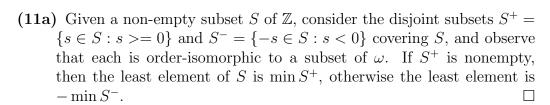
Luke Palmer MATH 4730 2006-04-16



- **(11b)** E(3) = 3.  $E(-1) = \omega$ .  $E(-2) = \omega + 1$ . rng  $E = \omega + \omega$ .
- (14) Given  $a, b \in A$  such that a < b, show  $F(a) \subset F(b)$ . Notice that  $a \in F(a)$  but  $b \notin F(a)$ , so  $F(a) \neq F(b)$ . If  $F(a) = \emptyset$ , then  $F(a) \subseteq F(b)$ . So assume that  $F(a) \neq \emptyset$  and  $a' \in F(a)$ .  $a' \leq a < b$ , so  $a' \in F(b)$ .  $\square$
- (16) The following three cases follow from Theorem 7K about well-orderings:
  - $\alpha^+ = \operatorname{seg} b$  for some  $b \in \beta$ : Then  $\alpha^+ \in \beta$  and therefore  $\alpha^+ \in \beta^+$ .
  - $\alpha^+ = \beta$ : Then  $\alpha^+ \in \beta \cup \{\beta\} = \beta^+$ .
  - $\beta = \operatorname{seg} a$  for some  $a \in \alpha^+$ : Then  $\beta \in \alpha^+$ , so  $\beta \in \alpha \subseteq \beta$  or  $\beta = \alpha \in \beta$ , a contradiction.
- (18) If  $\bigcup S = \alpha^+$  then for every element s of S,  $s \subseteq \alpha^+$ . Since s is an ordinal, either  $s \in \alpha^+$  or  $s = \alpha^+$ . Also,  $\bigcup S = \alpha^+ \in S$ , so  $\alpha^+$  is the greatest element. This proves part (i) by contrapositive, and its negation trivially implies part (ii).
- (20) Suppose that S were infinite. We can construct an infinitely ascending sequence of R by induction. However, this sequence will be an infinitely descending sequence of  $R^{-1}$ , contradicting its being a well-ordering.  $\square$