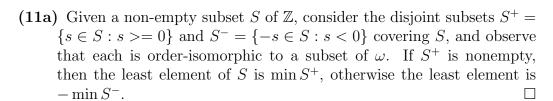
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- **(11b)** E(3) = 3. $E(-1) = \omega$. $E(-2) = \omega + 1$. rng $E = \omega + \omega$.
- (14) Given $a, b \in A$ such that a < b, show $F(a) \subset F(b)$. Notice that $a \in F(a)$ but $b \notin F(a)$, so $F(a) \neq F(b)$. If $F(a) = \emptyset$, then $F(a) \subseteq F(b)$. So assume that $F(a) \neq \emptyset$ and $a' \in F(a)$. $a' \leq a < b$, so $a' \in F(b)$. \square
- (16) The following three cases follow from Theorem 7K about well-orderings:
 - $\alpha^+ = \operatorname{seg} b$ for some $b \in \beta$: Then $\alpha^+ \in \beta$ and therefore $\alpha^+ \in \beta^+$.
 - $\alpha^+ = \beta$: Then $\alpha^+ \in \beta \cup \{\beta\} = \beta^+$.
 - $\beta = \text{seg } a \text{ for some } a \in \alpha^+$: Then $\beta \in \alpha^+$, so $\beta \in \alpha \subseteq \beta$ or $\beta = \alpha \in \beta$, a contradiction.
- (18) If $\bigcup S = \alpha^+$ then for every element s of S, $s \subseteq \alpha^+$. Since s is an ordinal, either $s \in \alpha^+$ or $s = \alpha^+$. Also, $\bigcup S = \alpha^+ \in S$, so α^+ is the greatest element. This proves part (i) by contrapositive, and its negation trivially implies part (ii).
- (20) S has no infinitely descending sequence, and S has no infinitely ascending sequence under R. Since R is a linear ordering, S must be finite. \square