Introduction to the Finite Difference Method: Filling and Draining a Cylindrical Tube

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Chapter 1

Filling a Tank

1.1 Water Tank Exercises

1.1.1 Rectangular Prism

Create a model for the filling of a water tank that is a rectangular prism with a width of $10~\mathrm{cm}$ and a length of $5~\mathrm{cm}$.

Next paragraph.

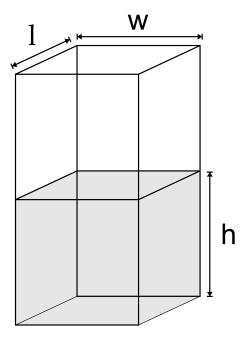


Figure 1.1: Water Tank

Chapter 2

Solving Differential Equations

What we've been doing in the preceding chapters is, basically, solving differential equations. You're given an equation for the rate of change of something (height changes with time), and an initial value, and then have to see how the system evolves over time.

2.1 Constant change

What if the rate of change is constant: our quantity changes at the same rate everywhere. For example: (see Eqn. 2.3 and Table 2.1)

$$\frac{dy}{dx} = \frac{1}{2} \tag{2.1}$$

$$y = \sqrt{5x} \tag{2.2}$$

$$y = 5x^2 \tag{2.3}$$

Code 2.1: Model of a filling tube without graphing (water-filling-fd-noGraph.py)

```
import math
    # Finite Difference Model

# PARAMETERS

tdt = 1.
    nsteps = 20
```

1 0 6.57 7 10 16.00	
7 10 16.00	1
	7
12 20 23.86	12
16.8 30 31.72	16.8
21.5 40 39.58	21.5
26.2 50 45.87	26.2

Table 2.1: Combined time, measured data, and modeled data.

```
r\ =\ 2.25
                  # radius (cm)
                   # Volume inflow rate: (cubic cm / s)
     Q = 5
9
             # Initial height (cm)
     h = 0
10
11
     print(0, h) # print initial values
12
13
     # TIME LOOP
14
     for t in range(1, nsteps):
15
16
          modelTime = t * dt
17
          \mathrm{dh} = \mathrm{Q} * \mathrm{dt} \ / \ (\mathrm{math.\,pi} * \mathrm{r} * * 2) \hspace{1cm} \# \ \mathit{find} \ \ \mathit{the} \ \ \mathit{change} \ \ \mathit{in} \ \ \mathit{height}
18
          h = h + dh
                                                       # update height
19
20
           print(modelTime, h)
21
```