Luke Palmer
MATH 4650
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2.4.4

```
nriter[f_] := Function[x,
      f[x] f'[x]
  ─────────────────────
  f'[x]^2 - f[x] f''[x]  ]
```

(a)

```
parta = nriter[Function[x, 1 - 4 x Cos[x] + 2 x^2 + Cos[2 x]]];

NestList[parta, 0.5, 10]

{0.5, -0.221664, -0.525737, -0.477432, -0.490425,
 -0.487101, -0.487964, -0.487741, -0.487798, -0.487784, -0.487787}

parta =.
```

(b)

```
partb = nriter[Function[x, x^2 + 6 x^5 + 9 x^4 - 2 x^3 - 6 x^2 + 1]];

NestList[partb, -2.5, 10]

{-2.5, -1.72231, -0.613917, 0.0186786, -0.0184556,
 0.018647, -0.0184249, 0.0186157, -0.0183943, 0.0185845, -0.0183638}

partb =.
```

(c)

```
partc = nriter[
   Function[x, Sin[3 x] + 3 E^(-2 x) Sin[x] - 3 E^(-x) Sin[2 x] - E^(-3 x)]];

NestList[partc, 3.5, 10]

{3.5, 0.139504, -2.30723, 17.8549, 0.0517161, -4.97792,
 -237.549, ComplexInfinity, Indeterminate, Indeterminate, Indeterminate}

(* Ooh, it diverges *)
```

```
NestList[partc, 3.0, 20]
```

```
{3., -0.128134, 2.27809, 0.193452, -2.33352, 15.8661, 0.135454,
 -2.3361, 15.6996, -0.00832061, 30.0672, -0.161956, 1.96924, -0.005446,
 45.9192, -0.135065, 2.19738, 0.178921, -2.2445, 26.623, 0.0774449}
```

```
(* Doesn't seem to be converging *)
```

```
partc =.
```

■ (d)

```
partd = nriter[Function[x, E^(3 x) - 27 x^6 + 27 x^4 E^x - 9 x^2 E^(2 x)]];
```

$$\text{partd} = \text{nriter}[\text{Function}[x, E^{3x} - 27x^6 + 27x^4 E^x - 9x^2 E^{2x}]];$$

```
NestList[partd, 4.0, 50]
```

```
{4., 0.327964, -0.117642, 0.196599, 0.00787283, 0.162607, 0.0413423, 0.142682,
 0.0603288, 0.129568, 0.072455, 0.12056, 0.0805839, 0.114262, 0.0861605, 0.109825,
 0.0900322, 0.106692, 0.0927376, 0.104477, 0.094635, 0.102912, 0.0959686,
 0.101805, 0.0969071, 0.101024, 0.0975681, 0.100472, 0.098034, 0.100083,
 0.0983624, 0.0998075, 0.098594, 0.0996135, 0.0987573, 0.0994765, 0.0988725,
 0.0993799, 0.0989537, 0.0993117, 0.099011, 0.0992636, 0.0990515, 0.0992296,
 0.09908, 0.0992057, 0.0991001, 0.0991888, 0.0991143, 0.0991768, 0.0991243}
```

```
(*Not converging very fast at all.*)
```

```
partd =.
```

2.4.6

■ (a)

$$\text{Show} \lim_{n\to\infty} \frac{\text{Abs}[\frac{1}{n+1}]}{\text{Abs}[\frac{1}{n}]} < 1$$

$1/n > 0$ **whenever** $n > 0$, **so :**

$$\text{Show} \lim_{n\to\infty} \frac{n}{n+1} < 1$$

But $\lambda == 1$, **and is not** < 1, **so it's not linearly convergent formally.**

■ (b)

$$\text{Show} \lim_{n\to\infty} \frac{\text{Abs}[\frac{1}{(n+1)^2}]}{\text{Abs}[\frac{1}{n^2}]} < 1$$

$1/n^2 > 0$ whenever $n \neq 0$, so:

Show $\lim_{n\to\infty} \frac{n^2}{(n+1)^2} == \lim_{n\to\infty} \frac{2n}{2(n+1)} == 1 < 1$

Again, λ is not < 1, so it's not linearly convergent.

2.4.8

■ (a)

Show $\lim_{n\to\infty} \frac{\left|10^{-2^{n+1}}\right|}{\left|10^{-2^n}\right|^2}$ **is finite**

These quantities are always positive, so:

$\lim_{n\to\infty} 10^{2*2^n-2^{n+1}} == \lim_{n\to\infty} 10^0 == 1$

(b)

Show $\lim_{n\to\infty} \frac{\left|10^{-(n+1)^k}\right|}{\left|10^{-n^k}\right|^2}$ **is not finite**

It suffices to show that $\lim_{n\to\infty} 2n^k - (n+1)^k$ **goes to** $+\infty$**.**

The k'th order term in $(n+1)^k$ will always be monic,
so $2n^k - (n+1)^k$ will always look like n^k − (some other stuff that depends on powers $< k$ of n). In the limit,
the k'th order term will dominate, so that expression diverges to $+\infty$.

