{L}} = \lambda$, $\bar{\bar{M}} = \mu$, and $L \cap M = \emptyset$. Given a function $f : L \cup M \mapsto K$. Construct $g : L \mapsto K = f \restriction L$ and $h : M \mapsto K = f \restriction M$. So $\kappa^{\lambda+\mu} \leq \kappa^\lambda \cdot \kappa^\mu$.

Given a pair of functions $g : L \mapsto K$ and $h : M \mapsto K$. Construct $f : L \cup M \mapsto K = g \cup h$. g and h have disjoint domains, so f is a function. Therefore $\kappa^\lambda \cdot \kappa^\mu \leq \kappa^{\lambda+\mu}$, and therefore they are equal. $\square$

(2) $2^{\aleph_0} \neq \aleph_{\omega_1 \cdot 49+\omega}$

Proof. Let $\beta = \omega_1 \cdot 49 + \omega$. β is a limit ordinal, so $\operatorname{cf} \aleph_\beta = \operatorname{cf} \beta = \aleph_0$ (by Enderton's Theorem 9N). However, by König's theorem, $\aleph_0 < \operatorname{cf} 2^{\aleph_0}$. $\square$

(3) If A is a set well-ordered by $(<)$ and $f : A \mapsto A$ satisfies $x < y \Rightarrow f(x) < f(y)$, then for all $x \in A$, $x \leq f(x)$.

Proof. If not, then there is a least x such that $f(x) < x$. Construct by recursion $R : \omega \mapsto A$ where $R(0) = x$; $R(a + 1) = f(R(a))$. $R(1) < R(0)$, and given $R(n+1) < R(n)$, $f(R(n+1)) < f(R(n))$, so $R(n+2) < R(n + 1)$. Therefore R is an infinitely descending sequence in A, a contradiction. $\square$

(7) Given a set P of $\aleph_0$-many people. There is an infinite subset of that set such that all people have met each other or all people have not met each other.

Proof. Let $M \cup \tilde{M} = P^{(2)}$, where $P^{(2)}$ is the set of 2-element subsets of P. M represents the set of pairs of people who have met, and $\tilde{M}$ represents the set of pairs of people who have not. Then by Ramsey's theorem, there exists a $P' \subseteq P$ where $\bar{\bar{P}}' = \aleph_0$ and either $P'^{(2)} \subseteq M$ or $P'^{(2)} \subseteq \tilde{M}$. $\square$