2.5.1a

aitken[s_, n_] := $s[n] - \frac{(s[n+1]-s[n])^2}{s[n+2]-2s[n+1]+s[n]}$

seq[0] := 0.5

seq[n_] := $3^{-\text{seq}[n-1]}$

Map[aitken[seq, #] &, Range[5]]

{0.547915, 0.547847, 0.547823, 0.547814, 0.54781}

2.5.4

```
stef[f_] := Function[p0,
   Module[{p1, p2},
     p1 = f[p0];
     p2 = f[p1];
     p0 - (p1 - p0)^2/(p2 - 2 p1 + p0)]]
```

$$p0 - \frac{(p1 - p0)^2}{p2 - 2\,p1 + p0}$$

stef[1 + Sin[#]^2 &][1.]

2.1529

stef[1 + Sin[#]^2 &][%]

1.87346

$p_0^{(1)}$ **== 2.1529 and** $p_0^{(2)}$ **== 1.87346**

2.5.12

(a)

NestList[stef[2 + Sin[#] &], 1., 5]

{1., 2.42044, 2.55201, 2.5542, 2.5542, 2.5542}

(b)

NestList[stef[#^3 − # − 5 &], 3., 20]

{3., 2.96235, 2.92325, 2.88256, 2.84014, 2.79582, 2.74943, 2.70075, 2.64957, 2.59565, 2.53876, 2.47874, 2.41555, 2.34953, 2.28183, 2.21534, 2.15622, 2.11447, 2.09703, 2.09459, 2.09455}

(c)

NestList[stef[3 #^2 − ⅇ^# + # &], 1., 5]

{1., 0.92397, 0.910443, 0.910008, 0.910008, 0.910008}

▪ (d)

NestList[stef[cos(#) &], 1., 4]

{1., 0.72801, 0.739067, 0.739085, 0.739085}

3.1.3c

$$\begin{pmatrix} k & x & f(x) & & \\ 0 & 0 & 0 & 0.7833 & -0.123 \\ 1 & 0.6 & 0.47 & 0.6726 & \\ 2 & 0.9 & 0.6418 & & \end{pmatrix}$$

Order 1 polynomial:

$f_1(x) == 0.7833\, x;$
$f_1(0.45) == 0.3524;$
$\ln(0.45 + 1) == 0.3716;$

$$\text{Error} == \left| \frac{1}{2} \frac{-1}{(\xi(0.45) + 1)^2} (0.45 - 0)(0.45 - 0.6) \right|$$

(∗ Where $\xi(0.45) \in (0,0.6)$. It takes on its greatest value when $\xi(0.45) == 0$, and the error is 0.0337∗)

Order 2 polynomial:

$f_2(x) == 0.7833\, x - 0.123\, x\, (x - 0.6);$
$f_2(0.45) == 0.3608;$
$\ln(0.45 + 1) == 0.3716;$

$$\text{Error} == \left| \frac{1}{6} \frac{2}{(\xi(0.45) + 1)^3} (0.45 - 0)(0.45 - 0.6)(0.45 - 0.9) \right|$$

(∗Where $\xi(0.45) \in (0,0.9)$. It takes on its greatest value when $\xi(0.45) == 0$, and the error is 0.0101∗)

3.1.4c

$$\partial_{xxx} \frac{\log(3\, x - 1)}{\log(10)}$$

$$\frac{54}{(3\, x - 1)^3 \log(10)}$$

$$\text{Error} == \left| \frac{1}{6} \frac{54}{(3\, \xi(1.4) - 1)^3 \log(10)} (1.4 - 1.0)(1.4 - 1.25)(1.4 - 1.6) \right|$$

Where $\xi(1.4) \in (1.0, 1.6)$. This attains its maximum value when $\xi(1.4) == 1.0$, and the error is :

$$\left|\frac{1}{6}\,\frac{54}{(3*1-1)^3\log(10)}\,(1.4-1.0)\,(1.4-1.25)\right|$$

0.0293149

3.1.13

$$\begin{pmatrix} k & x & f(x) & & & \\ 0 & 0 & 0 & 2y & 6-4y & \frac{(4y-14)/3-6+4y}{2} \\ 1 & 0.5 & y & 6-2y & \frac{4y-14}{3} & \\ 2 & 1 & 3 & -1 & & \\ 3 & 2 & 2 & & & \end{pmatrix}$$

$$\text{Solve}\left[\frac{(4y-14)/3-6+4y}{2} == 3,\ y\right]$$

$$\left\{\left\{y \to \frac{25}{8}\right\}\right\}$$

And there you have it.

3.1.18

$$\frac{8-P_2}{0.75-0.5} == 2.4$$

$$8-\frac{2.4}{4} == P_2$$

$$7.4 == P_2$$